

TEACHERS VIEWS ON MATHEMATICS,
MATHEMATICS TEACHING AND THEIR
EXISTING PRACTISES

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EXECUTIVE SUMMARY

This research project describes teachers current views on school mathematics and classroom teaching in relation to the new curriculum requirements.

We address three main questions: (a) What views on mathematics and mathematical activity appear to be prevalent among teachers? (b) What views of teaching mathematics that would facilitate learning are used in classrooms? and (c) What teaching strategies are employed by these teachers in their classroom? To address and synthesise these questions we constructed a theoretical framework around teacher's views on mathematics and that of teaching, in relation to Curriculum 2005, using data from the eight grade 3 teachers'.

An ethnographic research design is used, as its qualitative methods enabled the researchers' sufficient flexibility for describing, interpreting, exploring and explaining the views teachers have of mathematics and their teaching. The research data was gathered through (a) direct observation and (b) indepth interviews. The research analysis draws on the twenty-four classroom observations and sixteen pre- and post- interviews.

Prime importance was placed upon the authority of each participating teacher, to account for their own classroom practice. Since there is always the possibility of speculating, every effort was made to authenticate the qualitative data collected from the classroom lesson observations. These observations were written first as individual episodes and then into a story reflecting the teachers' views of mathematics and teaching.

The views on mathematics and teaching held by the teachers can be categorized into three groups: (a) transmission, (b) empirical and (c) connected. They are by no means water tight categories, as there is some overlap in teachers' views however, it helps us to identify the dominant views held by a specific teacher.

The views of the teachers involved in this study about mathematics and mathematical activities are in direct conflict with a pedagogical practice articulated in Curriculum 2005 (C2005), which offers learners opportunities to engage in problem-solving, logical thinking, recognising patterns, and implementing a pedagogy that focuses on conjecture, conceptual exploration and reflective, critical discussion. The predominant views of mathematics and mathematics teaching among the subjects of this study is that, of a system of algorithm transmitted by teachers to be committed to memory by their students.

Through a process of systematic observation of classroom interactions and interview it was possible to identify teaching styles that do not accord with the expectation of the C2005.

This revelation calls for a degree of 'unlearning' the mathematics, teachers know thus enabling them to acquire a new way of thinking about mathematics and a new approach to learning it. In the final chapter of this report we allude to recommendations, which are by no means exhaustive for teacher transformation.

ABBREVIATIONS

C2005	Curriculum 2005
CO	Critical Outcomes
GV	Grundvorstellungen
II	Individual Images
INSET	In-service Education for Teachers
MLMMS	Mathematical Literacy, Mathematics and Mathematical Sciences
OBE	Outcomes Based Education
PEI	President's Education Initiative
PNIP	Primary Mathematics Project
SO	Specific Outcomes
UWC	University of the Western Cape

CHAPTER 1: Relevance, Purpose and Expected Outcomes

INTRODUCTION

This study is about investigating existing practices of grade three teacher's prior to the implementation of Curriculum 2005 with an Outcomes Based Education approach.

1.1 RELEVANCE

It has relevance to investigate and describe current mathematics practices for several reasons:

- ? It is the basis from which to assist teachers in seeing and using alternatives in terms of materials, teaching style and activities, content and organization hereof, etc. As teachers' have to take into consideration the current situation of their students, so must educators take into consideration the current situation of teachers, too. Thus, an understanding of current practices is relevant to speculations on developing practice.
- ? It is a way to determine what is working or is not working within practices, especially those which have hitherto not been well described.
- ? It is a means to develop methodology in describing practices. This is necessary for further work on describing how practices change, making it relevant in terms of determining the success of Curriculum 2005, Outcomes Based Education (OBE), and other initiatives.
- ? It is necessary as a basis for the planning, implementation, and evaluation of actual initiatives in pre- and in-service training. Thus it is relevant as a basis for actions directed towards developing practice.

1.2 PURPOSE

The first purpose of the research is to describe current practices. It must, however, be recognized that teachers' actions only form part of the practice. Behind any action is a system of decisions. This decision making reflects teachers' knowledge-in-practice (Schon, 1983), but it cannot be captured through observing the teachers' practice. In order to see how the teachers' pedagogical content knowledge and the richness of their interactive decision making effect classroom practice, it is necessary to go beyond what can be observed directly. Naturally, this has methodological implications. The existing practices can then be compared to the demands of Curriculum 2005 with several purposes:

- ? To find any possible contradictions between current practices and demands of Curriculum 2005;
- ? To suggest changes necessary to meet the demands;

- ? To create input for a more general discussion of what may be understood as teaching practices which are consonant with the demands of OBE;
- ? To offer suggestions for further development of theory, formal curricula, as well as teachers' practices.

The second purpose is to use the descriptions of current practices in offering suggestions for theory, practice, as well as formal curricula.

Due to constraints in resources, the current research project will address mainly the first purpose. We find that a careful and respectful description of current practices form the best basis for offering suggestions. Hopefully this research will position us to address the second purpose with due respect to the complexity of classroom practice.

To allow for a more in-depth analysis of current practices, we have narrowed our research focus further. As we address in the following chapter, there is an intrinsic complexity of aspects of a mathematics classroom. In this particular project, we focus on those which are most directly concerned with the teachers' facilitation of students mathematical learning. Since approaches may vary significantly with the grade level, we have chosen to work only with grade 3 mathematics teachers. grade 3 was chosen, because issues prevalent in the starting of school may be less influential at this grade, while a mathematical practice is still in the process of being established.

1.3 RESEARCH QUESTIONS

We are interested in what third grade teachers do that could facilitate mathematical learning in their classrooms. The researchers deliberately chose the teachers' current practices, and not the learner outcomes, as the point of departure for this research. No attempt is made at understanding the reasons that teachers have for their actions. Instead, we look at teaching from the teachers' perspective by investigating the teachers' actions in class.

The main question that guides this research is: What do third grade mathematics teachers in the Western Cape schools do that facilitates mathematical learning?

Stemming from this, we look at:

1. What views on mathematics and mathematical activity appear to be prevalent among these teachers?
2. What views of teaching mathematics that, would facilitate learning are used in these classrooms?
3. What teaching strategies are employed by these teachers in their classrooms?

These questions determine to a large extent the research methodology. We believe that these questions are best pursued through observation and analysis of teaching/learning situations.

CHAPTER 2: Theoretical Framework and Background

2.1 INTRODUCTION

This project addresses the relation between existing practices of school mathematics teaching and curriculum requirements. In order to be as open as possible in our analyses of classroom practice, we would like to address the possible views that could influence, guide and limit our observations.

This study is also about determining exemplary practices among Mathematics teachers. This calls for a particular stance on the issue of effective Mathematics teaching practices. Initially the study of effective teaching practice was approached by studying a particular component of teaching in isolation (Koehler & Grouws, 1992). However, the realisation of the complexity of viewing teaching led to calls, to pair research on teaching with that on learning. For example, Romberg & Carpenter (1988) called for the integration of the two domains. The study of teaching should preferably therefore not be studied in isolation from that of learning.

Another aspect that has gained prominence in shaping teachers practice is the teacher's subject matter knowledge (Shulman, 1986). The teacher's subject matter knowledge has been described as the ideas, theories and frameworks of Mathematics, as well as the ways of knowing that are characteristic of Mathematics (McDiarmid & Ball, 1992). Various studies have highlighted the fact that the opportunities that teachers create in class depend in part on their view of what Mathematics is all about (Ball, 1989; Evan & Lappan, 1994).

Since we are concerned with teaching and learning of mathematics in South African schools, we want to address all of these aspects in turn. We will therefore discuss the nature of mathematics in order to guide our classifications of something as mathematical activity. Next, we discuss views on learning and teaching and to what extent these can actually be observed. To provide some background, we will however, start with a brief outline of the new South African curriculum.

2.2 AN OVERVIEW OF THE NEW SOUTH AFRICAN CURRICULUM

Curriculum 2005 proposes a very different approach to what most South African teachers and learners have experienced in classrooms. The previous syllabus emphasized content knowledge rather than integrated classroom learning experiences of knowledge, skills and attitudes. The Outcomes-Based Education (OBE) framework defines the essential knowledge, competencies, attitudes and values which learners in different learning areas should acquire, develop and demonstrate.

In the South African OBE system there are three different kinds of outcomes:

- (a) Critical Outcomes - these are broad cross-curricular outcomes which are statements of intent which give direction and guidance to the statement of more specific outcomes.
- (b) Learning Area Outcomes - OBE fosters a more holistic approach where integration of learning content is emphasised. In order to facilitate integration, the new curriculum is developed on the basis of learning areas. Each learning area has its own specific outcomes.
- (c) Specific Outcomes - refers to the specific knowledge, attitudes, proficiency and competencies which should be demonstrated in the context of a particular learning area.

The Assessment Criteria have been put in place to "provide evidence that the learner has achieved the specific outcome." The assessment criteria tells us what to look for in the classroom when learners are engaged in an activity. The assessment criteria also give teachers direction in explaining what the specific outcome means in terms of the particular context in which they are working.

The Range Statement is another element which plays an important role in the OBE framework. It tells teachers how deep, how complex and far to go with the content. It is not intended to prescribe to teachers what they must do, but rather to assist them.

The Performance Indicators are another important aspect in the OBE learning programme. These give teachers much more detailed information about what learners should know and be able to do in order to show achievement. They also provide teachers and learners with the levels to be reached in the process of achieving the outcome.

The terms introduced and discussed, viz., critical and the specific outcomes provide us with an indication of what learners are expected to achieve in terms of values, knowledge, competencies and skills. The assessment criteria tell us how to assess learners' evidence of achievement. The range statements give practical ideas of the possible complexity of an activity and the suggested content.

An important feature of OBE is that all learners are expected to learn and to succeed (Spady & Marshall, 1991). This places a tremendous responsibility on the teacher to be creative and innovative in his/her teaching and develop means in order for all learners to be successful. One way of addressing this is by fostering different teaching and learning styles. This issue is taken up in much greater depth in this report under the headings of views on mathematics and views on teaching.

Source: Department of National Education (1997) Curriculum 2005.

2.3 THE TEACHER'S VIEWS ON MATHEMATICS

Many educated persons, especially scientists and engineers, harbor an image of mathematics as akin to a tree of knowledge: formulas, theorems, and results hang like ripe fruits to be plucked bypassing scientists to nourish their theories. Mathematicians, in contrast, see their field as a rapidly growing rain forest, nourished and shaped by forces outside mathematics while contributing to human civilization a rich and ever-changing variety of intellectual flora and fauna. (Dossey, 1992,p. 39)

2.3.1 Products, Processes and Context - a Pluralism of perspectives

Reading over the Specific Outcomes (SO) for MLMMS (Mathematical Literacy, Mathematics and Mathematical Sciences), one finds several statements describing mathematics:

"The development of the number concept is an integral part of mathematics." (SO #1)

"Mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships." (SO #2)

"Mathematics is a human activity." (SO #3)

"Mathematics is used as an instrument to express ideas from a wide range of other fields." (SO #4)

"Mathematics enhances and helps to formalize the ability to grasp, visualize and represent the space in which we live." (SO #7)

"Mathematics is a language that uses notations, symbols, terminology, conventions, models and expressions to process and communicate information." (SO #9)

"Reasoning is fundamental to mathematical activity." (SO #10)

Together, these statements reflect a broad and inclusive philosophy of mathematics. It is a modern view which emphasizes the contexts and processes of mathematical activity rather than the end-products of this activity. It is a view which is open to fallibilism and not promoting absolutism (Ernest, 1991). However, the Assessment Criteria and Range Statements show - through the repeated focus on evidence and demonstration of knowledge - that also the products of mathematics are considered in OBE.

We would claim, with Skovsmose (1990b), that mathematics cannot be described through one perspective only. It is best captured through what he calls a pluralism of perspectives, or the 'movement' from one perspective to others. Mathematics is such a rich discipline that to give it full credit, it must entail a product perspective, a process perspective, and a contextual perspective (cf. Skovsmose, 1990b).

In the Assessment Criteria and Range Statements, we see - as already mentioned - a focus on the accepted statements of mathematics. In SO # 9, we see the mathematical language stressed, including the learners' use of it. Reasoning is the focus of SO #10, which also addresses the questions of mathematics through its mentioning of forming conjectures and experimenting. The students are familiarized with the meta-mathematical views indirectly through all of the Specific Outcomes, and explicitly in SO #3 on the historical development of mathematics in various social and cultural contexts.

2.3.2 Mathematical enculturation

Reading through the Specific Outcomes, we see that the students are not introduced to the present mathematical practice(s) as passive receivers. Rather, they are supposed to engage in the activity which is characteristic to these practices, thus leading to a mathematical enculturation, a term coined by Bishop (1988/1991). Bishop claims that there are six such fundamental activities: counting, measuring, localizing, designing, playing, and explaining. (Most of) these are reflected in the Specific Outcomes. Some examples:

Counting underlies SO #1 with the focus on the development of the number concept.

Measuring is addressed in SO #5: measure with competence and confidence in a variety of contexts.

Localizing is implicit in SO #7: describe and represent experiences with shape, space, time and motion; and #8: analyze natural forms, cultural products and processes as representations of shape, space, and time.

Explaining (and perhaps also designing and playing) could be seen as part of the activity in relation to, among others, SO #2: "Mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships. .. Mathematics offers a way of thinking, of structuring, organizing and making sense of the world."

Also, SO #10: "Use various logical processes to ... justify conjectures. ... Learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others."

By focusing on activity, Bishop addresses the process aspects of doing mathematics. This can also be done by looking at the types of activities across Bishop's fundamental activities. This could be problem solving, discovering, investigating, reflecting, generalizing, proving, etc. With the more inclusive view on mathematics from which we are working here, these aspects must be considered part of mathematical activity.

2.3.3 A multi-cultural view of mathematics

Bishop distinguishes between 'mathematics' and 'Mathematics' : mathematics is a generic term since "There are, clearly, different mathematics ..." (1988/91, p. 56). Mathematics is the internationalized discipline and it

"... is certainly not the product of one culture, nor is it the result of the activities of one cultural group. ... Mathematics is therefore not just a subset of all the mathematics which different cultures have developed, it is a particular line of knowledge development which has been cultivated by certain cultural groups until it 'has reached the particular form which we know today. " (op. cit., p. 57)

The Specific Outcomes reflect this multi-cultural understanding of mathematics. It is clearest in SO #3, but is also evident in SO #5: "Measurement in mathematics is a skill for universal communication.."

Going further, Bishop distinguishes three pairs of values in the Mathematical culture, though it is the synergetic effect of these values which comprise the culture. One value is that in Mathematics, impersonal, rational, abstract, and deductive thinking is valued and images of material objects are held in higher esteem than relations and processes. At the same time, progress - the strive towards increased organization and knowledge plus an open attitude towards the development of alternatives - and control over the surroundings are valued. Mathematics is a tool in obtaining control, taken in a broad sense by Bishop. Thus, Mathematics is used in explanations and predictions of natural phenomena and as such comprise powerful knowledge (op.cit., p. 70). Furthermore, Mathematical knowledge offers a security which leads to a feeling of control, just as "The feeding of Mathematical ideas back into society via technological developments is another example of this desire for control ..." (op. cit., p. 71)

It appears to us, that nowhere in the Specific Outcomes nor in the 7 critical outcomes, although appearing as 1 of the 5 social outcomes, are the values of Mathematics overtly or covertly addressed.

This is the first indication that an important perspective on mathematics is missing from the Specific Outcomes. We deliberately use 'missing' rather than simply noting this perspective absent, as we find that a curriculum which does not address the values inherent in the discipline, is bound to be influenced by the values implicit in the discipline within the scientific community and/or within society in a broader

sense.

2.3.4 The role of applications

Finally the position of applications is considered. In the state-delivered syllabus, content was reinterpreted and couched in application terms. However, in this type of applications, mathematics is applied after the relevant content has been dealt with. We are concerned also with the changes when applications based (or led) mathematics are to drive classroom activity, learning and instruction. (See Julie, 1993).

Attention might be paid to applications, but only after mastering an algorithm and most learners do not come this far. The Netherlands has a mathematics curriculum based on applications first. A rationale for this is given by Freudenthal (1973) that the learner should recapitulate the learning process of mankind. Learning should not start with the formal system, which is in fact a final product. The real-world situation or problem is explored intuitively for the purpose of mathematising it. Relevance is more easily understood in these terms. Niehaus et al (1997) describes how this approach has been taken forward in South Africa by the Realistic Mathematics Education joint project of UWC and the Freudenthal Institute.

Christiansen (1998b) distinguishes application situations on the basis of how much of the modelling process is included. The most inclusive category is where an evaluation of the model or the application is included. If reality is a starting point, but no actual evaluation takes place, it is a different situation. If the teacher or the problem statement provides students with a system description, so that a simplification in comparison to reality has already taken place, we see this as a different type of activity. Finally, the mathematizing may have been almost completed and given in the problem/exercise. We have chosen to distinguish between word problems that are not extremely liken to something which has already been covered, and exercises which are very similar to 'already covered' types of problems.

What we have found is that mathematics can only be described through a conglomerate of perspectives. It would be very limiting to only look for, say, conceptual understanding, whether it be of mathematical concepts or of concepts in relation to the use of mathematics (such as the technocratic transformation). Rather, the coding and analysis must be geared towards inclusion of mathematical activity of all sorts, addressing the values of mathematics, and so forth. This includes all the perspective of the Specific Outcomes and more.

2.4 VIEWS ON LEARNING

The purpose of teaching is - or dare we say should be - to give learners the possibility to learn. Thus, we feel that we cannot address possible perspectives on teaching without addressing possible perspectives on learning. An important pointer in this regard is found in the seven critical cross-field outcomes:

- CO 1 : Identify and solve problems that display responsible decision-making using critical and creative thinking.
- CO 2 : Work effectively as a member of a team, in groups, community or organisation.
- CO 3 : Organise and manage oneself and one's activities responsibly and effectively.
- CO 4 : Collect, analyse, organise and critically evaluate information.
- CO 5 : Communicate effectively using visual, mathematical and/ or language skills in the modes of oral and/ or written presentation.

Although OBE neither requires nor prohibits specific learning theories as long as they are consistent with the meaning and content of the key elements of (Spady, 1996), the critical outcomes point towards learning theories which emphasize learner autonomy, critical reflection and social interaction. In the following section, two learning theories are discussed which emphasize these aspects.

2.4.1 Constructivism

The view of learning as stocking up on knowledge and of teaching, and as transferring such knowledge to the empty vessels alias students have been thoroughly criticized. The notion of constructivism and Piaget's ideas are often mentioned in this connection. As a theory of learning, constructivism holds the view that, the acquisition of knowledge takes place when the learner incorporates new experiences into existing mental structures and reorganizes those structures to handle more problematic experiences (Kilpatrick, 1998). Knowledge is not passively received from others, or from authoritative sources. Rather, knowledge is constructed as the learner makes sense of the experiential world.

Applying these ideas to Mathematics, mathematical knowledge is seen as a creation of the human mind. Piaget's notions of accommodation and assimilation are applicable here. Assimilation takes place when the new experiences are incorporated into existing mental structures, and accommodation when these structures are reorganized to handle more general experiences.

The constructivist assumptions about learning determines a different set of actions which are desirable to achieve these assumptions. For example, the goal of teaching changes from developing pedagogical structures to help learners acquire mathematical knowledge to one where the facilitation of learner engagement with the task becomes the focus. Although constructivism does not espouse a particular teaching practice, certain practices that encourage learners to become active participants have been associated with it. For example, conducting investigations, working in groups and handling concrete objects have come to be characterized as "constructivist teaching".

2.4.2 Learning as Legitimate Peripheral Participation

In constructivism and related theories of learning, learning is viewed as a process which happens internally in the learner. This view is contrasted by theories which see learning as a process in the interplay between the acting/learning person, the activity undertaken, and the surroundings in general. These three elements are seen as mutually formative through this interplay. Thus, the distinction between outer and inner is to a large extent done away with (Wedeg, 1998, p. 6-7).

The view on learning as legitimate peripheral participation (Lave & Wenger, 1991) uses apprenticeship as the standard metaphor. Within this view, the learner is connected to a certain practice but is not as yet a full participant. This is, however, accepted, so that the learner gets the opportunity to participate in the practice in a more peripheral sense, but gradually moves towards full participation. This movement from the periphery towards the core or "community of practitioners", is called learning.

This view on learning is also applicable to the school situation. Within this perspective it is perceived that the students are situating themselves on the periphery in acquiring a particular school (mathematics) practice. "It is possible to see much mathematics instruction as an insertion of students into the mathematics classroom discursive practice rather than actual teaching of mathematical methods or concepts." (Lave, 1988, p. 176).

2.4.3 Theories as metaphors

We do not know what actually takes place when a person learns. Whether we have a view on knowledge as piled up information on the inner shelves, as cognitive structures, or as 'knowledge-in-action', these are only metaphors for what it means to know. Likewise, whether we view learning as stocking up on knowledge, as active construction of connections between experiences, or as going from peripheral to full participation in a practice, these are only metaphors for the actual learning process. These metaphors are useful in describing aspects of what takes place in a learning or teaching situation, but they can never capture the entire situation (Kilpatrick, 1987, p. 13).

A theory of learning should not become an occupation of the mind (Christiansen, 1998b). Instead, we must encourage the seeing of alternatives in both theory and practice. Still, theories of learning can guide our awareness to possible alternatives. This we have taken into consideration in the design of our research instruments.

2.5 THE TEACHERS' VIEWS ON TEACHING

2.5.1 Effective Teaching

Effective teaching is essentially concerned about how best to bring about the desired learning by some educational activity (Kyriacou, 1990). Initial research on this topic focused narrowly on particular characteristics of teachers or specific components of teaching. The characterization of teachers' actions as rational and reflective, spurred research onto the thinking, planning and theorizing that teachers do (Clarke & Peterson, 1986). Teachers are called upon to make decisions while teaching (Brown & McIntyre, 1995). This focus on interactive decision-making has been supplemented by research on teachers' routines, rules and patterns of practice (Elbaz, 1983; Leinhardt, 1987).

The increasing emphasis on integration of research on teaching and learning has added another dimension. According to Koehler & Grouws (1992) effective teaching is now viewed as a double lense where the outcomes of learning are determined by the learners' actions and thinking, whilst these actions and thinking are largely determined by what the teacher does or says in the classroom. In our analysis of the teachers views on effective teaching we have included the interactive decision making, practical knowledge as well as the actions of the learners. The focus on learners' actions will be discussed separately under the heading mathematical activities.

2.5.2 The shift towards learner/ learning centredness

A major trend that has been developing within teaching has been the shift away from authoritarianism and towards learner-centredness and learning-centredness. Whilst teacher-centredness, from where historically we come, is clear, there is a plethora of alternative theories that are taking us forward. Some are in harmony, some are compatible, and some not so compatible.

Abel (1997) gives the label of "Mediational/experiential" to a consistent grouping of theories, which embrace Feuerstein, Vygotsky and other scholars from a cognitive base.

What is important in the table below is not the label, but the trend the characteristics indicate.

	Traditional	Mediational/experiential
PURPOSE	Knowledge acquisition.	Attitudinal / cognitive / behavioural change.
ASSUMPTIONS	Change : rational through standardisation.	Change : rational, emotional, socio-cultural through awareness and experience.
CONTENT	Public, given, content knowledge and espoused theories. Theory and practice : Discrete and implicit.	Besides traditional, also personal, problematic, process-orientated knowledge; Espoused and theories-in-use together with outcomes. Theory and practice : Explicit and integrated.
PROCESS	Didactic and abstract. Non-life related. Individual. Instructor as expert. Learner passive and subordinate. One-directional. Not supported by appropriate text.	Dialectic, mediational and experiential. Life-related. Collaborative, holistic and personal. Instructor as facilitator/mediator. Learner active agent. Multi-directional. Supported by appropriate mediational texts.

Table 1 : Traditional Approaches and Mediational/Experiential Approaches

In some ways, the changes we are going through, particularly moving towards Outcomes Based Education and Curriculum 2005, are independent of any coherent driving theories. Indeed there are many supportive ideas, but it is almost as if there is a march away from hegemony towards socialism, which captures in essence, many (minor) supporting theories.

2.5.3 Teaching according to goals, control and content organisation

Clearly, we must come to terms with the fact that there are many possible teaching styles, and that not one will reasonably stand out as a 'best practice'. However, in accordance with Ernest (1991), we find that certain teaching styles are better in accordance with certain views of mathematics, of the child, etc. Ernest combines these in a classification of five approaches to teaching mathematics which are again linked to an ideological stance on society and the purpose of teaching mathematics. We find this classification very useful, but also find that in general a whole range of motives can direct the teacher's activities. We suggest a classification according to three parameters more internal to mathematics education.

This classification is based on Illeris (1995), who has the following model:

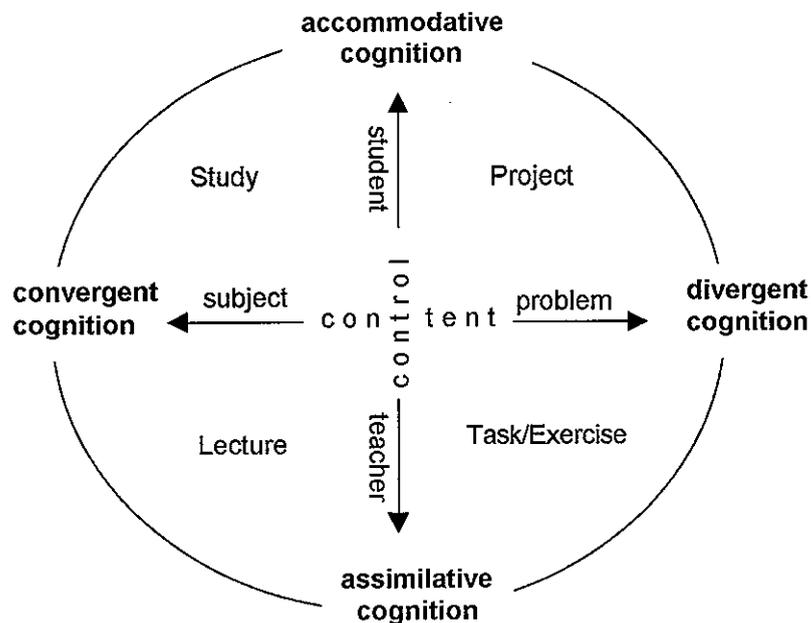


Figure 1 : Classification of Teaching Styles

Illeris distinguishes between a teacher and a student controlled learning/teaching situation, between subject organized and problem organized content. If the student is in control and the content organized in a manner usual to a given discipline, the typical situation will be that of a student studying her way through a course material or a book. If the teacher is in control but the content still structured according to the discipline, the typical situation will be that of a lecture. If instead the content is structured around problems, the situation will take the form of tasks set for students or exercises. Finally, if the student(s) is/are in control in a problem oriented approach, he talks about project work.

2.6 MATHEMATICAL ACTIVITIES

The constructive element in Piaget's genetic theory of cognition as well as many other theories have supported the idea that students should become active in the classroom - the basis of mathematical knowledge is exactly an activity, not information. We will not question the importance of activating the students. However, we find it important to consider what should be meant by 'activity' in this connection - as well as how activity leads to mathematical knowledge.

One could say that any type of activity could lead to mathematical insight, if the activity becomes the basis of reflective abstraction. We find this view a bit too inclusive for practical purposes. An illustration of this is, students may engage in activities which are not related to the instructional design. For instance, through interaction with peers, the student activity for thinking could be redirected by the students. Or they may be engaged in practical work such as finding materials, etc. A complete understanding of the practice of mathematics instruction is not possible without taking these aspects into consideration, and we will return to this when considering the aspect of goals pursued by students. But first we want to address the types of activity that are more overtly oriented towards mathematics. Here, we find it useful to consider the perspectives within the school of action-oriented developmental theories.

They consider students' sensory-motor as well as their conceptual activity as the source of their mathematical knowledge (cf. Cobb et al., 1997, p. 260). We find the inclusion of conceptual activity very important, to avoid the misunderstanding that all activity which could lead to mathematical knowledge has to be of a sensory-motor type. Furthermore, this school assumes "that meaningful mathematical activity is characterized by the creation and conceptual manipulation of experientially real mathematical objects" (Cobb et al., 1997, p. 260). This assumption not only allows for conceptual activity, it also points to the fact that for the activity to be meaningful to the students, the objects constructed or manipulated must be experienced as real (and it opens the possibility that not all mathematical activity is of this type and thus is not meaningful!).

Since we are concerned with the teaching practice, we focus on those types of activities which the teacher has initiated, that is, activity when students are on task. This is not to say that students do not construct mathematical knowledge or lay the ground for such construction in other situations. This limitation is made only with reference to the purpose of our research.

We will make a first distinction between

- a) practical and organizational activity,
- b) procedural, factual and rule-bound activity,
- c) problem-solving activity, and
- d) theorizing/explanatory activity.

To refine this classification, we recall one of the most used and most criticized learning taxonomies, namely Bloom's (Bloom, 1959). The levels in his taxonomy are: knowing, understanding, applying, analyzing, synthesizing, and reflecting critically. His point is that the latter levels must succeed the former. We are, however, not concerned with the possible prescriptive uses of the taxonomy; we mainly want to let it inspire our classification of student activity. Doing so, we find that the formerly mentioned mathematical activities (see preceding section) can be included. Curiously, we also note that proving, which is generally considered a difficult activity, can belong to several levels.

This leads us to the following classification:

- ? practical and organizational activity; not necessarily working with mathematical objects,
- ? procedural, factual and rule-bound activity (including ritual or symbolic proving); unclear whether or not the students see the mathematical objects as experientially real,
- ? sense-making, explaining procedures or concepts (may include empirical proving); mathematical objects seen as experientially real,
- ? applying know procedures or concepts to new situations; possibly manipulating experientially real mathematical objects,
- ? discovering, investigating, open-ended problem solving; clearly creating or manipulating experientially real mathematical objects,
- ? reflecting in order to formulate 'rules' or 'theorems' or concepts, generalizing, abstracting, proving empirically or analytically; clearly creating or manipulating experientially real mathematical objects,
- ? reflecting critically.

In a previous section, we addressed Bishop's categories of fundamental activities underlying mathematics in all cultures. Obviously, this would also provide a possible classification of students' activity in the classroom. Considering this aspect would give some indication whether students engage in the whole range of activity, thereby laying the foundation for a broad mathematical experience. We find, however, that this would be more useful to consider in relation to the curriculum and in looking at

the content in the mathematics instruction over longer time spans.

One disadvantage with the above classification is the focus on observable activity.

2.7 BASIC NOTIONS AND INDIVIDUAL IMAGES

Above, we introduced the notion of meaningful mathematical activity from Cobb et al. (1997). We find that more should be said about what makes mathematical objects experientially real to students. With the continued focus on contextualisation in mathematics education as a means to assist students in making sense of their school mathematics (cf. Arcavi, 1998), this calls for a critical investigation.

In some cases, students create useful images themselves, which may guide them in their mathematical activity. In other cases, the mathematics makes little sense to them. Then, the teacher may consider what basic ideas - or, since this is a vague notion in English: Grundvorstellungen or GV's for short (vom Hofe, 1995) - underlies the mathematical concept. The mathematical concepts are more general and theoretical constructs than the GV's which again carry the isomorphism of a whole set of phenomena. For instance, one GV connected to multiplication is the putting together of things already grouped in equally sized groups. Another is continuous enlargement or reduction. A third is a combinatorial GV.

The teacher may use GV's in a prescriptive sense. When she wants students to make sense of some mathematical notion, she can specify the GV's connected to this notion. These are then transposed into learning contexts so as to make them accessible to students. This is all the teacher is doing.

When the students meet the learning context, it activates some individual images (R's) in them. Thus, the R's are the descriptive parallel to the GV's. The U's can be different from student to student, as they are connected to one or more individual areas of experience. The student will try to grasp the learning context using her U's. This may lead to the situation where a student develops GV's in accordance with the teacher's plan, or it may lead to an understanding of the learning context different from the one intended by the teacher. Thus, this notion highlights the situation where the student actually makes sense of the mathematical concepts in the way intended by the teacher.

The notion of GV's is useful in understanding students' sense-making activity, but it is beyond the scope of our project. However, GV's and H's can be useful to the teacher in planning and evaluating teaching activities. We will use the notions to address the teacher's thinking about her own activity as well as her way of dealing with students' apparent U's.

The notion of GV's in this research context provides a useful tool in addressing the link between phenomena and the development of mathematical concepts. Together, GV's and H's make a clearer distinction between the prescriptive and the descriptive elements (cf. vom Hofe, 1997).

CHAPTER 3: Methodology and Research Instruments

3.1 THE RESEARCH DESIGN

The research data used in this study comes from eight consenting primary school teachers in the Western Cape Province. They are all grade 3 teachers with between three and thirty years teaching experience. Their formal qualifications range from Junior Primary Teaching Certificates to a Bachelor of Arts Degree.

The researchers chose an ethnographic research design (Hammersley & Atkinson, 1995) for this study, as its qualitative methods provides sufficient flexibility for describing, interpreting, exploring and explaining the process and products of teaching and learning.

This research design enabled us to observe the eight participants practices by sharing in the conditions of their classrooms. Also dialoguing with participants through interviews to "reveal the nuances of meaning from which their perspectives and definitions are continually forged". (Kalnin, 1986)

The qualitative research information was gathered through (a) direct observation and (b) in-depth interviewing. Firstly, the researchers drew on the twenty four completed classroom observations and sixteen pre and post interviews.

During the observation the researchers engaged in systematic noting which included holistic recording of events behaviours and resources in the classroom. To facilitate the noting of field notes, an observation schedule was constructed and piloted in one classroom by two independent researchers. Minor changes were made with regards to the amount of categories being used. The first part of the observation schedule collected general information regarding class size, setting, desk arrangements, date and time. The observations were also audio-taped, then transcribed. The researchers used two tape recorders. One pocket tape-recorder with a lapel microphone, monitoring the teachers' conversations. Then a table, tape recorder monitoring all student conversations within the range of the tape recorder. Tape recording transcripts enabled the researchers to reconstruct the lessons observed. This provided us with factual information, leaving interpretation until a discussion with the participant of the lesson, by the researchers.

We do recognize that there are some limitations and weaknesses using tape recordings with regards to: (a) it does not record silent activities, (b) provides no visual account of activities, (c) continuity can be disturbed by the practical problems of operating.

The researchers were conscious of the fact that the tape recordings and even note-taking could interfere with, inhibit, or in some ways impact upon the classroom setting and the participants. The observational method also required a great deal from us as researchers in trying to manage the difficulty of a relatively unobstructive role and the challenge to identify the "big picture" while observing huge amounts of fast-moving and complex behaviour (Evertson & Green, 1985). However, the qualitative nature of the observations enabled us to discover the complex interactions in the lessons observed.

Before each teacher began teaching their lesson, a pre-instructional schedule was given to them to complete. This schedule included (a) stating the title of the lesson, (b) the intended mathematical/cross curricula content, (c) the aims of the lesson, (d) intention to meet aims, (e) how to check meeting aims, (f) how the lesson relates to earlier lessons, and (g) ideas/concepts students will find difficult and or easy. There was also a post instruction schedule which teachers were asked to complete. This schedule included aspects on: (a) were aims achieved, (b) what did you like about the lesson, (c) what did you not like about the lesson, and (d) any other comments.

Secondly, the interviews with the eight teachers were unstructured as described by (Brown & Dowling, 1998) nonetheless, it was "a conversation with a purpose" (Kahn & Connell, 1957) where we explored a few general topics to help uncover the teachers' meaning perspective. It was hoped that by not structuring the pre and post interviews too closely, what teachers deemed as important would emerge. The interviews also had particular strengths, in that it provided the researchers with large amounts of data quickly. It also provided us with an opportunity for immediate follow-up and clarification on what teachers were saying.

The researchers recognize that there were some limitations and weaknesses using interviews as a research design (Marshall & Rossman, 1995) with regards to: (a) it was dependent upon the full cooperation of the eight participants, (b) some of the participants' were uncomfortable sharing all that was hoped to be explored. (One participant, constantly switched off the tape recorder, when feeling uncomfortable.) Then (c) there is also a possibility that elements of the interview responses may not have been properly comprehended by the researchers.

With regards to the quality of the data, the combination of observation and interview data enabled a degree of objectivity in the assumptions and analysis. It helped the researchers to avoid oversimplification in the descriptions and analysis, because of its narrative nature. The combination also allowed us to understand the meanings teachers hold of everyday mathematics perspectives and teaching perspectives.

The combination of observations and interviews also enabled us to "witness events which particularly preoccupy the hosts, or indicating special symbolic importance to them" (Schatzman & Strauss, 1973). The process of preserving the data and meanings on tape and the combined transcriptions greatly increased the efficiency of the data analysis. The initial decisions about the data analysis was too broad and unmanageable, this led the researchers to recast the entire research endeavour. A balance between efficiency considerations and design flexibility was struck. We were guided by initial concepts, but shifts occurred as the data was collected and analysed to the extent of discarding some of the initial concepts.

The way in which the researchers brought order, structure and meaning to the thick narrative data, was to search for general statements about the relationships among the categories of data. The category generation phase of the data analysis was the most difficult and complex process. The process used in category generation involved noting regularities in the setting and of the participants chosen for the study. The analysis became more complete when the critical categories of (a) Teachers' Mathematics Views and (b) Teachers' Teaching Views were defined. The relationships among them were established and they were integrated into grounded theory as described by (Miles and Huberman, 1993).

The analytic procedures used were to: (a) organise the data; generate categories, theme and patterns; testing the emerging hypotheses against the data and searching for alternative explanations of the

data, which led to this report writing. Each of the phases of data analysis mentioned, went through a data reduction process, as the "thick" data was interpreted. The researchers paid careful attention to how the data was being reduced throughout the research endeavour. In some analysis there was a direct transfer of data onto pre-developed data recording memos. This helped us to streamline the data management and ensuring reliability across several researchers.

As the categories of meaning emerged, the researchers searched to identify the salient, grounded categories of meaning held by the participants with regards to: (a) mathematical views prevalent amongst the teachers, (b) teaching views to facilitate learning and (c) the teaching strategies the teachers employ.

3.2 ANALYZING DATA FROM CLASSROOM OBSERVATIONS

Selecting special episodes from the data according to a very well-defined research question or perspective and then analyzing these episodes in great detail, gives a useful insight in the finer details of the constitution of classroom activity. (Christiansen,1997). However, it may not be the best basis for understanding the totality of the complex interplay of factors at stake in the mathematics classroom.

Instead, we chose to develop a coding system for determining the type of mathematical learning pursued. This includes :(a) The teachers' views on mathematics and the prevalent mathematical activity among them, (b) the teachers' views of teaching mathematics to facilitate learning, and (b) the teaching strategies employed by the teachers. Thus we have chosen not to focus on disciplinary or management issues, among others.

3.3 CODING THE MATHEMATICAL CONTENT

3.3.1 Developing the Coding System for Mathematical Content The first attempt to develop a coding system worked with nine categories which were merely listed, not ordered. For instance, the first category concerned itself with whether the focus of the activity was on mathematical products, processes or contexts. The third category then addressed the focus within the product perspective. This composed a simple coding system, which reflected that, the perspectives on mathematics are not necessarily ordered in any straight forward manner. However, it camouflaged how some categories may be viewed as sub-categories of others, and it meant that there was overlap between categories.

To overcome this, a tree structure of categories was developed, as illustrated in the figure below.

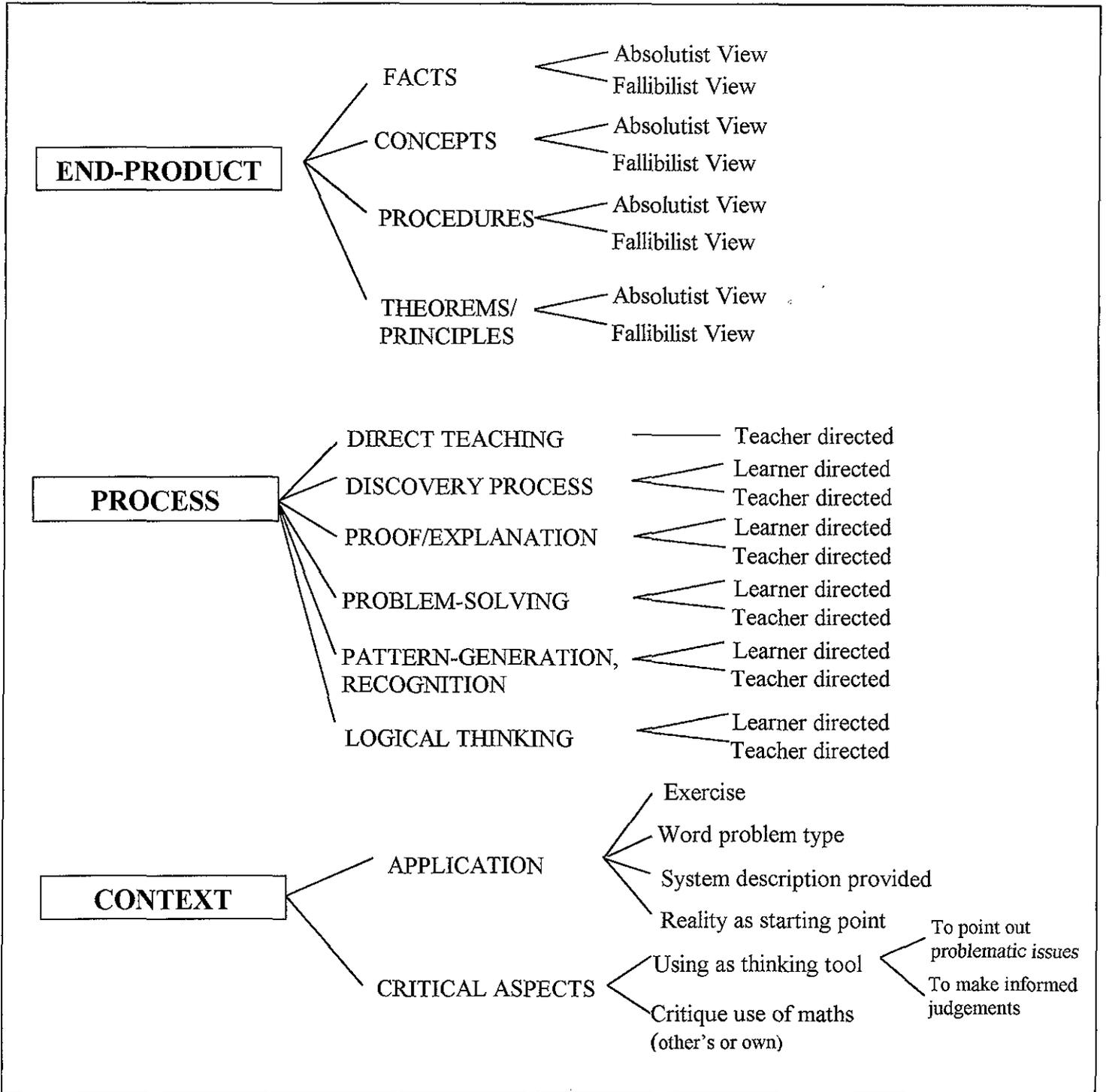


Table 1 : Coding Scheme for Analyzing Views on Mathematics

The categories necessitate some clarification of certain concepts such as 'concept' versus 'theorem' and 'procedures'. It becomes necessary to clarify what is to be considered as markers of one particular category. This is discussed in more detail below.

It also appears that the mathematics content is to some extent intertwined with the teaching approach, at least in this coding system. For instance, we have distinguished between references to the fallibilism

of mathematics and exercising the fallibilist view in the classroom. This does not have a theoretical basis, simply a pragmatic one; it would be too much to develop parallel categories under teaching approaches.

Interesting, we did not find it necessary to add categories to this part of the coding beyond what was suggested by the theoretical framework. We take this as an indication of the inclusiveness of our theoretical analysis, though it would be equally valid to refer it, to the closure of the values which we carry concerning mathematics.

The pilot study was coded using this system, and we found that the kinds of mathematical activity present in the pilot study were covered by the categories. We did not find that there were marked differences within categories which the coding did not allow for, Consequently, this part of the coding system was not altered after the first encounter with practice.

A slight change was made after working with the coding of teaching activities. Thus, the types of reflections on a problem and its solution and on the contexts of a problem were moved from being categorized in connection to theories of learning mathematics to being placed under the context perspective of the mathematical content. This is where it should have been in the first place, in the theoretical framework as well.

3.3.2 MARKERS OF TYPE OF MATHEMATICAL CONTENT

By end-product, we mean activity where the focus is on mathematics as it has been developed outside of and prior to the activity in the classroom.

Mathematical facts are statements which are considered to be true. An example would be that 3 is a prime number.

Standard procedures and algorithms are recognized by a strong focus on what to do, how to do it, following certain steps.

By theorems, we refer to statements which could be submitted to a truth evaluation, and which are considered true in the particular situation. Examples are: "Every other natural number is an even number." "The order of the addends does not effect the sum."

Mathematical concepts are not easily defined nor determined in the classroom situation; for instance, the teacher may ask students to perform repeated addition of a given number in order to develop the multiplication concepts. We have not gotten closer than to say that when some activity appears to (and this requires analysis to determine) have as a goal to develop reifications and is not directed towards the learning of an algorithm as such, or when a definition is developed, implicitly or explicitly, we will say that the focus is on concepts. Given this vague 'definition', we would be surprised if this category will not have to be developed in the course of the analysis.

We also expect that a certain activity may be considered as having a focus on procedures at one time, while it would be classified as having a focus on theorems or concepts at a later time, due to some development during the time in between. If this is so, we will have to consider the coding as well as our interpretation of progression in the classroom.

By fallibilist, we mean that the mathematics is treated as being the result of a human process and thus open to change, further development, etc. Indicators of fallibilism would be references to historical

discussion of possible alternatives, or that the mathematics is developed through a process of discovery, construction, reasoning and refutation.

By absolutistic, we mean that the mathematics is treated as if given by an authority, either specific or unnamed. Markers are clear references to authorities or statements stating the mathematics to be given and is unquestionable.

By focus on process, we mean that there are references to or that students engage in mathematical activity such as discovering, abstracting, generalizing, proving, and refuting.

It is a long-standing discussion whether mathematics is discovered or invented. We wanted to allow for the possible presence of both aspects. The focus is on the discovery process, if students are working to find patterns or connections that have not yet been formulated or the teacher is demonstrating such a process. An example is students trying to find out what happens to the area of a plane figure when both 'length' and 'width' are doubled.

The focus is on proof/explanation if a pattern or connection has already been stated - whether it is in accordance with an accepted theorem or not - and the task is to justify the given statement. Markers would be asking and answering, why-questions and discussing the certainty of the statement ("how can you be sure?").

The sub-categories of modelling/application have been developed in Christiansen (1998b). In marking the critical aspects, we will use the categories developed in chapter 2 as guidelines.

3.4 DEVELOPING THE CODING SYSTEM CONCERNING TEACHING

The first attempt in developing a coding system concerning teaching was an unsystematic list of factors to consider in order to get an inclusive perception of the teaching activity. It was quickly abandoned for its lack of theoretical underpinning and structure. It was, however, a necessary step, as it made it compelling for us to reconsider the discussion of teaching and learning in our theoretical framework. Thus, the framework was definitely sharpened by the need to develop a functional coding system.

Our coding system contained several categories which concerned the contexts of teaching mathematics; relevant and of great influence to the outcome, yet it would be too much to insist on including all aspects in our coding. This led us to abandon otherwise very relevant categories such as those describing how the teacher responds to or promote students' questions, the classroom management, the use of praise and punishment, the teacher's general consistency between spoken statements and actions, and the behavioural demands made on students'. Some categories were made into sub-categories of others. For instance, part of the reflective activity had already been moved so as to be part of the categorization of mathematical content. Left was a part on reflective discourse encouraged by the teacher, which was made a sub-category of student activity encouraged by the teacher, and a part on reflections on the instructional situation which was made a sub-category of purpose/motivation. As we are interested in the teaching practice, we decided to leave out the issue of assessment except for continuous assessment which was or could be used in the teaching situation. Thereby, questions of assessment were closely connected to the issues in relation to individual images, and subsequently will only be considered in that connection. Finally, we decided to make a category concerning the teacher's organization of the content, with learning styles and forms of representation as just two sub-categories hereof. This would make it easier, we hope, to address the teacher's organization of the content as a whole, forming the basis for the dialogue with the teacher concerning her instructional choices.

These reflections led us to the system of coding illustrated in the figure below.

BASIC NOTIONS AND INDIVIDUAL IMAGES	<ul style="list-style-type: none"> Basic notions Teacher elicits learners' individual images Learner offers individual images Whole class teaching
CONNECTIONS	<ul style="list-style-type: none"> Abrupt shifts between approaches or topics With previously done work Interdisciplinary Ground laid for future connections
LEARNER ACTIVITY ENCOURAGED BY TEACHER	<ul style="list-style-type: none"> Organisational Procedural Sense-making Application Discovery / Investigation Reflecting Underground activity
ORGANIZATION OF CONTENT	<ul style="list-style-type: none"> According to textbook Building on learners' in-school experiences Building on learners' out-of-school experiences Discovery oriented Other criteria
PURPOSE /MOTIVATION	<ul style="list-style-type: none"> No direct references to motive or purpose of activity Purpose is purely authoritarian; 'because I say so' Purpose is stated by teacher Purpose is life-related References to future use Purpose is negotiated among students' and teacher

Table 2 : Coding Teaching – Categories and Sub-Categories

The coding may still not appear satisfactory. It is not sufficiently strict, in the sense that much is left open to interpretation by the researcher. For instance, the researcher will have to decide if basic ideas are at play, something which itself can only be done as the result of some analysis. There are also problems in distinguishing between answers which are not what the teacher considers correct and individual images. In general, there will be times where it is difficult to decide how to categorize an activity. We see this not entirely as a problem but also as an indication that we are indeed touching upon non-trivial issues.

The categorization of the organization of the content will to a larger extent than is the case for the other categories address the lesson as a whole. This implies that the researcher must state how s/he reached the categorization. We are aware of this necessity to document the coding and will apply this in the actual analysis. This is also a general methodological issue of interdisciplinary objectivity and reproducibility.

3.4.1 MARKERS OF TEACHING ASPECTS

First we identify markers for the type of activities encouraged by the teacher:

- ? In any classroom, it is necessary to organize activity, passing out papers, directing students to the desired seating, etc. These activities are necessary but not of a mathematical character. These, we have categorized as practical and organizational activity.
- ? Procedural, factual and rule-bound activity includes any stating of already acquired information/knowledge and any execution of algorithms already mastered or stated by the teacher in steps that can be carried out according to the teacher's directions.
- ? When students engage in addressing why a certain algorithm works ("you carry one, because it is like having one more ten, and that is the ten's column"), what is to be understood by some definition ("an even number is one where if you had that many pieces, you could divide it into two equal heaps, without having to half any of the pieces"), or in other ways extent, the mere execution of procedures or stating of facts, we talk about sense-making activity. As we are looking for the activity encouraged by the teacher, we will categorize it as such, when the teacher asks for explanations or reasons for why 'something works'.
- ? If students' are left to work on a situation that cannot be handled simply by following steps in an already familiar algorithm, but where they are on the other hand supposed to be knowledgeable about concepts and procedures with which the situation could be addressed, they will have to apply these concepts or procedures in slightly unfamiliar ways. When the teacher encourages such activity, we will talk about applying known procedures or concepts to new situations.
- ? At times, the teacher may encourage students' to work on problems or investigating situations or issues where there is no direct method given by the teacher. When the teacher appears to know what she wants students' to find as their answer, their new method, their new 'theorem' or the like, we talk about discovering or problem-solving with strong teacher guidance. Of course, it is hard to be sure just from observing the teacher, whether or not she knew the outcome of the activity. The teacher's guidance is not a strong enough indicator, as she may also guide students' in situations where she is not certain of the outcome. For this reason, we asked the teachers to state the purpose of their lesson in advance on the pre-observation questionnaire.
- ? We distinguish between investigation without the teacher being certain of outcomes and open-ended problem-solving; in the former, there is no specific problem stated, which is the case in the latter type of activity. For instance, asking students to state as much as they can about triangles would be considered an investigation, while asking students to find out how a class of 43 students can be divided into groups of seating would be considered as an open-ended problem-solving activity. Each of these would be distinguished from the teacher-guided situation, where students are asked whether the class of 43 can be divided into

- ? equal groups. This is a category where boundaries between types of activity are not quite sharp.
- ? There is a difference between the teacher encouraging the students to reflect in order to formulate rules or concepts, generalizing, etc. and the sense-making activity mentioned above. In the latter, the generalizing may already have been done or the concepts or rules stated by the teacher. In the reflective activity, the teacher will encourage students to formulate theorems, concepts, etc. or to describe procedures. In the theoretical framework, we have put some emphasis on the role of reflective discourse. To follow up on this, we have decided to make explicit note of when the former activity is made an object of discourse.

We have discussed the notion of basic ideas in the theoretical framework. Depending on the type of content, the basic ideas can vary greatly. It will be necessary to discuss possible basic ideas in relation to the content actually taught in the observed lessons and then note which of these the teacher may have drawn on or sought to develop.

When it comes to the individual images - the students' counterpart of the basic ideas - we have chosen to abstain from analyzing the individual images; that would be very useful knowledge indeed, but it would be besides the point of characterizing teaching practices. Our focus here will be the way the teacher relates to the individual images that occur. As already noted in the previous section, it is difficult to distinguish between students' answers and the underlying individual images. Two students may both give $\frac{7}{11}$ as the answer to $\frac{2}{3} + \frac{5}{8}$, but one student may simply have manipulated the symbols without considering the meaning, while the other may have thought of ratios and adding these - such as how many games won out of a total. We are especially interested in the extent to which, and the way in which the teacher may address students' individual images, rather than in her way of handling 'wrong' answers. However, it may still give a useful indication to note how the teacher tackles answers that are not in accordance with what she wants.

We have chosen to distinguish among situations:

- 1) where the teacher asks students to tell how they are thinking, what reasoning may have taken them to a particular answer or other type of statement, etc. - in which case we will say that the teacher was searching for student's 11's,
- 2) where the students' are the active part in directing the teacher, to be aware of the students' understanding,
- 3) where the teacher addresses the class as a whole but still with the purpose of finding out what students' may be thinking.

In some cases, the teacher may simply make a mental note that, not all students have the knowledge or insight she would like, but otherwise ignore the 'wrong answers or the individual images. The teacher may also indicate that she is aware of the divergence, by pointing out that not all arrived at the same answer or by collecting a list of conjectures. We say that she has simply acknowledges the situation.

It is also a common possibility that the teacher will simply praise, correct or state an answer or an explanation as being right or wrong. This we have categorized as the teacher evaluating answers or individual images. Of course, the teacher may decide to develop her teaching according to her observation of students. For instance, she could repeat an activity, give explanations, give additional tasks to explore particular issues, or the like. This type of activity could be directed towards all, one or a group of students. If the teacher stops the teaching of the class to address one student in particular, we would not talk about differentiated instruction. Differentiated instruction refers to situations where

the teacher directs different students. To engage in different activities, typically of the same type but on different levels (pupil differentiation) or to engage in activities which allow for different learning styles, preferred forms of representations, working from different individual images or the like.

Classifying the organization of content is an attempt to say something about how the teacher organized the lesson as a whole. We have chosen to look for types of organization with very different origins:

? Following a textbook may be based on theories of learning and teaching which the teacher found reflected in the book, or it may be an entirely pragmatic choice.

? Organizing the content so that it starts being close to students' daily life experiences and then generalizing or abstracting may be based on general learning theories, but it may also find ground in theories of developing mathematical knowledge and understanding in particular. These issues will not be determinable through analysis of the classroom practice, but it will be something to pursue in the later interviews with the teachers. For that, we will limit ourselves to this rather pragmatic categorization for the time being.

?

Connections refer to the connections which the teacher makes between different topics, parts of the lesson, this and other lessons, or the content and some out-of-school situation or practice:

? If the teacher shifts from one topic or one approach to another without stating that this is now taking place or in other ways addressing the shift, we consider it an abrupt shift.

? If, on the other hand, the teacher states that she is now moving on to something else or to work with the same topic in a different way, we would say that she has made a link. We also consider it a link, if the teacher refers to former or future lessons explicitly, for instance stating that "this is just as when..." or "we will get back to this when ...". At times, the researcher may find that the teacher was making a solid foundation for later work, either in the same or in another lesson. This will naturally be hard to confirm, unless a reference is actually made later on, but we will note such possibilities in order to form another basis for discussing teachers' intentions behind their organization of activity.

? We will say that there was possibly ground laid for later connections. We recognize that we may also see activity which is the result of such earlier groundwork. We say that the teacher makes use of formerly introduced concepts, methods, etc. The teacher may also make explicit references to earlier work. These two categories are thus different mainly due to how explicit the connection was made. While all of these connections are connections within the mathematics classroom setting, it is also possible to have connections across school subjects.

? We have not distinguished between interdisciplinary, trans-disciplinary, and cross-disciplinary approaches, since our focus is on the mathematical content. But we recognize that non-mathematical content could be drawn in, and we have decided to categorize it as interdisciplinary activity. Needless to say, this means that we need to be able to distinguish mathematical content from that not so. While this may well feel rather easy, because 'we know mathematics when we see it', it is rather problematic to want to put this in more definite terms. It becomes even more muddled, considering the inclusive view on mathematics which we have presented in the theoretical framework. Without getting into a lengthy analysis, we will simply say that the activity is interdisciplinary if there is some focus on contextual meaning or some shifts away from the mathematical meaning.

- ? Since teachers may decide to teach more subjects within the same lesson, we have found it necessary to distinguish between situations where the non- or extra-mathematical content is connected to or used in developing the mathematics and when it is separate, that is not connected to or used in developing the mathematics. Unfortunately, a more detailed analysis could show that some content was used to develop the mathematics in a way which was not determinable at face value. We recognize this problem and will analyze those situations in more detail should we find that it would add to the analysis. Also, this will be a point which will be considered in the interviews with the teachers' about the reasons and incentives for their practice.

The categorization of purpose or motivation stated by the teacher is relatively straight forward, as it mainly concerns the explicit statements which the teacher makes:

- ? We have included a category of not making any reference to the motive or purpose of the activity, which will only be applied by the introduction or start of a new activity.
- ? We have chosen to distinguish between general life-related purposes and references to the future of particular students. This distinction may well turn out to be artificial or impractical, but we included it for the purpose of addressing possible references to particular groups of students - a necessary consideration in South Africa, which has just started the transition away from the unfairness, promoted by the apartheid system.
- ? Finally, we have made a distinction between those purposes stated by the teacher and those purposes negotiated among students and teacher. In the latter, we have made a sub-category of reflections on the instructional situation, as outlined in the theoretical framework.

It is rather evident from this discussion of the coding categories, that it is impossible to state markers of the categories in such a way that the subjective interpretation of the researcher is removed completely. What is more, this is not simply impossible in this particular piece of research; it is epistemologically impossible. We recognize this and thus embrace that the coding does not comprise the analysis but assists in pointing to episodes that would require further analysis in order to address our research questions.

The coding developed and underwent refinement as a result of our encounter with the fieldwork.

CHAPTER 4: Teachers Views on Mathematics and Teaching

This chapter tells the story of each teacher observed in this research project. In each situation we consider the teachers' view of mathematics and that of teaching.

Although the stories all reflect the experiences of a selected group of teachers, in a specific province with its own unique peculiarities, strengths and weaknesses, we do put forward a case that lessons can be drawn from this sample of teachers observed. In that each exemplifies aspects of mathematics teaching and learning which is generic to teachers who are poised to change their practices, which are consonant to OBE. In this way we feel that the mathematical and teaching issues will be accessible to a wider audience. The fact that all the teachers we write about are women is a reflection of the low percentage of male Foundation Phase teachers in South Africa.

The data used in composing these stories included the preliminary and post interviews, the audiotaped classroom lessons and the written field notes for each lesson observed. The researchers were faced with the problem of eliciting, analysing and authenticating the accounts of episodes in an unbiased manner. The account - gathering method proposed by Brown and Sime (1981) were considered for purposes of authenticity. Stringent checks were made on how the information from participants were being transformed into accounts of episodes. Checks on the authenticity of the accounts were again examined in checking on the coding reliability, of the researchers involved in the transformation of the text.

In the light of the weakness in account gathering and analysis, Kitswood (1977) suggests some safeguards which we considered in our data collection. First, he calls for cross-checking between researchers as a precaution against consistent but unrecognised bias in the interviews themselves. Second, he recommends that unresolved problems should be taken back to the participants themselves for comments. We did this in the post observation interviews. Only in this way we can be sure that we understand the participants' own actions. Cross-checking amongst researchers remained a constant exercise during the transformation of data process. Only when all these stringent checks for authenticity were taken into account was the data considered as scientific data (Brown & Sime,1977).

The real names of the participants are not used, instead it is substituted by pseudonyms. The order in which these stories are recorded does not represent any particular pecking order or priority. The word learner/s is used when speaking about the student/s in the participating classrooms.

CATHY'S STORY

INTRODUCTION

Cathy teaches at a school which opened during the first term of 1998. Its setting is in a newly developed urban area with low cost housing. Learners in Cathy's class live within walking distance from the school. Their ages range from eight to thirteen years.

She teaches a multigrade class with twenty-seven grade three students and nineteen grade four students. While the classroom instruction is mainly in English, Cathy allows Afrikaans (the learners' mother tongue) as a language of clarification for her learners.

Cathy is twenty-two years old, and started her teaching career in 1995. She is a temporary teacher, substituting for colleagues requiring leave. She has been in the current school since January 1998. She taught grade two for the first school term of 1998 and has since been placed in a multigrade class for grade three's and four's as mentioned earlier.

Cathy does express some discomfort teaching in a multigrade class. She finds difficulty in making right decisions with regards to 'what to teach' in a 'whole class' setting or as separate grades. She sees herself more as a Senior Primary teacher than a Junior Primary teacher.

VIEWS OF MATHEMATICS

Cathy has a contextual understanding of time. She starts her lesson with everyday human activities, by asking her learners to mention the times of regular happenings such as, what time they go to bed and the time they play, among others.

Cathy values building up a step-by-step procedure by introducing learners to the numbers and its position on the clock-face. She places importance on mastering the procedures for reading and recognising time. Learners are introduced to the function of the short and large hand including that of the second hand.

Once Cathy had established all the functions of the clock, she then went onto the interpretation of time. Her basic notation of time is something that goes 'round and round and round.

All the classroom activities were teacher directed and gave very little attention and space for learners' own methods to be shared. She works towards establishing the one way of doing and interpreting the idea of 'past' and 'to', linking it mainly to 'quarter past' and 'quarter to'. Cathy also introduces learners to the symbolic language of time which has its own symbols. For example,

"T: Write your numbers and make a dot.. T : Erase.. erase. First make your twelve and then your three and then your six and then fill in the numbers in between. "(lesson 2)

VIEWS ON TEACHING

Cathy introduces the concept of time to learners by getting them to identify regular happenings at home and school. For example,

"T: How do you know when the school starts?" "P: Eight o' clock. " "T: What time must you leave your house.. go to school?" "P: Seven o'clock " (lesson 1)

Cathy then refers the class to a plastic clock-face where they show the times of daily happenings. The different numbers on the clock-face is discussed such as,

"T: What is directly under the twelve? P : Six T : Six, and what will be on the right hand side of the watch? P : Nine T : ... on the left P : Three " (lesson 1)

Orientating learners' to the position of all twelve numbers. Cathy then moves on to discussing the small and large hands, until learners are familiar with them. Her explanation for the function of the two

hands is captured in the following dialogue with learners.

"T: When we have the short arm on any number and the long arm... is on the twelve.. it shows that is the hour. "(lesson 2)

The idea of the second hand is introduced by eliciting learners responses.

"T: What do we call that orange thing here? P : A arm. T : Yes... it's also an arm... but P: It goes round T: It goes around and around.. what does it show us? P : Seconds. " (lesson 2)

Cathy immediately moves to establishing connections between seconds and minutes.

"T: What comes before minutes? P : Seconds. T : There's sixty what in a minute? P : Sixty minutes. T : Sixty what in a minute? T : Who heard of seconds T : We have sixty seconds in a minute " (lesson 3)

By the dialogue one is able to deduce that the learners did not have a good grasp of either the minute or second hand. This lead Cathy to get learner's to repeat several times that there are sixty seconds in one minute, without necessarily understanding the concept seconds and minutes.

She then asks learner's

" T : Who heard how many hours in a day?" (lesson 1)

The learners' were able to give a quick response of twenty-four hours. However, Cathy never linked seconds and minutes to hours. Instead she immediately drew learners' attention to the fact that they have stated there are twenty-four hours in a day but that there were only twelve numbers on the clock-face.

"T: Where's the other twelve?" (lesson 1)

This discussion leads Cathy and her learner's to the concept of A.M. and P.M. The repetition of the set of numbers from 1 to 12 is placed in the context of regular happenings. For example,

"T: ... we at school from which hours? P : Eight o'clock.

T : What time do we go home? P : Half past one teacher. " (lesson 2)

She engages the whole class at this stage by counting on from eight o'clock to one o'clock, but the latter is read as thirteen instead. The learner's are expected to interchange twelve hours times and twenty-four hour's times. Cathy gets learners to practice in showing twenty-four hours times by identifying how long they play in the afternoons and what time they go sleep at night. This exercise also leads them to the recognition of hour intervals. For example, they are lead to count the amount of hours they play in the afternoons. They start off by counting on from two o'clock, in hour intervals up to seven o'clock.

Cathy introduces learners to the use of quarter-hours. This brings in the new idea of 'past' and 'to' (a quarter past two, a quarter to three). The idea of moving the hand a quarter of a turn is discussed, making sure that learners understand that after a quarter-turn the hand points to the three, which is quarter 'past'. Another quarter-turn is shown, this introduced learner's to the notion of equivalence of one half and two quarters.

"T: ... we have two quarters and two quarters is equal to... P: Half. T : A half. " (lesson 3)

Another quarter-turn is introduced which brings the minute hand to the nine and Cathy says to the learners that the clock shows 'three-quarters past two'. She then spends a good few minutes reinforcing the notion of 'quarter past' and 'quarter to'.

Cathy also discusses the idea of half-past the hour. She demonstrates, on the same large plastic clock-face, how the large hand makes a complete turn each hour. She insists that learners be aware that, in half an hour, the small hand moves halfway to the next numeral. When learners offer her an

incorrect response she elicits the help of those she knows will have the correct answer.

"T: Can somebody help her... between which two numbers.

Mark (lesson 2)

Cathy engages learners in further activities with a variety of clock-faces including that of Roman numerals.

DAPHNE'S STORY

INTRODUCTION

Daphne did not disclose her age. She teaches in a well-established school located in a middle income area. The school is more than seventy-five years old.

Daphne has a class enrolment of thirty-eight grade three's. The ages of her learners range from seven to eleven years. Their home language includes Xhosa, Afrikaans and English. Many of the learners have to travel to school from surrounding areas. Daphne has three learners who are repeating grade three for the second time.

Daphne is currently completing her last subject to gain her Senior Certificate. She is sensitive about her personal circumstances and makes reference to her unqualified status with the Education Department. She holds a Primary Teacher's Diploma and hopes to correct her unqualified status when completing her senior certificate examinations.

Daphne attended three different INSET courses in Mathematics, Basic Handwork and Handicraft about three years ago. More recently she attended a two day workshop on OBE arranged by the Western Cape Education Department.

VIEW OF MATHEMATICS

Daphne commences her lessons with some real world experience as a starting point. For example, she uses smarties as a context for counting in two's, three's and four's.

"T... lets count the smarties. P.- Two, four, six, etc. " (lesson 1)

The learners are asked to clap on every fourth number while counting, thus generating the four times table. She uses paper-folding for introducing the operator meaning of fractions. To establish halves, quarters and a whole, an apple is used.

She sees practical activities as an essential part of any mathematics lesson. Her assumption is that, by engaging learners with empirical work, they will learn faster. However, Daphne doesn't make any real connections between the different practical contexts used in the three lessons. This gives one the impression that the context doesn't necessarily have to make sense to learners, as long as it steers and directs them towards finding the end-product.

The practical activities in each lesson is not consolidated in any form of written work. Daphne did however, give learners a worksheet at the end of the third lesson observation. The worksheet dealt with renaming shapes such as squares, rectangles and triangles by halving it. It had no significant relevance to the notion of a fraction.

Daphne does allow learners to generate their own mathematics for example, sharing smarties amongst imaginary children.

"T: You have sixteen smarties.. share the sixteen smarties among four children... I want to see how you are going to do that.. ". (lesson 1)

Even though learner's generate the mathematics, it is through guided elicitation by Daphne. For example,

"T: ...share the twelve smarties among the four. P: I'm done teacher. T : What did you do? P: Shared it in half... and.. in half again... ". (lesson 1)

However, there is no discussion on the different methods employed by the learners. This is because Daphne is focused on the notion that there must be one final correct answer, and if the learner's method generates the correct answer, then the method must be acceptable.

This learner autonomy is not valued as Daphne only accepts one right answer. When a response is incorrect, she ignores the response and shifts to another learner whose hand is up, in the hope of getting the desired answer.

VIEWS OF TEACHING

Each of the three lesson observations done in Daphne's class had a different focus. The first lesson observation dealt with revision, counting in two's, three's and four's, then introducing fractions as the concept of sharing. She makes an abrupt change to paper folding into quarters and comparing fractions. The final episode in the first lesson introduces the concept of more than one whole.

In these lessons, it is the use of different representations that stands out. The teacher starts off by making links to earlier work on the three times table. They are then told to count and clap on every fourth number. Learners then count smarties given to them by the teacher one-by-one, in two's and three's. Learners' are then instructed by the teacher to share their smarties amongst four imaginary children. They are asked to explain how they did it, but not much is offered. When learners are not able to give correct answers, they are referred to the smarties to perform the operation. Daphne makes a shift to working with a rectangle folded in half and half again. This introduces the notion of 'quarter' which Daphne readily explains to the learners. An activity sheet is given to them where they work on the relation between quarters, halves and wholes.

Daphne attempts an integrated approach in this lesson by incorporating ideas on using senses, colours, taste and language. However, she doesn't succeed in making the links clear. This was further complicated, in that there was a lot of mathematical content in the first lesson, but it is not made clear to learner's how it is linked. She progresses with care but occasionally makes a leap that learner's clearly have problems following. (eg She shifts from having learner's work on sharing and then jumping to having them name a shape, she has passed out to everyone.) Daphne uses a technique of going over answers in detail with the class, where they have to chorus answer the four and five times, as a form of reinforcement.

In more than one instance Daphne seemingly goes from the more difficult to the easier. For example, learners are asked to divide sixteen amongst four, then twelve amongst four, and then she asks them orally only how much each child gets if you share four among four. A similar thing happens later when Daphne first asks how many quarters in a half and in a whole with clear reference to a drawing. She moves on to ask how many quarters in two wholes. Then, she makes a double shift, asking learners how many wholes are cut up if one ends up with eight quarters. She shifts abruptly in passing out a shape. Learners have to name the shape and properties. They must fold and shade in $\frac{2}{4}$. There is a

clear explanation of notation (why a quarter is written as one over four) by the teacher.

The lesson ends with no main basic idea or purpose stated. What stands out most in Daphne's lessons is that she has put extra effort into her teaching to accommodate the researcher. For one, she modelled her lessons on the grade one, Western Cape Education Department's two day training programme. Where many ideas were given to teachers in attendance but it lacked focus and reflection, which we also found in Daphne's lesson. Her attitude also comes out in conversation with one of the researchers after the lesson observations.

"T: Did you..... Did I meet up with..... what you expected " (lesson 3)

She does make the point that "using this method"(OBE) cannot be used everyday because it requires more resources. This gives another indication that this is not her everyday lesson.

DENISE'S STORY

INTRODUCTION

Denise teaches at the same school as Greta. The school opened in 1998 with the required furniture in place but no teaching supplies. At the time of the PEI research project they were short of six teachers and some auxiliary staff. The total school enrolment is eight hundred and seventy-five learners.

The principal described the learner community as being mostly from the Eastern Cape (Transkei and Ciskei) with little urban experience. Denise is thirty-two years old and has been teaching for the past two years. She completed her teacher training at one of our local colleges in 1996. She holds a Junior Primary Teacher's Diploma.

There is an enrolment of one hundred and forty-one grade three's, of which Denise has forty-eight learners. Their ages range from eight to twelve years. The learners' language of instruction is both Xhosa and English. However, the language of explanation is Xhosa. She has ten mathematics periods per week. Denise keeps a well disciplined classroom.

VIEW OF MATHEMATICS

Throughout the three lessons Denise taught, she focuses on establishing a procedure to deal with multiplication with two digit numbers. She wants learners to understand that, if they have the procedural knowledge for multiplying two-digits, they shouldn't have a problem in achieving an answer. This leads to the teaching of a standard algorithm resulting in a step-by-step approach. When doing the algorithm for multiplication, in her view there is only one correct way, the vertical form. Her main goal is, for the learner's to master the algorithm for multiplying with two-digit numbers.

Denise engages learners in a process of establishing the steps for the standard algorithm for long multiplication under her control and guidance. She uses the concept of place value to lead the learner's onto the correct sequence. For example, in her example of 17×12 , the sequence is 7×2 ; 7×1 ; 1×1 . When the teacher-directed explanation of one example proves to be lacking the desired results, she simply goes on to explain the next example. The underlying belief seems to be that repetition will lead to better comprehension.

The classwork exercises done by the learners, were deemed complete only when all groups had a correct answer. Errors were corrected within the groups where learners sat. If an error persisted,

Denise would then throw it open to the whole class, building on the elicited responses from learners.

"T.. Where do we put 14? P: That side. T... then what do we do next? P: Underneath 2. T.. Where do we go from there then?" (lesson 2)

In the lessons' observed, Denise introduces the long multiplication content without any context. No attempt was made to discuss the purpose or relevance of multiplication with two-digit numbers. Denise did not encourage or allow individual methods even though she had learner's working independently in groups. Routine steps and answers were being drilled.

Denise establishes an important link throughout the work on multiplication, in that learners are constantly reminded of the link between repeated addition and multiplication. She insists from the start they should understand that, 2×3 is another way of thinking about $3 + 3$. They should know that any multiplication can be done by repeated addition. For example, the answer to 99×3 can be found by the addition of three, ninety-nines. When the learners know the multiplication facts and the use of place-value is understood, she stresses the point that the answers are found quicker by multiplication.

Denise engages learners in double recording, emphasising the link between addition and multiplication of two-digit numbers. The process of double recording clearly benefits some of the less able learners. However, the more able learner's who used the shorter method, Denise discourages them in doing so. Her teaching process is driven by looking and finding correct answers to the problems, for example,

"T: what is your answer?" "T: Add and get the answer." (lesson 3)

VIEW OF TEACHING

Denise uses the basic idea of multiplication as repeated addition to explain the meaning of multiplication. She tells learners that multiplication is the same as addition for example, (2×3) or $(3 + 3)$. She elicits individual learners images on the meaning of multiplication as repeated addition on numerous occasions.

A great deal of her teaching time is spent on repeating the steps for doing the long multiplication algorithm, at which time she encourages chorused responses from learner's. Denise uses repetition in two ways, sometimes she uses it to reinforce the steps or to urge learners to reconsider what they have said. In a few instances in her lessons she asked individual learners for justification in response to an answer given ("Is she correct?", "is it true?"). She feels strongly that all her learners learn the same algorithms. This is driven by the motivation that she wants them to see "how easy it is" if they follow the set procedure correctly. Denise provides little if any explanation why this procedure is important for example, (12 is put aside without any explanation). Denise's teaching approach can be summarised in the following way; set procedures were introduced, then practiced by learners, reinforced by the teacher and then repeated for sense-making.

Denise has her learners sitting in eight groups of six, having the same academic ability in mathematics. However, she gives all learners the same problems to solve with an expectation for the same procedure to be followed. It appears that she uses this particular group setting mainly to get learners to work together, in correcting each other in the application of the algorithm. She has a rigid approach to what she will allow in her classroom. She displays a strong belief that learners learn and understand best when they follow a set procedure which results in rote learning. This is confirmed in that, her explanations are limited to the steps in the procedure. Denise's teaching style can be described as

transmission within a cooperative setting.

Denise does encourage some learner reflection but this is restricted to remembering what the next step is in a set procedure. This is illustrated by the type of questions she asks,

"T.. What number do you think you can multiply by now? Tina, what do you think?, Can anyone help?". (lesson 2)

Her teaching is further characterised by shifting from one learner to another for correct responses and ignores those learners who gives an incorrect answer. An example in her dialogue is,

"T: Who else wants to try? That is wrong. " (lesson 2)

In contrast, Denise's response to correct answers were greeted with "very good", "Let us give applause!".

In establishing procedural knowledge in multiplying two-digit numbers learners are showered with questions such as ("where do we start?", "What do we do?", "What is the answer?"). In the first lesson observation the set procedure is established and reinforced in the remaining observations.

FREDAS STORY

INTRODUCTION

Freda teaches at a school situated in a well established urban area. The community consists of private home owners. On the periphery of the area there is a very old housing township which accommodates many displaced families from the then District Six in Cape Town.

Freda has thirty-six learners in her class and counts herself lucky to have such a low enrolment. She has a multicultural classroom where the learners collectively have more than one home language. Freda doesn't have any difficulties with language, as all her learners started in grade one at her school, so they have the necessary vocabulary.

Freda has been teaching for the past fifteen years. She spent fourteen years at her current school and one year at a neighbouring school. She taught grade one's and two's for most of her teaching experience and has only been teaching grade three since this year (1998).

Freda has been married for the past fourteen years. She has two girls aged eleven and nine years old, who attends the school at which she teaches. Freda lives in close proximity to the school, in a neighbouring area.

She enjoys teaching, and takes a lot of pride in what she does. She doesn't use any conventional textbook but gathers materials and ideas from different sources e.g. magazines, newspapers, television and so forth. She does however mention that, they don't really have an appropriate budget to purchase new textbooks.

She expressed some discomfort with a researcher observing her teaching. She feels that, she is not herself because she loves to crack jokes and clown about at times, but when an adult is in her classroom, she tends to be a bit inhibited. Freda also expressed a feeling of inadequacy. She says that it stems from the fact that six to eight years ago she knew exactly how to go about things in her classroom. Now, she is unsure of what is expected with regards to what approach is right and what

isn't right. She says that at times when she is halfway through a lesson, she decides to abort the approach she is using because learners don't respond to it. This makes her apprehensive to have an observer in the class.

VIEWS OF MATHEMATICS

The learning context is important to Freda. She places emphasis on learner's real-life experiences and draws on it to build up an understanding for working with fractions. For example, the context of money is used to show how a big amount can be made up with a multiple of smaller monetary units. An investigative context such as paper-folding is used to introduce the meaning of a third. She uses lollies for the operator meaning of fractions. From the context she attempts to develop the mathematics. Drawing on the assumption, that the richness of the context in itself is sufficient to develop the mathematics she wishes her class to learn.

The nature of the context used in Freda's lessons draws on greater learner interactivity with the materials and therefore the process becomes a highly interactive one. For example,

"T.. Pick up your square. P.- Got it T.. Divide... that square... into thirds... three equal parts RI didn't do it right teacher. T: Okay try again, T.- I want thirds... discover his own... way." (lesson 1)

Learners' own methods are also important to Freda inspire of her controlling and directing the learning process, which is to direct learners to the end-product.

Freda introduces the procedure of applying the operator meaning of a fraction. This is done by eliciting learners' responses and then building on them step-by-step, developing a procedure to enable learner's to grasp and achieve a product with the fraction as operator. For example,

"T.. When we want to find a quarter of a number... What do we do?... What are the different things that we can do? ". She also repeats the responses which learners feedback to her, seeking justification for the responses. She asks the class, "T· Haw many TEN RAND NOTES do I get... my FIFTYRAND NOTE?... R Five. T.. Is he correct...?"

R Yes, teacher. T.- Why? How do you know he's correct? ... ". (lesson 3)

She illuminates aspects of the lesson which she wants learners to make-sense of and justify.

She also introduces them to the procedure of identifying and writing the symbolic notation of a fraction, such as thirds, quarters and halves.

Freda teaches for conceptual understanding. She draws on the different concepts covered in the three lessons which dealt with multiplication, geometry, division and money. For example,

"T : A rectangle.. how many sides does that have? P : Four. T :... fold that rectangle.. into THREE EQUAL parts... " (lesson 1)

VIEWS OF TEACHING

Freda makes connections between real life experiences by incorporating money and lollies as a context for learning place-value and fractions. She engages learners with geometric shapes such as, squares and rectangles, which they have to sub-divide into halves, thirds and quarters, in order to recognise fractions in paper-folding. She takes the liberty in asking learners to name the properties of the geometrical shapes, which they eagerly respond to.

The "student activity" in Freda's lesson is characterised by establishing procedures for example, writing down a fraction and giving the meaning of the fractional parts. The three lessons had many different representations of a fraction, which were woven together to give meaning to a fraction. Freda used the question and answer technique in a positive way attracting a large percentage of learner participation in the lessons. The less abled learner had no hesitation to offer an answer, even if it was incorrect. She makes learners' feel that their input is valued. If they do give an incorrect answer, Freda embarks on a course of eliciting their individual images in order to correct their understanding. Freda repeated the same content and followed the same procedure in all three lessons, teaching three different groups. She got learners to count in two's, three's, and four's. Where they had difficulty in counting the multiples, Freda encouraged them to count on, using their fingers.

"T.. One hundred and five ... carry on ... plus three more Count on... use your fingers ... count on".
(lesson 2)

This exercise was done to reinforce their previous knowledge on multiplication. She wants learners to later make the connection that, a number like fifty is obtained from multiples of smaller numbers, including money. Freda gives learners money problems to solve, laying the ground for a fraction as operator. For example,

"T.. How many ONE RAND CONS... if I go to the bankteller... with twenty rand? P.. Twenty",
"Five rand... how many fifty cent pieces?" (lesson 2)

Freda's basic idea of a fraction is that of operator. She engages learners in the various exercises asking them, how many fifty cent coins in two rand; how many tens and units there are in ninety-seven and so forth.

Freda's lesson takes an abrupt shift where she reinforces composing and decomposing of numbers. She writes the number four hundred and twenty-four on the board and engages learners in the following dialogue.

"T.. Can you... break up the number?... A Four hundreds and... seven tens and four units': (lesson 1)

(Teacher refers to units as loose ones). Freda repeats this exercise a few times using different examples, in the hope that the learners will understand the underlying concepts of place-value.

Freda establishes a pattern where she repeats the elicited responses which learners' feedback to her, in an effort to use it as a sense-making technique. For example,

"T. What is the third of fifteen? P.. Five T. Five So what did you do in your mind, what are you doing? P. Teacher, I counted in ... fives ". (lesson 2)

GRETAL'S STORY

INTRODUCTION

Gretal is a twenty-six year old teacher, expecting her second baby, and teaches at the same school as Denise. Gretal holds a Junior Primary Teacher's Diploma.

The school opened recently and was unfinished at the time of the classroom observations. It is located in a relatively small township approximately forty kilometres from the University of the Western Cape. The school opened at the beginning of the school year in 1998.

Gretal has forty-three learners' in her class. Their ages range from eight to twelve years. Her classroom instruction is mainly done in the learner's home language, which is Xhosa. The grade three's have a total of ten mathematics periods per week, which is scheduled between 08H30 and 11H00 each day.

Gretal believes in rewarding successful answers. She has developed a system whereby she offers learner's incentives such as sweets and a hand clap. This is her first year teaching grade three's, as she taught grade one's previously.

VIEW OF MATHEMATICS

Gretal clearly values procedural knowledge. She engages learners in activities where they are taught the procedures for finding equal parts in fractions, the procedure for adding or subtracting fractions and so forth. A lot of emphasis is placed on knowing the procedures for writing fractions with symbols, in pictures and words. For example,

"T..16 divided by 4 oranges... show as by picture Deon. " (lesson 3)

The explanations offered by Gretal to her learner's are rule-based. For example, she tells them that, if the denominators are the same, one only has to add or subtract the numerators. She sets them up to learn by using steps to come together to add or subtract, resulting in cumulative knowledge build-up, to establish a single rule. She uses this developmental approach to derive at the products she wants learner's to achieve. She introduces the terms numerator and denominator as words to know, and does not engage learner's in its meaning.

Gretal provides learners with very drawn-out explanations. For example, she spends much time explaining the concept of a whole fraction, making the connection to something that is full. "When a square is full in maths, it is called a whole". Gretal elicits responses from her learner's on which she builds the next procedure to be learnt.

"T. ... When you cut a piece from ... the whole, what is it?" "P. A fraction. T. Yes, a fraction ... are pieces. " (lesson 1)

Although Gretal favours a procedural approach, she often introduces investigatory approaches. For example, she uses paper-folding to introduce the equivalence of a whole, two halves and four quarters. In another instance she refers to real world objects like oranges, and chocolates to introduce the operator meaning of fractions. For example,

"T. ... two oranges is equal to how many quarters? P.- Eight T. 16 quarters is equal to how many oranges?" (lesson 2)

The manipulatives are not used to foster conceptual understanding, rather to establish procedures. Gretal's views on mathematics can be described as favouring a mechanistic build-up, this is interspersed with empirical methods.

VIEW OF TEACHING

Gretal has two basic ideas for fractions - a part of a whole and as equal parts of a collection of objects (operator meaning). When using the basic notion of part-whole, she sees fractions and pieces as the same thing. The representations offered to her learner's is that a fraction starts from a whole and a whole is something full. Her idea of a half is that both sides are equal and that the two equal parts form a whole. A quarter is introduced as a half of a half (four equal parts). When using the operator meaning, she uses the notion of division as sharing. For example, sharing a slab of chocolate with four children.

Gretal emphasizes the formal characteristics of adding and subtracting fractions. Learners are coaxed

into getting the ONE way of doing the algorithm. An instrumental understanding of fractions with regards to what must be done, is communicated to the learner's.

She presents the terms numerator as the top number and the denominator as the bottom number. Another representation of colouring in blocks is used to understand the function of denominator and numerator. Learners are asked to count the number of blocks the whole is divided into, this is reinforced to represent the denominator. They have to count the number of shaded blocks and they are told that this represents the numerator. She introduces the names numerator and denominator as terms which learners have to master when doing addition and subtraction of fractions. The relational understanding of its meaning and function is ignored. To her learning is, imitating those procedures learnt.

Gretal also links fractions to whole geometric shapes eg. squares, circles, triangles. Learners' everyday experiences are also connected to finding the stated fraction of a whole apple, orange, chocolate and pink and white cats. Connections are made to the minute hand when it is half and quarter time, a very poor connection though.

She links counting in fours to multiplication and division. Gretal builds on links made to previous lessons and learner's in-school experiences such as ("Is it a square?", "There are other shapes as well, we did them in Sub A.") and repetition of earlier work.

Most of Gretal's lessons start off with a practical activity, she then engages learners in the process of solving the problem, and it is through this process that Gretal hopes to achieve success with her learner's in understanding the underlying concepts.

The content is organised around learning procedures for addition and subtraction of fractions. Gretal abruptly shifts to greater than, smaller than and equal to in fractions. She puts a lot of emphasis on learner's getting to know and understand the procedure for addition and subtraction of fractions. Therefore, most of her energies are spent on sense-making, recall and repetition of the procedure. In sense-making she wants her learners to know that fractions are pieces of a whole, that two halves is two over two in written notation. This view is consistent with the explanation which she offers to her learner's right at the beginning of her first lesson. The function of the numerator, denominator, establishing a rule for adding and subtracting fractions serves as the main goals for her three lessons observed.

The question and answer technique which Gretal uses is mainly for recall and finding out if learner's know the next procedure. Gretal states a fact and then seeks learner agreement. For example, all learner responses are repeated by the teacher to confirm a right answer or response. She uses the same 'repeating' technique to question an incorrect answer.

Gretal doesn't make it explicit to learner's that they are adding or subtracting numerators of the same kind when she asks them to add $\frac{1}{4} + \frac{1}{4}$. Then she gives them a subtraction sum where the numerators are greater than one. For example, $\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$. An important aspect which is missing in all of Gretal's lessons is the element of inspection. The answers found by learners is stated as a matter of fact. With the missing element of inspection, Gretal gives her learners the understanding that to add or subtract a fraction one has to only apply the rule to get the answer. Too often rules are introduced without learners ever understanding the first ideas of fractions. No learner reflection is encouraged in any of Gretal's lessons.

HAZEL'S STORY

INTRODUCTION

Hazel is twenty-three years old. She teaches the only English grade three class in the school, while the rest of the tuition in the school is done in Afrikaans. Hazel is also fluent in Xhosa. She holds a three year Diploma in Education for Senior Primary and is unmarried. She completed matric mathematics. The learners' ages in her class range from eight to ten years.

Hazel's school has a total enrolment of one hundred and fifty grade three's, of which ten learners are repeating grade three for the second time. The school is situated in one of many newly developed subsidized housing projects along one of our national roads, The learners come from low-income families, where unemployment is very prevalent.

For the duration of the PEI research project I was introduced to four different principals. The school is experiencing real serious difficulties. The morale of the teachers at the school is low. On one of the visits to the school nine teachers were absent. The school was in total disarray, as some staff members present refused to supervise some of the classes that had no teacher. The school experiences a high staff absenteeism rate on a daily basis. They also have their fair measure of burglaries which also has a negative impact on the everyday functioning of the school. The school building is well constructed and modern. The facilities available are adequate but the school is unable to provide appropriate security to combat the crime they are experiencing.

Hazel enjoyed having her lessons observed. She expressed that she always wanted someone, other than a staff member, to observe her teaching and give her an objective opinion of her classroom practice. This was the first time someone observed her teaching since qualifying as a teacher. Hazel enjoys teaching. She sees each day as a challenge.

VIEWS OF MATHEMATICS

Hazel starts off her lessons with practical activities. She uses apples and oranges to introduce the concept of a fraction. However, the practical activities is not a means to an end, as she shifts the learner's focus very quickly to the board, where she writes down the fractional notation for the cut apples or oranges. The practical activity ceases immediately and learners are asked to read and repeat the symbolic representation of the fraction notation. The practical activities instigated by Hazel have a pretense of realism, but she does not exploit or extract the mathematics in it.

The assumption which Hazel makes in using this view of hands-on practical activities is that, she perceives that it is through empirical work, her learners will learn concepts and procedures faster. This wasn't always the case, as many procedures were stated as a matter of fact, instead of as a result of inspection from the practical work learners were engaging in.

The learners were given space to discover concepts and procedures for themselves, but it did not bear much fruit, as Hazel tightly controlled the kinds of answers she expected from her class. So she failed to elicit their responses. When a learner responds incorrectly she automatically shifts to another learner in search of the correct answer, without correcting the previous response. The learners responses were clearly limited and directed towards the end product.

Hazel's view of a fraction is that, it is a part of a whole. This view is repeated many times when she deals with different aspects in fractions. For example,

"T: One apple.. divide between... five children. One whole orange.. cut into two pieces. One apple cut ... into four pieces. " (lesson 3)

As soon as learners give her a correct answer, she draws their attention to the board and writes the symbolic notation for the fractions.

VIEWS OF TEACHING

Hazel introduces learners to the concept of having no remainders when dividing in the context of fractions.

"T. What is five divided by two people ? P Is equal to two remain one P-who else.-reached another answer on that one.. . P. Two and a half. T. Excellent .. where does the half come from..." (lesson 1)

At first the learners struggled to grasp what Hazel was trying to share with them. But then she changed the context and based it on a practical problem. They were then able to better understand what was expected of them. For example,

"T ...let's call this oranges... divide the five oranges between two people " (lesson 1)

Learners realized that there could be no remainders as they needed to give equal parts to the two people. With Hazel placing this problem in a practical context, learners immediately knew what type of division they needed to engage in. This enabled them to give the answer as five divided by two, equals two and a half, instead of five divided by two, equals two remainder one.

Hazel develops and builds her lessons on elicited responses from her learners, which she controls. The idea of notation of a fraction is introduced, basing it on practical activities. Learners cut apples and oranges into two halves, and she introduces the notation for one-half and writes $1/2$ on the board. Hazel repeats the process of learners cutting apples and oranges for the other notations such as, quarters ($1/4$), thirds ($1/3$), and fifths ($1/5$), writing down the notation for each fraction. She gets her learners to chorus count the equal parts to make sure that there are for example, four parts when referring to quarters. Hazel then holds up one of the four equal parts and says,

"T.. This is one-quarter of an apple " (lesson 1)

She repeats this for each of the other fractional parts. She also holds up all four parts and says,

"T.. Four quarters make a whole " (Lesson 1)

She writes the notation $1/a$ on the board. Hazel's basic idea of a fraction notation can be summed up by what she tells her class,

"T ... a half says... one apple.. the one above the line.. it's cut into two pieces... that is the two underneath the line.. that is how a fraction looks. " (lesson 2)

Hazel has difficulty in getting learners to obtain thirds by cutting their apples and oranges into equal parts. This context did not provide them with the ability to engage in exact measurements, so much was left to the learners' own intuition of what was a fair equal part.

She then introduces the words numerator and denominator, telling them that they would have to remember these words for the rest of their lives. Her definition for the numerator is that, it is the number on top of the line of a fraction, and the denominator is the number underneath the line. The words numerator and denominator is reinforced by repetition. Hazel asks learners to give arbitrary numbers which can be written as a fraction, identifying which is the numerator and the denominator.

"T.. ... one.. can give me the numerator,... one can give me the denominator. .. put it together and you have your fraction ". (lesson 1)

Learners are also asked to close their eyes and repeat the words "numerators" and "denominators".

Hazel's basic idea of a fraction is that, it is a part of a whole. Fractions are gotten by division.

"T.....I must divide it between two. " (lesson 1)

She asks learners to create their own fraction problems by division. One of the learners responds saying,

" P. Five apples... divide by three children ". (lesson 1) The teacher responds to this saying,

"T. Five apples... divide by three children doesn't sound interesting". (lesson 1)

She dismisses this problem posed by the learner with no further explanation or justification, as to why she is saying that it is not an interesting problem.

Hazel introduces addition of fractions by asking learners, how one apple can be put together. She encourages them to think of a sum and therefore establishes a 'rule' for doing addition of fractions. The 'rule' for adding two halves was very quickly established by learner's. Hazel then led them to add quarters and thirds using the same procedure. They did this with relative ease.

The notion of a common denominator is introduced at this stage by using small numbers. Her basic idea of a common denominator is

"T. ... a denominator that is COMMON it CAN divide into. .. a half... and a quarter. It is common to both..." (lesson 3)

Hazel then leads her learners through a step-by-step process to introduce learners to the formal algorithm of finding the common denominator. Telling them that it is found by multiplying the denominator of the first fraction by that of the second fraction. The fractions are written separately as $1/4 + 1/2 = 2/8 + 4/8$.

PAT'S STORY

INTRODUCTION

Pat, is sixty years old. She holds a Junior Primary Teacher's Certificate. She is a mother of seven children, all are married. Her eldest granddaughter is twenty years old and matriculated last year. Another grandchild matriculates the end of this year (1998). She took the 'package' at the beginning of 1995. She had a year's break from classroom teaching but in 1996, this small Christian school invited her to join the teaching staff where she has been for the past three years.

The school is situated in a middle income area in one of the suburbs which is approximately fourteen kilometers from the University of the Western Cape. It used to be a fruit plantation in much earlier days. Pat teaches a multigrade classroom with sixteen grade one's and twelve grade three's. She finds this a very awkward combination because the syllabi for the two grades are completely different. Their ages range from eight to nine years. The learners' home language includes English, Afrikaans and Xhosa, however, the language of instruction is English.

Most of the learners attending the school lives far from the school, some travel up to eighty kilometers per day one way. There is a special school bus which assists learners to travel to school while others are brought by their parents or teachers.

Pat uses a modular system. There are eight modules for grade three's for the year. At the time of the

research she was doing module three. The grade one's occupy the front tables while the grade three's sit at the tables at the back of the classroom. The grade one's leave at 13h15 and the grade three's leave at 14h15, so Pat is able to do any 'catch up' work with them during this hour, which she is very happy for.

VIEW OF MATHEMATICS

Pat starts her lesson with mental arithmetic. Learners count in two's and three's. They draw on their multiplication and division facts in order to say, how many two's in six, eight and so forth. The dominant view which emerges from the three lessons observed in Pat's classroom is that she engages learners in a step-by-step procedure for multiplication, decomposition of numbers, division by four and making a clock-face. The three lessons dealt with different issues in mathematics but could possibly be summed up under one heading viz., Number facts.

Pat first engages learners in mastering the procedures they need to build on, enabling them to complete the classroom exercises 'successfully'. For example, she wants learners to create story sums with multiples of five. So using the hundreds number chart, she gets them to first count in five's, while pointing to every fifth number. Here she stresses the ordinal aspects of a number. She repeats this process again, then moves them to the next activity, chorus reading the five times table.

Once Pat is satisfied that learners have a good grasp of the five times table, she asks them to give her a story sum with the multiples of five. She shows learners an example of the kind of response she expects from them.

"T: Five boxes, each box has four candles. How many candles?" (lesson 1)

The sum is therefore $5 \times 4 = 20$. Multiplication is viewed as repeated addition by Pat. She tells learner's that 5×4 can also be viewed as $4 + 4 + 4 + 4 + 4$, but states that multiplication is seen as an easier way, and therefore only elicits multiplication sums from the learner's.

Pat elicits and values learners' responses but she carefully directs them to the end product. This is confirmed by the many times she asks them "so what is the answer", after they have given her the procedure for solving a problem. For example,

"T: ... who's very clever... give me the correct answers T : ... put your answer there... $2 \times 4 = 8$. " (lesson I)

To enable learners to calculate quickly and correctly, Pat's view is that they must know the number facts. Where, they are able to give correct answers, for example, $72 - 10$, $6 - 3$, 40×2 , $70 + 11$. These number facts were built up over a period of time with lots of repeated exercises and memorizing.

VIEW OF TEACIHNG

Pat's objective for the three lessons is for learners to understand and learn number facts. She does this by building up the facts in many different ways.

In the first lesson, Pat takes learners through a procedure of counting in two's, three's and five's. She then makes them aware of the link that exists between multiplication and division. For example, $5 \times 4 = 20$ can also be shown as $4 + 4 + 4 + 4 + 4 = 20$. She gives this example to learners in the hope that they would make the connection. Pat also makes them aware of the link that exists between

multiplication and division. She draws their attention to the example of (4×4 and $16 - 4$). This was later linked to sharing sweets among four children. Pat wants her learners to move freely from multiplication facts to a corresponding division fact as shown above. This procedure does assist learners in building up an understanding of the commutative property of multiplication (e.g. $5 \times 8 = 8 \times 5$) used later in the lesson.

When Pat feels that learners have covered the multiplication facts of the five times table 'successfully', she introduces the idea of number 'stories'. For example, Pat takes $5 \times 4 = 20$ and tells them that she has "T. five boxes, each box has four candles. How many candles?" (lesson 1)

After she has given learners an example of what she expects, they are then asked to suggest their own 'stories'. After much prompting one of the learner's suggests

"T.-I have.. seven trees... and on each tree I have FIVE apples. How many apples altogether. "
(lesson 1)

Pat also introduces the first ideas of place-value. She engages learners in activities where they have to give her a two-digit number, identifying the tens plus the units. One learner says,

"76 is um it.. seven tens teacher plus six.. loose ones " (lesson 1)

The exercise is repeated with many different numbers in the hope that learners will understand the idea of place-value.

The place-value exercises lead learners to the next set of ideas on the composition and decomposition of numbers. Pat does this in a number of ways, where learners have to count on by adding 11 to 65 to make 76, add 10 to 66 to make 76. This included activities in finding the difference between 100 and 24 ($100 - 24 = 76$) and the difference between 99 and 11 ($99 - 11 = 89$). She provided them with many different numbers to deal with the idea of decomposition of numbers, to strengthen their idea of place-value.

In the final lesson she tasks learners to plan a birthday party helping them to build up a knowledge of facts in a pleasurable way. This indicates that she wants them to enjoy the activities as far as possible. With the assumption that if they enjoy the activities, instead of being under stress, they are more likely to learn and apply the facts.

SALLY'S STORY

INTRODUCTION

Sally is forty-four years old. She has been teaching since 1973. She taught in Ciskei for the first six years, then she moved to Cape Town. She has been teaching at the same school for the past seventeen years. She holds a Primary Teacher's Certificate and a Junior Primary Teacher's Diploma. Sally's home language is Xhosa.

Sally's school is in old established township, in a relatively developed urban area. Sally has thirty-nine learners of which ten are males and twenty-nine females. Their ages range from eight to eleven years. During the three classroom observations the average class attendance was twenty learners. The teacher's reason for this was that a number of her learners and their families were involved in relocating to another newly developed housing township along one of our national roads.

Sally's language of instruction is English but she uses Xhosa as the language of explanation in her classroom. She has seven mathematics periods per week. Sally uses the mathematics textbooks to guide her teaching.

VIEW OF MATHEMATICS

Sally starts her lesson with mental arithmetic. She gets learners to count in two's and three's. They have to identify how many groups of two in ten and other even numbers.

Sally makes learners understand that division has two interpretations, viz., grouping and sharing by division. Yet she only uses the idea of grouping in these lessons, as if it is the only correct understanding of division.

She also engages learners in doing many different activities using real world objectives. For example, finger counting, using counters, dividing slices of bread and bananas. However, Sally made no real connections between the different practical activities. It had no direct relevance on what she was trying to achieve, which was to establish the notion of an algorithm, and the practical activity could therefore be seen as mindless activities.

Sally provided her learners with the opportunity to talk in small groups, which gives learner's the impression that mathematics is something to be discovered. However, Sally tightly controlled this process, keeping them on task in establishing a set standard algorithm.

VIEW OF TEACHING

Although Sally is aware of the two interpretations of division, viz., grouping and sharing she doesn't use it. She only elicits learners' input on one, grouping. Only this one meaning is constantly conveyed through the examples she uses. Learners are asked to group after chanting and chorsing, counting in two's and three's ("how many two's are there in ten"). She gives learners a paper problem on grouping, laying the ground for division as grouping. At this time she reminds them that division is grouping. Sally provides learners with further activities for reinforcement by giving an everyday example, where they have to divide three bananas amongst three boys, and six slices of bread amongst three boys. She then proceeds to replace the bananas with the counters. Her justification to her learners for this action was, that they may not always have bananas available to work with.

She introduces the algorithm for long division by asking the learners to think in terms of grouping:

"T: So we are going to ask this question... How many times... how many groups of two are there in twelve... how many groups of two. How many groups of two. " (Lesson 1)

She proceeds by introducing the division symbol, where she again makes mention to learners that, division means grouping and sharing. But until then, the class has only worked with grouping. The teacher wants learners to think of division as both grouping and sharing, but they are left to make these connections and interpretations. So far all classroom activities are mainly teacher directed. Sally is doing all the explaining, and the tasks she gives learners are all of such a nature that they can be done by following a routine already presented in the class.

Sally does however, build on learner responses in order to present the next set of instructions.

"T.. Okay now we have divided, now the whole, the one big piece of paper... into four parts right... into how many? P.. Four parts" (Lesson 2)

She repeats learner responses a number of times.

"P. I have four equal parts T.- Four equal parts How many parts do you have? P.- Four equal parts" (lesson 2)

This is perhaps done to ensure that individual learners "take in" what is being done, as it is a prerequisite for the next explanation. She makes learners memorize the steps for the algorithm as she goes through the different processes.

Her teaching process is dominated by applying known/taught procedures to each new number, and engaging learners in a lot of reinforcement work. She provides lengthy explanations with emphasis on certain words,

"T.. Whole is ONE okay ". "T. • Okay one is a whole. " "T.. One whole part has been divided into Two Equal parts " " T.. Two equal parts ... called half each part " (lesson 2)

which could be terms which she wants her learners to remember.

CHAPTER 5: SUMMARY AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter we provide a summary of the teachers' views of mathematics and of teaching. The teachers' views are categorized into three groups - transmission, empirical and connected. These categories are not water tight compartments and there is overlapping of teachers' views into two or more categories. However, it is possible to use the categories for the views that are predominantly held by a specific teacher.

In summarizing the teachers' views, we begin with a description of the kind of views that we believe are required for effective teaching practice in OBE. The identification of these criteria serve the purpose of a benchmark against which the teacher's existing views on mathematics and teaching can be evaluated.

5.2 THE TEACHERS' VIEWS OF MATHEMATICS

In chapter 2 we argued that OBE calls for a pluralistic view of mathematics. Mathematics should not be seen in terms of a singular perspective, like end-products or be occupied with processes. Rather, the teacher's pluralist perspective should focus on the context, the processes and also the products of mathematics.

A further description of the kind of mathematical view that is required for OBE, is found in the definition of mathematics which appears in all the Phase documents:

"Mathematics is the construction of knowledge that deals with quantitative and qualitative relationships of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking, etc. in an attempt to understand the world and make sense of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction. [Department of National Education 1997].

It is our contention that this definition as well as the specific outcomes and assessment criteria call for very specific processes, a very specific context and specific end-products. The identification of this context, processes and products provide us with benchmark against which the participants' views of mathematics can be measured.

The reference to mathematical understanding as contested, clearly implies a fallibilist view of mathematical knowledge. Some of the mathematical processes associated with this view is the formulation of mathematical ideas, as well as discussion, testing and revising these ideas. The definition also calls for a specific context in its reference to mathematics as a human activity. This points towards activities that arise from human need and curiosity and which lead to the solution of problems from the environment. The processes associated with the emphasis on this context is problem-solving, logical thinking, and so forth. As a result of the solution of these problems, mathematical products (concepts, procedures, etc.) evolve. The solutions to these problems are expressed in symbolic language.

Most of the participants in this study hold mathematical views that can be described as absolutist with the teacher being the final arbiter on what is acceptable or not. Within this absolutistic view, three different strands can be identified. The transmission absolutist holds the view that mathematics is a set of rules and correct procedures that the learners must master. These rules and procedures are taught in a decontextualized way. No reference is made of the functional nature of the rules or procedures in real life, other than that it is needed to do mathematics. The end product is an algorithm and the process is usually imitation and repetition of what the teacher has done. The conversation pattern in this class entirely emanates from the teacher with learners' participating only in response to a direct question from the teacher.

A second strand in the teachers' views on mathematics is what we describe as empirical absolutism. Here the teacher allows multiple answers and use multiple ways of getting to the answer. The learners are allowed some space for discovery but the process is still tightly controlled by the teacher. Two outstanding features of this approach is the passing references to real life phenomena and the emphasis that is placed on practical work. The belief seems to be that a stimulating environment will help with the development of learner cognition. These teachers have a wide repertoire of empirical representations which provides the context for the lesson. The type of processes favoured in this class are investigation, experimentation and discovery. Although learner participation is encouraged, there is still a step-by-step build up of the rule or algorithm. The conversation pattern in this class is different from the pure absolutist in the sense that there is a lot more communication between teacher and learner. Usually when the learners' response does not match the teacher's expectations, the teacher shifts to another learner.

The third strand that can be identified is the connected absolutist view. The context for these type of lessons is some real life experiences of the learners which is taken as the starting point. Learners are engaged in the process of sense-making, in a more interactive environment where learner-learner interaction is encouraged. The teacher encourages multiple approaches and multiple solution strategies. There are several distinguishing characteristics of this mathematical view. Greater emphasis is placed on the active involvement of the learners and building on the current state of the learners' knowledge and skills. Also there is a shift away from procedural knowledge towards more conceptual understanding through the numerous connections that are made.

The different form of conversation in this type of lesson, the orientation towards real world experiences and the multiple routes towards solutions all point towards a fallibilist view. However, the fact that the teacher is still regarded as the authority for determining right or wrong, and that learner justification or refutation is not encouraged, make it absolutist as well.

This classification has much in common with the one by Perry (1970). The transmission view is similar to his dualistic view, the empirical view to his multiplistic one and the connected view to his relativistic view.

5.3 THE TEACHERS' VIEWS OF TEACHING

In this section the teachers' views of teaching will be summarized with reference to the type of learner activities that are encouraged, the type of planning for instruction and the different representations that the teacher uses for instruction. The analysis of these components again serve as a benchmark to measure the participants' views of teaching.

Based on the definition for mathematics referred to above, the teacher should strive to create a particular environment for learners to learn. This involves that the teacher should create both the opportunities and the challenges for the learners to learn. This environment should foster the conceptual understanding of mathematical ideas through active learner participation. The teachers perception of her role can therefore be identified from the type of activities structured for the learners'. It is our contention that the type of learner activities most compatible with the definition of mathematics are learner exploration and experimentation, engaging learner's in mathematical discourse and creating opportunities for them to work both collaboratively and individually.

In summarizing the teachers' views on planning for instruction, we grouped together the coding for the motivation and the organisation of the content. It is our view that the type of planning that is required should be flexible, rather than rigid. This is an indication that the planning is based on the learners' understanding and not on covering the text. Planning should also take into consideration the current understanding of the learner. It should therefore be directed at eliciting the existing knowledge and skills of the learners. Finally, the teachers' planning should make possible the type of learning that is active and participatory.

A clear indication of the teachers' ability to make the mathematics comprehensible to the learners' is the repertoire of different representations that the teacher possesses. This repertoire of representations is determined by the teacher's basic notions (*grundvorstellungen*) of the mathematical concepts, the type of individual images elicited by the teacher and the type of connections that are made. As far as the required view is concerned the reference to the real world stand out for both connections and basic notions.

The teacher with a transmission view of teaching focuses on learner activities that promotes memorization and rote learning. The teacher would typically explain a single procedure, which the learners are expected to imitate. The time spent on teacher explanations and the number of examples are not considered when a particular routine is explained. This is borne out by the fact that Denise spent three periods on explaining the procedure for multiplication with two digit numbers. Despite the lack of success with the teacher dominated explanation, she never adapted, nor changed the teaching style. The planning is very rigid with the single purpose being, conveying the rules for a particular procedure. The teacher's scope of representations for the content is limited and connections with real world experiences are non-existent.

The teacher with an empirical view values practical experiences and regard it as embodying the mathematical ideas that learners must discover. Much attention is paid to the selection of activities that will stimulate the development of cognition in the learner. The learner activities favoured by these teachers are hands-on ones, like paper-folding, cutting, etc. which will help the learners to discover the mathematics. The teacher's planning is based on the idea that the mastery of mathematical concepts is a prerequisite for applying these concepts. Applications is only introduced after the practical activities. This teacher makes far more connections and has more basic notions than the transmission teacher. In the lessons on division both the 'partitive' and 'quotitive' meanings were introduced, and in the lesson on fractions both the part-whole and the set-subset meanings were introduced through practical activities. However, the connection between these representations were not highlighted.

The teacher with a connected view places more emphasis on contextual problems. Learner exploration of the context is encouraged and more often in working with concrete materials, serves as the point of departure. The learners' work in groups and learner-learner interaction is actively encouraged. The teacher's planning is far less rigid and opportunities are created to elicit the learners' ideas and to build on it. Although extensive use is made of classroom discourse, both between teacher

and learner and among learners, the teachers continued to present mathematical ideas that they perceived to be too complex for the learners. The teachers with these views are more skilful in eliciting the individual images that learners have about mathematical ideas and to build on these ideas. This also implies that the teacher has a wider range of basic notions for concepts, being able to recognise the learners' individual images. The very approach which emphasizes the context, also implies that a lot more connections are made.

5.4 A CLASSIFICATION OF TEACHERS' VIEWS

Based on the above descriptions, we can categorize the participants in this study according to the predominant views on mathematics and mathematics teaching that they hold.

Transmission	Empirical	Connected
Denise	Cathy Daphne Gretal Sally Pat	Freda Hazel

Table 3 : Classification of Teachers' Views of Mathematics and Mathematics Teaching

Denise's views can be described as strongly transmission which is consistent with a view of mathematics as the acquisition of a set of routines and procedures, and teaching as the conveyance of these routines and procedures. Within the empirical group, the views of the participants range from weak to strong empirical, depending on the extent to which conceptual understanding by learners is viewed. Although all of the teachers view mathematics as practical activities, the extent to which these practical activities are used within the context of the lesson and the functional nature of its use, distinguishes the weaker from the strong empirical view. Pat would therefore be classified as holding weak empirical views, while Gretal would be classified as highly empirical. Teachers with a connected view certainly values conceptual understanding the most. The difference between Freda and Hazel lies in the extent to which conceptual links are established in the classroom. Freda with her greater emphasis on real-life experiences and conceptual links is therefore classified as holding highly connected views.

CHAPTER 6: Recommendations

6.1 INTRODUCTION

It has been argued that a pedagogy that reflects the principles of OBE should facilitate learning in a context which will create understanding and meaning making. This calls for specific views on the part of the teachers on, inter alia, mathematics and mathematics teaching. Mathematics should not be viewed as a static body of knowledge that learners must master, but as a dynamic and expanding human creation. As the product of human creation, it is changing and fallible. Similarly, OBE calls for teaching approaches that emphasize problem-solving so that learners can construct appropriate knowledge.

However, this is not the case in the classrooms of the participants in this study. The current views of the participants on mathematics and the teaching of mathematics do not foster the kind of conceptual understanding that is required. The predominant view of mathematics and mathematics teaching in this study is that of a system of rules which needs to be taught directly (transmission view) or camouflaged in practical activities (empirical view).

If teachers are expected to teach mathematics for understanding the world, to be experienced by learners as a human activity, to be developed and contested through language, symbols and social interaction, then teachers themselves must be helped to form qualitatively different views of all aspects that impact on their practices. In this regard, Sarason (1990) argues that schools should be viewed as places that exist equally for the development of students and teachers. We need to remind ourselves that the teachers we are asking to become catalyst for classroom transformation, are also the products of the system that they are being asked to change. We agree with Schifter (1993) that the kind of reform being asked of teachers is not only very difficult from what they have been doing in the past, but that it is also much harder.

In structuring experiences for teachers to change, we should guard against INSET which is interpreted in terms of past practices or the introduction of a cadre of jargon that leaves teachers with no understanding of what to do. According to Lampert (1988) this type of experience leads to attempts by teachers to enact what they have learned in the INSET programme based on the new teaching paradigm. However, in the process they reduce it to the latest fashionable strategies, for example using physical resources or group work. What is called for is INSET provision that will help teachers lead to deep seated change and help them to formulate a qualitative different view of learning. This however cannot occur without first of all dealing with well established beliefs held by teachers over time. Beliefs are not easy to change, nor do habits evaporate overnight. The paradigmatic shift being required of the teacher entail a carefully designed professional upgrading and long-term support.

6.2 RECOMMENDATIONS

Our recommendations are by no means exhaustive, however, we trust that policymakers will be able to draw on the data in this report and the experiences which we have built up over a period of twelve years in the provision of INSET for mathematics teachers. Much of the recent INSET-focusing on Curriculum 2005 has been concerned with awareness raising. Many Foundation Phase teachers are being left with their awareness highly elevated, and not much else.

Awareness being raised, the next step is to define exactly what is necessary to do. A needs analysis, (which the PEI research project could be identified as doing) should be conducted before any INSET training is undertaken to determine the present context of teachers, identifying the real needs of the school and of the individuals within it.

A conceptual underpinning should be evident where continued planning and delivery of INSET should be within coherent frameworks, to be repeated at agreed periods every year. Currently, conceptualization in teacher in-service education is generally fragmented. Diamond (1991), for instance, argues that the current impact of in-service for teachers demonstrate that the process rarely produces positive outcomes. Joyce et al. (1976) also provide a discouraging account of many in-service programmes. The findings reported were negative, describing the course as weak and a failure.

There is the temptation to present short instant guides for INSET. This approach makes it difficult for teachers to translate something from the programme into professional practice. Posh (1985) draws on a medical analogy - calling this scenario "tissue rejection". He sights that in an organ transplant tissue rejection may occur if it does not correspond to that of the recipient. The same is true in INSET, because the supplied information may not correspond to neither the previous experience of the teacher nor the situations in which teachers have to act. This revelation calls for a degree of "unlearning" the mathematics, teachers know. This "unlearning" will enable them to begin to acquire a new way of thinking about mathematics, and a new approach to learning it.

The impact upon learners' achievement is reasonably presumed to arise from the significant professional development experienced by their teachers. For effective INSET in the development of appropriate views on mathematics and mathematics teaching, the following principles should be considered

- ? INSET which is cognitively demanding, rigorous and effective and, flexible to enable different models of implementation which must result in positive, substantive professional development
- ? Teachers must be sensitised to be able to recognise and critically analyze the process of learning they need to foster in their own classrooms, the nature of mathematics, and the kinds of classroom structure that will promote that goal. It is our belief that if teachers are expected to teach mathematics for understanding then by the same token they must become learners to embrace and experience the new thinking.
- ? A more profound and longer-lasting impact on changed practices can only be realized when programmes are based on the same pedagogical principles as the intended mathematics instruction.

- ? A coherent INSET framework should be set in motion because greater exposure leads to significant greater development of the participants.
- ? INSET programmes will have to dig deeper, to provide participants with opportunities to construct for themselves more powerful, alternative understandings of learning and teaching, as well as mathematical knowledge . Teachers must be confronted to work through such experiences in order to facilitate the process of cognitive re-organisation.
- ? As long as there remains a clash of vision between what is proposed by Curriculum 2005, the OBE approach, and where teachers are now, no one-off INSET programme will address this.
- ? The most important resource teachers bring with them to training programmes is their experience. Therefore, the programme design should create learning experiences which provide opportunities to build on teachers' existing knowledge about practice. Furthermore, the process of critical reflection to make sense of both past and present experiences, is seen as crucial for continued professional development.
- ? Our study encountered the anxiety and threat felt by many teachers out there, no change is likely to develop from this situation. If transformation is to occur, it is only possible when there is a supportive framework within which it can take place. No teacher is voluntarily going to risk conceptual confusion or even relinquish life-long views on teaching and learning mathematics.
- ? If therefore, becomes imperative that attention has to be given to the process, as well as the content in INSET.

6.3 THE PRIMARY MATHEMATICS PROJECT AT UWC

In conclusion we discuss the professional teacher development model used by the Primary Mathematics Project, University of the Western Cape. Our target audience is primary mathematics teachers in the Western Cape and its surrounds. We have been initiators and supporting teacher development since 1981 to the current period.

The overriding philosophy of the project has been to improve the quality of Mathematics teaching in primary schools through the development of people. We have a history of engaging teachers in long-term coherent frameworks of quality intervention, bearing relevance on ever changing curriculum demands. This is achieved by first engaging teachers in a sixteen hour 'awareness' workshop, dealing with subject specific knowledge and pedagogy. For example, Transformation Geometry, Fractions, Assessment, etc. These teachers are then invited to enrol on a three month certificate course. Here the focus is on, engaging teachers in an experiential setting, which is followed by reflective analysis. In this way we perceive that qualitative changes in teachers' knowledge base and pedagogy can grow and develop. Then, teachers who have gone through one of our many programmes, is offered enrolment on the Further Diploma in Education Mathematics (FDE : Mathematics) programme. This course is offered part-time over two years. It is accredited by the University of the Western Cape. The outcomes of the course is to enhance teachers' knowledge base and conceptual understanding in mathematics and technology.

Another important feature of the programme is the effective development of "key" teachers.

Here we help teachers develop a sense of value and purpose within their own understanding and in empowering them to share this with others. We believe that collaboration among teachers is essential to the process of reform. It is through the development of 'key' teachers that the project is able to support and sustain on going activities in schools.

In summary the Primary Mathematics Project's teacher development programme is based on the following guiding principles

1. Both intervention and support are crucial elements in teachers' continued professional development (Schifter, 1993; Schifter & Fosnot, 1995).
2. It should provide extended opportunities for networking, as well as for individual reflection on the learning experiences (Krainer, 1998).
3. Teachers' should develop a broader and more integrated understanding of mathematics. We argue that teachers with such conceptual understanding are more likely to have views of mathematics and mathematics teaching and learning which commensurate with the demands of OBE.

We would like to offer the following diagrammatic representation of the (PMP) model as a possible way forward in designing quality, relevant, effective, impacting and sustainable INSET.

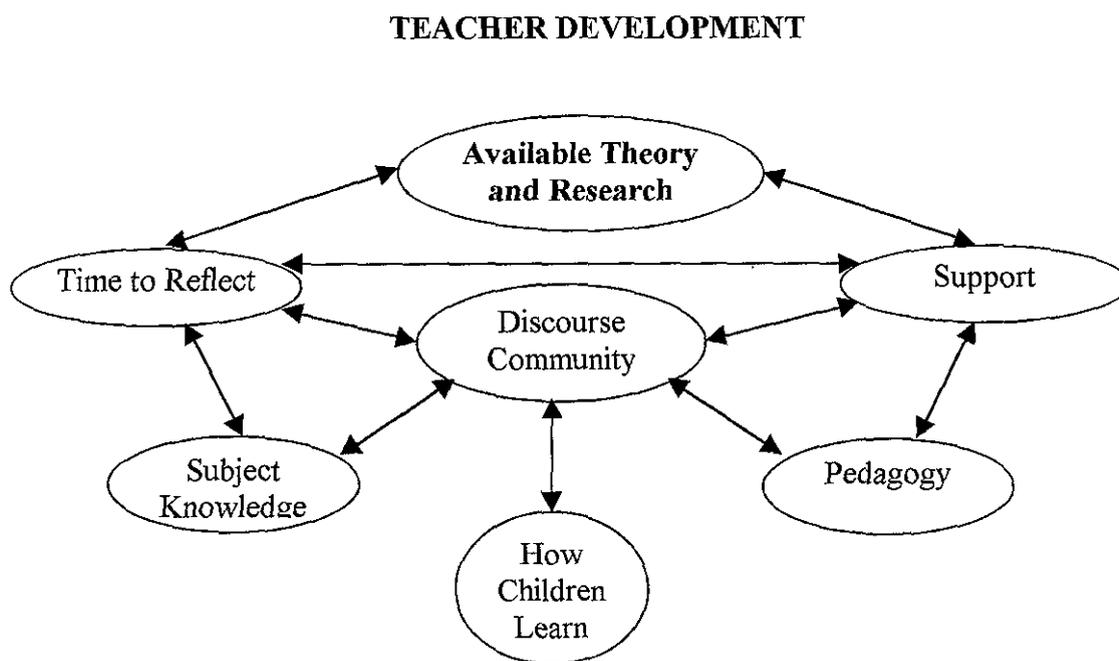


Figure 2 : PMP's Teacher Development Programme

The model in figure 2 has a history of research. Several publications on teachers' content and pedagogic knowledge, in primary mathematics have been produced (Rossouw, et.al 1997; Rossouw & Smith, 1998a; Rossouw & Smith, 1998b).

This model provides sufficient flexibility for teacher development programmes to remain in transition. It enables one to respond to curriculum reform, changing practices and address individual teacher's basic notions of mathematical concepts, procedures and processes in a sustained way. The model is dynamic in nature in that, it interacts with an ever changing teaching environment.

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