Grade 7

Mathematics Teacher Guide

ENGLISH

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

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Book 1

Contents Grade 7 Book 1

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Introduction to the workbooks

What are the workbooks?

The national Department of Basic Education is providing workbooks to every child in a public school in a number of subjects including mathematics. These workbooks are to be provided free of charge to every child.

Each and every child should have their own workbook. They should be allowed to take them home and they can (and indeed must) write in them.

These workbooks will help teachers to manage their teaching time and monitor the progress and performance of their learners.

The two books for Mathematics Grade 7 are available in English and Afrikaans.

The workbooks have been designed to be fully compliant with the National Curriculum Statement (NCS) and the Curriculum and Assessment Policy Statements (CAPS).

What is the place of these worksheets in teaching?

It is important to see what place the worksheets can play in your teaching of Grade 7 mathematics. They are not a substitute for your teaching the concepts and procedures of mathematics. What the worksheets are for is as a help in the practical work you give the learners to do. There are three very important components in every teaching interaction:

1. Firstly, it is important to have a knowledgeable teacher who is familiar with the **content knowledge** being taught.
2. Secondly, it is necessary for the knowledgeable teacher to **communicate this knowledge** so that the learners do not just memorise facts or formulae. Provide concrete (hands on) activities and semi-abstract activities such as making drawings. Good teaching requires an understanding of what the learners already know, building on it, and the skill to communicate in a way that the learners can understand easily, but still be kept interested and challenged.
3. Thirdly, for learning to be retained, learners must make it their own, and this requires **immediate practice**. It is this component the worksheets are designed for – to help the learners make the new knowledge and skills their own. The worksheets provide a well designed and sequenced set of practical exercises for the learners to use under your guidance. They will save you a lot of time(and money) having to write exercises on the board or photocopying your own worksheets.

The structure of the worksheets

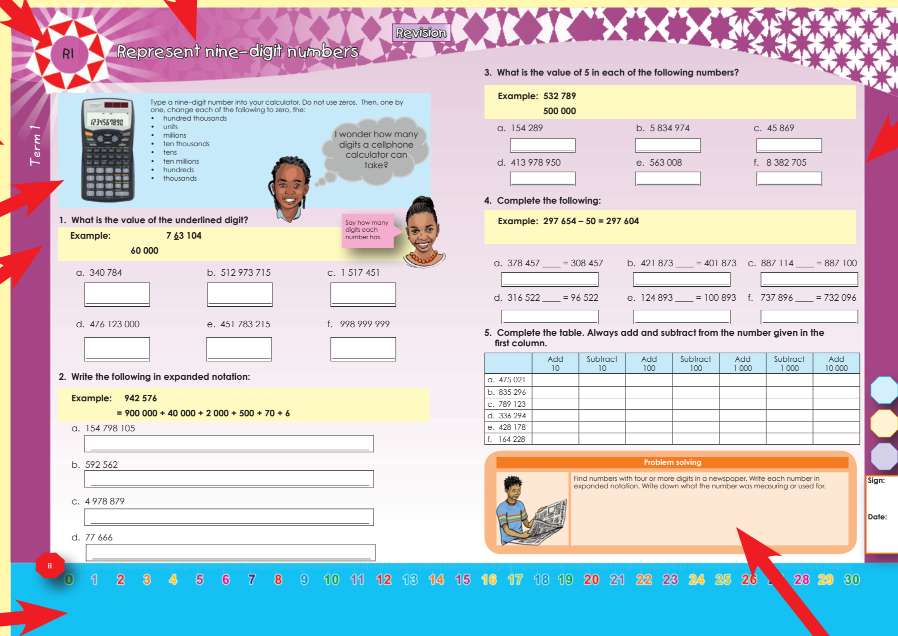
Worksheet number

Worksheet title

Term indicator
(32 worksheets per term)

Topic introduction frame
(A graphic, text and examples for pre-worksheet discussion)

Questions



Language colour code:
Afrikaans (Red), English (Turquoise)

Colour coded content area

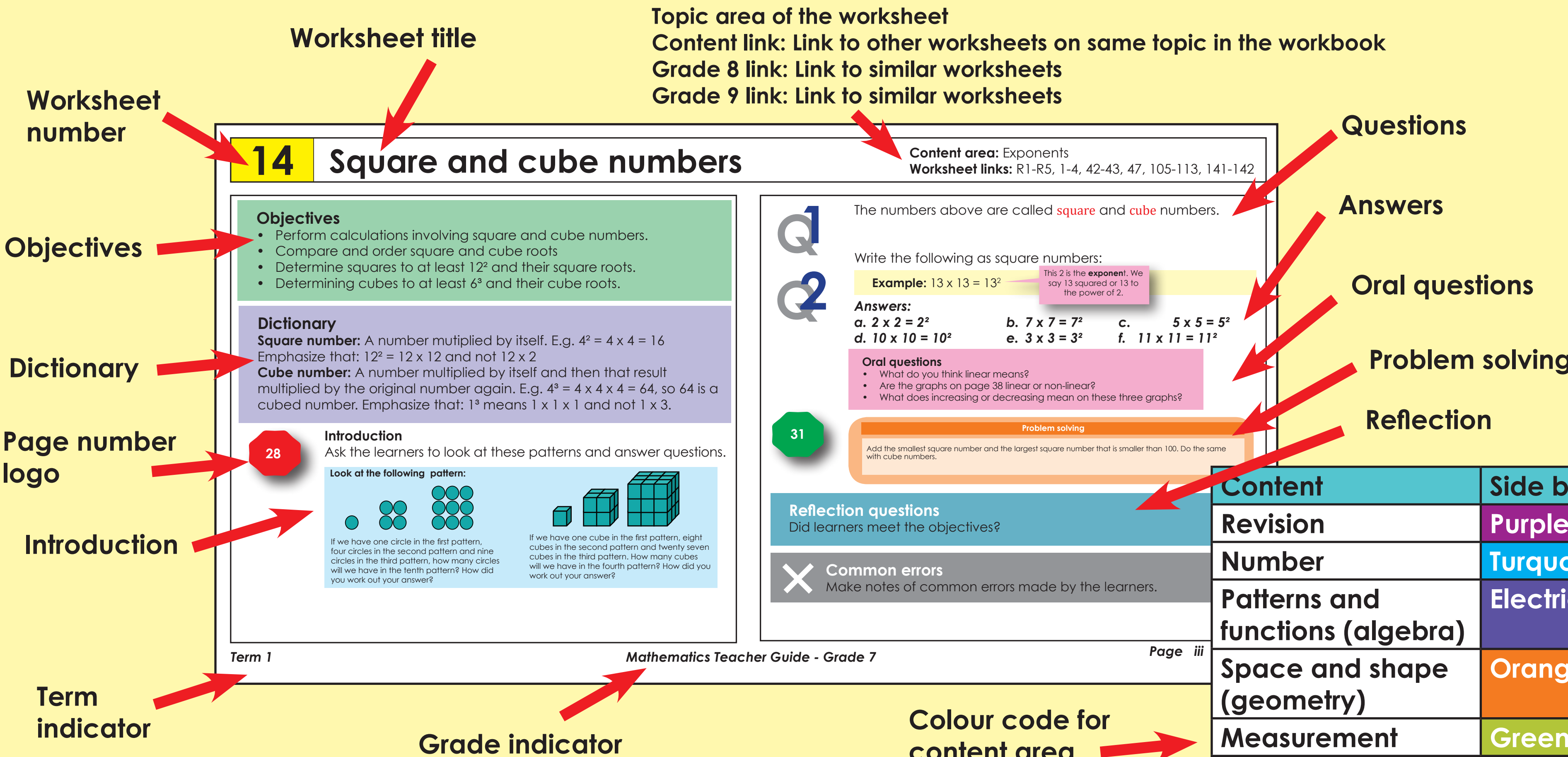
Content	Side bar colour
Revision	Purple
Number	Turquoise
Patterns and functions (algebra)	Electric blue
Space and shape (geometry)	Orange
Measurement	Green
Data handling	Red

Teacher assessment rating

Place for teacher to sign and date

Problem solving activity

The structure of the Teacher Guide



Content	Side bar colour
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Measurement	Green
Data handling	Red

More notes on the structure of the Teacher Guide pages

Content link

The content link refers to the main concepts that we are dealing with in the Foundation Phase. For example, if we are describing how to measure a flat surface, the content link will be other worksheets dealing with measurement of area and volume of shapes and objects.

Resources

Note that sometimes you need additional resources and this needs careful preparation. E.g. if you need to use Cut-outs or any other resources, you have to ask yourself: "Do I have the resources in my class? Can I make it from recyclables? Can I ask the children to bring things from home?" Making sure you have the resources ready is in addition to the normal preparation that you need to make before any lesson. You should always have read the worksheet and worked through it yourself before using it.

Introduction

The introduction links to the Introduction in the worksheet in the learner's book. This could be:

- A fun activity to get the learner's attention
- A problem activity to get the learner involved and thinking
- A revision activity to revise some important concepts needed to further develop the concept in this lesson further

Oral questions

These are questions you can pose for learners after they have been doing a question or two in their workbooks to check their understanding.

Problem solving

This is an activity that can be done by those who have finished the worksheet before the others or it can be used as a homework activity. It is meant to be challenging and/or fun.

Reflection

These are the questions that you need to ask yourself after the lesson. If you cannot answer "Yes" to all the questions you pose to yourself about whether the learners have reached the objectives of the worksheet, you should plan to revise or cover those concepts again in the next lesson.

Common Errors

We can improve our teaching and learners' learning if we know what kind of mistakes are being made. You should keep a journal of common errors and how you can correct them. Only through identifying the cause of the problem can you correct it.

The concrete-to-representational-to-abstract sequence

What is the purpose of the "Concrete-to-representational-to-abstract" (CRA) sequence?

The purpose of teaching through a concrete-to-representational-to-abstract sequence of instruction is to ensure learners have a thorough understanding of the mathematical concepts and skills while they are learning.

What is this sequence?

Concrete level

The concrete level of understanding is the most basic level of mathematical understanding. This level is the crucial beginning for the development of conceptual understanding of mathematics.

Each mathematical skill and knowledge is first modelled with concrete materials. Children should be provided with many opportunities to practice and master mathematical skills and knowledge using concrete materials.

Concrete level learning occurs when children have opportunities to manipulate concrete objects to solve problems.

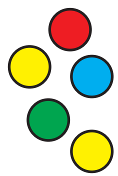
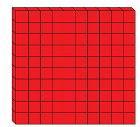
The concrete objects you use in a classroom lesson can include everyday objects (beans, sticks, matches, popsicle sticks or stones) or specially made objects (sometimes called manipulatives) designed so that a child can learn some mathematical concepts by actually handling it. The experience of using these concrete objects provides a way for children to learn concepts such as addition, subtraction, multiplication and division in a developmentally appropriate, hands-on way. Examples of specially made manipulatives are: counters, interlocking cubes, Cuisenaire rods, colour tiles, pattern blocks, base-ten blocks and rods, fraction strips, tangrams and geoboards.

There are two types of **concrete** objects we can use:

- **Discrete** concrete materials are those that are individual, distinct objects that can be counted.
- **Continuous** concrete materials are used in measurement, e.g. scales, rulers, measuring cups, trundle wheels.

In practice concrete objects are used most in the earlier grades, but in Grade 7 you will find that there are times when you need to use them too.

Discrete materials
Discrete materials can be easily manipulated through sight and touch. Children first need a lot of experience in the early grades with discrete materials before they will benefit from using continuous materials.

Unifix cubes	Counters	Pattern blocks	Beans	Stones	Base ten blocks
					

Continuous materials
There are concrete objects that can be used to do continuous measurements of other objects, such as scales, rulers and measuring cups, and clocks.

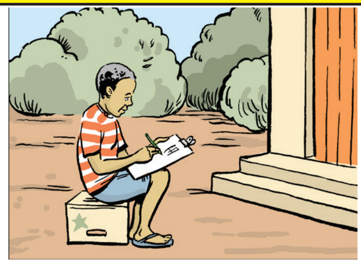
Digital bathroom scale	Analogue bathroom scale	Digital kitchen scale	Ruler	Measuring cups	Trundle wheel
					

The workbooks provide learners with opportunities to practice and demonstrate mastery using some concrete materials. Your task as a teacher is to make sure they have these items. Some of the Workbook Cut-outs provide such items.

Representational level

At the representational level of understanding children use or draw pictures of concrete objects when solving problems. As soon as children have mastered a particular mathematical concept or skill at the concrete level they should move to the representational level. When children draw solutions, children are crossing an intermediate step where they begin to transfer their concrete understanding toward an abstract level of understanding.

The representational level includes the semi-concrete and semi-abstract levels. The semi-concrete involves the representation of actual numbers with things such as dominoes, pictures on cards, dice, etc. and the semi-abstract involve drawing pictures that represent the concrete objects previously used. This includes the semi-concrete and semi-abstract levels.

Semi-concrete	Semi-abstract
	

The **semi-concrete** involves the representation of actual numbers with things such as dominoes, pictures on cards, dice, etc. Some cut-outs enable objects such as dice to be made.

The **semi-abstract** involves drawing pictures that represent the concrete objects previously used.

The **semi-abstract** involves drawing pictures that represent the concrete objects previously used.

The workbooks have a large number of pictures that the learners can use to solve problems.

Abstract level

After the learners have mastered the two previous levels they can move to the abstract level, using only numbers and mathematical symbols.

The child no longer uses concrete objects or drawings to solve problems. This is particularly so in the Senior Level.

When children solve problems using paper and pencil only, it is a common example of abstract level problem solving. Abstract understanding also enables us to do mental mathematics – 'doing maths in your head'.

Many opportunities in the workbooks are given on the abstract level to demonstrate and practice the concept before moving on to the next concept.

What if a child cannot solve problems at an abstract level?

We have these suggestions for you if a child is not successful at solving problems at an abstract level. Provide remedial instruction on the concept or skill at the:

- concrete level using appropriate concrete objects.
- representational level and provide opportunities for the child to practice by drawing solutions.
- abstract level giving the children the opportunity to explain their solutions and how they got them.

Mental mathematics

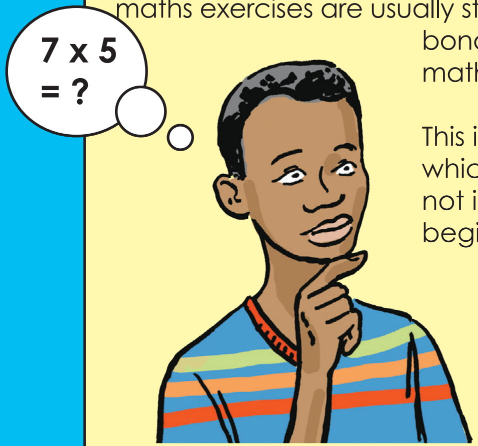
Mental mathematics is using knowledge of the basic mathematical facts to perform mental, as opposed to pen and paper, calculations. Mental maths calculations are done in one's head instead of using pencil and paper, calculators or other aids.

Do the workbooks have mental maths exercises?

No. The worksheets do not include mental maths exercises.

Why is this?

The reason is simple. The worksheets are pencil and paper exercises. They are often more complicated than mental maths exercises (and it would take a teacher a lot of time to design such exercises). By comparison mental maths exercises are usually straightforward and any teacher can test number bonds, knowledge of multiplication tables, and basic maths facts.



This is not to say that the lesson the teacher plans which includes the use of a worksheet should not include mental maths exercises (often at the beginning of a lesson as a way of 'warming up').

Also, mental maths skills will aid the learners as they do the worksheet.

What is mental mathematics?

Mental mathematics is using knowledge of the basic mathematics facts to perform mental calculations rather than using pen and paper or aids such as calculators or computers.

We use mental maths as a way to calculate (give exact answers) and estimate (give approximate answers) quickly, using the maths facts that we have committed to memory. These maths facts include such things as the rules of multiplication, division, etc. and bonds and times tables.

To use mental maths means being able to give an answer to a maths question after only thinking about it, rather than doing calculations on paper. Even if the calculation is such that one does need to use pen and paper (or a calculator), mental maths enables one to quickly judge the reasonableness of the answer so obtained.

For success in mental maths a learner needs a good number sense as he or she has to make sense of number combinations while going through the process of learning the basic mathematical facts. A mental mathematical calculation requires the learner to use a combination of maths factual knowledge and number sense.

An expanded conception of mental maths skills includes being able to truly understand maths concepts and solve problems in a logical, methodical way.

How does one learn to do mental maths?

Traditionally, training in doing mental calculations relied very heavily on 'learning by heart' such things as bonds and times tables, though this has limitations in developing true number sense. People can memorise things they do not understand. However, it is still important that learners do know their bonds and times tables.

A number of well known mathematics programmes have their own special mental mathematics teaching methods.

To become competent in mental maths one first has to learn the 100 or so number facts relating to the single digits 0 to 9 for each of the four operations.

When the learners have memorised and know these facts, they can quickly retrieve them from memory, they have instant recall. Through practice over time the learner will achieve automaticity. He or she will no longer have to work out a strategy in their head on how to answer the problem.

So good teachers should be developing the "mental maths" skills wherever and whenever appropriate. Mental mathematics is a necessary part of what a knowledgeable maths learner does. Fluency in the 'language' of numbers and the use of that 'language' does require some degree of automacity (which would obviously include thorough memorisation of bonds and multiplication tables as well as a basic conceptual understanding of the four operations.)

[Becoming a good reader requires a similar development of automacity – the beginning reader moves from sounding out words to reading instantly.]

What are the basic mathematical facts?

Number work	Comparing and ordering numbers
	Counting on
	Counting back
Addition	Number bonds
	Counting on
	Adding zero
	Number families
	Building up and breaking down numbers
	Doubling in addition
	Near doubles
	Filling up the tens
	Compensation
	Commutative property of addition
Subtraction	Taking away
	Halving in subtraction
	Doubling in subtraction
	Subtraction as the inverse operation of addition.

Multiplication	Skip counting (multiples)
	Multiplication by zero
	Multiplication tables
	Equal groups
	Repeated addition
	Commutative property of multiplication
	Place-value-change strategy for multiplying by 10, 100, 1000
Division	Sharing leading to division
	Grouping leading to division
	Halving in division

Teaching mental maths

A maths teacher needs to incorporate some aspect of mental maths in nearly every lesson. The actual time spent may often be very short – five minutes a day – though some lessons may focus more directly on mental maths.

To do mental maths learners need to know the number facts relating to the digits 0 to 9. Initially this involves learning and practice. With time the learner will be able to recall and use these facts automatically.

In the early years of mental maths development it is important to give the children short tests, mark them, and give the children feedback.

Mental maths tests can be oral or pencil and paper or you can have a combination of written and oral answers. Oral answers and explaining how they got the answer will be more valuable to you as teacher and the learners because they will hear and share different strategies.

When you for example ask "What is 7 times 5?" also ask what "7 times 5" means. They might answer "7 groups of 5". Then continue: "If 7 groups of 5 equals 35, how much will 8 groups of 5 be?" "6 groups of 5?", etc. Ask the children that gave the correct answer: "How did you get the answer?" and then ask the learners that got it incorrect: "How did you get the answer?"

Through their explanation not only can you assess them but the rest of the class also learn from them. You will notice that children will use a variety of strategies to calculate. The child that answered it incorrectly might correct him or herself when explaining how she or he got the answer or you as teacher can guide the child while giving feedback to the correct answer.

Help your learner to think mathematically using the workbooks

There are three kinds of knowledge: physical, social and conceptual knowledge.

Physical knowledge

Learners gain physical knowledge through touching, using, playing with, and acting on concrete/physical material. Learners need a lot of concrete experiences in the mathematics classroom to develop their physical knowledge of numbers and number patterns.

The workbooks provide a variety of ideas and pictures on how to use concrete resources. At the back of each workbook we include cut-outs that encourage the use of resources.

Teachers need to consider which concrete resources should go with each worksheet. The **Resources block** gives some suggestions. Find out if your school has these resources or whether you can make them yourself.

Social knowledge

Social knowledge is the words and conventions we need to know and remember if we are to be able to communicate with and interact with other people. For example, we need to be on time at school. It is a convention, it is a decision we have taken and all agree to. Below are example of some mathematics conventions that we will find throughout the workbooks:

- The way in which we write a number sentence.
- The way in which we write a number symbol.
- The way in which we use the equal sign to show equivalence.

We have agreed to use these conventions so that we can communicate mathematically with others. The teacher must help learners to put what they have learned in words or writing to explain it to the others.

Conceptual knowledge

When learners see relationships, patterns, regularities and irregularities when doing activities, they are constructing conceptual knowledge. A concept is a general idea we hold in our minds that helps us to understand real individual things in the world. We build up conceptual knowledge based on our experience.

What is your role as a teacher in developing conceptual knowledge when using the workbooks?

You should use the worksheets to assist the learners to build up their understanding of mathematics and to see the patterns in numbers. Encourage your learners to reflect on what they are doing and thinking when completing a worksheet.

You can ask them questions like:

- How did you get this answer?
- What did you do to complete this task?
- What is another way to solve this problem?
- Can you compare your thinking or solutions with your partner's?
- How can you show your thinking using, drawings, concrete resources, numbers and words?

R1 Represent nine-digit numbers

Topic: Whole numbers **Content links:** R2-R5
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

Objectives
Revise the following done in Grade 6:
- Order, compare and represent numbers to at least 9-digit numbers

Dictionary
Place value: The value of a digit depending on its place in a number.
E.g. 389 123: the value of the 8 is 80 000

ii

Introduction
Ask the learners to go to their workbooks on page ii. Ask the learners to type a nine-digit number into their calculators. Do not use zeros. Change the following to zero. Example: 364 281 193

• hundred thousands	364 **0**81 193
• units	364 081 19**0**
• millions	36**0** 081 190
• ten thousands	360 0**0**1 190
• tens	360 001 1**0**0
• ten millions	3**0**0 001 100
• hundreds	300 001 **0**00
• thousands	300 00**0** 000

Oral questions
Ask questions such as:
- How did you change a digit to zero? What happens to that place value?

Q1 What is the value of the underlined digit.

Example: 7<u>6</u>3 104
60 000

Answers:
a. 80 b. 3 000 c. 10 000 d. 70 000 000 e. 5 f. 90 000

Q2 Write the following in expanded notation:

Example: 942 576
= 900 000 + 40 000 + 2 000 + 500 + 70 + 6

Answers:
a. 100 000 000 + 50 000 000 + 4 000 000 + 700 000 + 90 000 + 8 000 + 100 + 5
b. 500 000 + 90 000 + 2 000 + 500 + 60 + 2
c. 4 000 000 + 900 000 + 70 000 + 8 000 + 800 + 70 + 9
d. 70 000 + 7 000 + 600 + 60 + 6
e. 500 000 + 40 000 + 9 000 + 300 + 20 + 7
f. 4 000 000 + 9

Q3 What is the value of 5 in each of the following numbers?

Example: 532 789
500 000

Answers:
a. 50 000 b. 5 000 000 c. 5 000
d. 50 e. 500 000 f. 5

R1 Represent nine-digit numbers *cont...*

Topic: Whole numbers **Content links:** R2-R5
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

Q4

Complete the following:

Example: 297 654______ = 297 604
297 654 – 50 = 297 604

Answers:
a. 378 457 – 70 000 = 308 457
b. 421 873 – 20 000 = 401 873
c. 887 114 – 14 = 887 100
d. 316 522 – 220 000 = 96 522
e. 124 893 – 24 000 = 100 893
f. 737 896 – 5 800 = 732 096

Q5

Always add and subtract from the number given in the first column.
Answers:

	Add 10	Subtract 10	Add 100	Subtract 100	Add 1 000	Subtract 1 000	Add 10 000
a. 475 021	475 031	475 011	475 121	474 921	476 021	474 021	485 021
b. 835 296	835 306	835 286	835 396	835 196	836 296	834 296	845 296
c. 789 123	789 133	789 133	789 223	789 023	790 123	788 123	799 123
d. 336 294	336 304	336 284	336 394	336 194	337 294	335 294	346 294
e. 428 178	428 188	428 168	428 278	428 078	429 178	429 178	429 178
f. 164 228	164 238	164 218	164 328	164 128	165 228	163 228	174 228

iii

Problem solving

Find numbers with four or more digits in a newspaper. Write each number in expanded notation. Write down what the number was measuring or used for.

Answer: Learner's own answer. For example:
12 475 = 10 000 + 2 000 + 400 + 70 + 5

Could be R12 475 for a suite of furniture

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

R2 Compare and order whole numbers

Topic: Whole numbers **Content links:** R1, R3-R5, 1
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

Objectives

Revise the following done in Grade 6:

- Order, compare and represent numbers to at least 9-digit numbers

Dictionary

Place value: Place value: The value of a digit depending on its place in a number. E.g. 389 123: the value of the 8 is 80 000

Ascending order: Arranged from smallest to largest. Increasing.

Descending order: Arranged from largest to smallest. Decreasing.

Interval: A set of real numbers between two numbers either including those two numbers, or excluding them, or including only one of them. It is a range of numbers.

iv

Introduction

Ask the learners to go to their workbooks on page iv.
Revise the following symbols with your learners: > < =
Ask learners to give five examples using the symbols. E.g.

- 256 < 265
- 265 > 256
- 265 = 265
- 65 < 265
- 605 > 265

Ask the learners what an interval is. Guide them by saying they will find it on a number line. When working with numbers, it is the numbers between two specific values.

Q1

Arrange these numbers in ascending order on the number line:

17 211 17 235 17 347 17 678 17 922 18 212

Answers:

a. 18 212 – 17 678 = 534
b. 17 347 + 17 922 = 35 269 ÷ 2 = 17 634,50
c. E.g. 17 900 d. 17 211 e. 18 212

Q2

Arrange these numbers in ascending order on this number line:

1 571 1 699 1 782 1 999 2 102 2 342

Answers:

a. 1 571 b. 2 342 c. 2 342 – 1 571 = 771
d. E.g. 1 570 e. E.g. 2 343 f. 1 699 + 1 999 = 3 698

Q3

Arrange these numbers in ascending order on the number line:

34 284 34 285 34 286 34 287 34 288 34 289

Topic: Whole numbers **Content links:** R1, R3-R5, 1
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

R2 Compare and order whole numbers *cont...*

Q3

Answers:
a. 34 284 b. 34 289 c. 34 289 – 34 284 = 5
d. E.g. 34 280 e. E.g. 34 290 f. 34 286 + 34 287 = 68 573

Q4

Fill in the missing numbers:

21 000	22 000	23 000	24 000	25 000	26 000	27 000	28 000	29 000	30 000
31 000	32 000	33 000	34 000	35 000	36 000	37 000	38 000	39 000	40 000
41 000	42 000	43 000	44 000	45 000	46 000	47 000	48 000	49 000	50 000
51 000	52 000	53 000	54 000	55 000	56 000	57 000	58 000	59 000	60 000
61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000

Q5

Which number is halfway?

Example:

471 340 471 345 471 350

Answers

a. 21 208 $\boxed{21\ 216}$ 21 224

b. 318 970 $\boxed{319\ 020}$ 319 080

c. 12 897 $\boxed{13\ 397}$ 13 897

Q6

Which number comes next? Answers:
a. 331 349 b. 549 323 c. 508 614

Q7

Write in ascending order:
Answers:
a. 421 178; 421 179; 421 180; 421 181; 421 182; 421 183
b. 543 687; 543 688, 543 689; 543 690; 543 691
c. 903 675; 903 676; 903 677; 903 678; 903 679

Q8

Write in descending order:
Answers:
a. 564 747; 564 746; 564 745; 564 744; 564 743
b. 907 570; 907 569; 907 568; 907 567; 907 566
c. 352 703; 352 702; 352 701; 352 700; 352 699

Q9

Fill in >, < or =:
Answers:
a. < b. < c. < d. > e. > f. >

Q10

Fill in >, < or =:
Answers:
a. < b. = c. < d. < e. > f. <

vii

Problem solving

Use each of the following digits only once to make the biggest eight–digit number possible, and then the smallest eight–digit number possible.

1 5 6 2 9 8 3 7

Answer: 98 765 321 and 12 356 789

R3 Prime numbers

Topic: Whole numbers **Content links:** R1, R3-R5, 1
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

Objectives

Revise the following done in Grade 6:

- Recognise and represent prime numbers to at least 100

Dictionary

Prime number: A number that can be divided evenly only by 1 or itself. The number must be greater than 1. E.g.: 7 can be divided evenly only by 1 or 7, so it is a prime number.

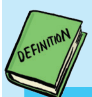
Composite number: If it is not a prime number it is called a composite number. E.g.: 6 can be divided evenly by 1, 2, 3 and 6 so it is a composite.

viii

Introduction

Fun question: When you add two prime numbers will it give you a prime number?

Which numbers smaller than 100 can only be divided by one or by the number itself?



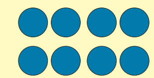
A **prime number** can be divided evenly only by 1 or itself. It has two, and only two, factors – 1 and itself. A prime number must be greater than 1.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

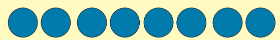
Q1

Use a drawing to show that the following numbers are not prime numbers but composite numbers.

Example: 8 can be divided by 1, 2, 4 and 8

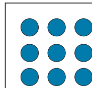


2 x 4 or 4 x 2



1 x 8 or 8 x 1

a. 9



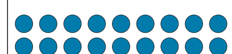
3 x 3

1 x 9 or 9 x 1

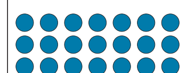
9 is not a prime number.
9 can be divided by 1, 3, and 9

b. 18

1 x 18 or 18 x 1



2 x 9 or 9 x 2



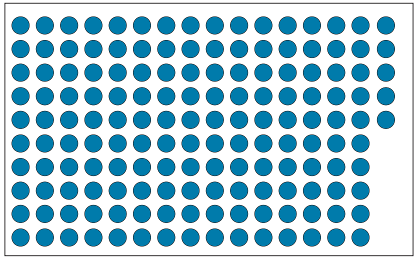
3 x 6 or 6 x 3

18 is not a prime number.
18 can be divided by 1, 2 3, 6 and 9

R3 Prime number *continued*

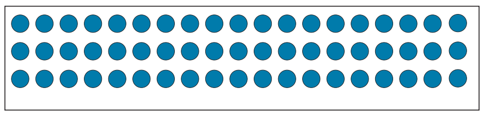
Topic: Whole numbers **Content links:** R1, R3-R5, 1
Grade 8 links: R1, R4, 1 **Grade 9 links:** R1, R10, 76-80

c.



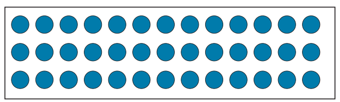
155 is not a prime number.
155 can be divided by 1, 5, 31 and 155.
155 is a composite number.

d.



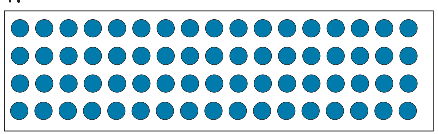
57 is not a prime number.
57 can be divided by 1, 3, 19 and 57.
It is a composite number.

e.



39 is a composite number.
39 can be divided by 1, 3, 13 and 39.

f.



68 is a composite number.
68 can be divided by 1, 2, 7, 17 and 68.

Q2 Identify all the prime numbers from 1–100.

Answers:

a. 2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53; 59; 61; 67; 71; 73; 79, 83; 89; 97

Q3 How would you write the following numbers as a product of prime numbers?

Answers:

a. 2 x 2 x 3 x 3 = 36
b. 2 x 2 x 3 x 5 = 60
c. 3 x 5 x 7 = 105
d. 2 x 2 x 3 x 5 x 7 = 420
e. 2 x 2 x 2 x 2 x 3 = 48
f. 2 x 2 x 2 x 3 x 3 x 5 x 5 = 1 800

Q4 What numbers are these? Why?

Answers:

a. All of the above are prime numbers, they can only be divided by 1 and themselves. Some are 1-digit prime numbers, some 2-digit and most 3-digit.

ix

Problem solving

How many three–digit prime numbers are there less than 1 000. 143

[6 rows of 19 = 114] + [1 row of 19 – 6 = 13] + [1 row of 19 – 3 = 16] = 143

Reflection questions

Did learners meet the objectives?

Topic: Whole numbers, Rounding **Content links:** R1-R3, R5, 1, 44, 47
Grade 8 links: R1, R4, R6, 72 **Grade 9 links:** R1, R6, R10, 17, 20, 76-80

R4 Rounding off to the nearest 5, 10, 100 and 1 000

Objectives
Revise the following done in Grade 6:

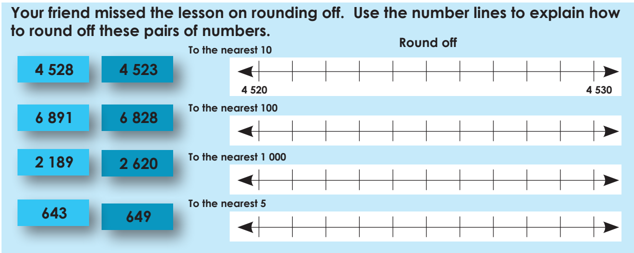
- Rounding off numbers to the nearest 5, 10, 100, or 1 000

Dictionary
Rounding: Rounding means reducing or increasing the digits in a number while trying to keep its value similar. The result is less accurate, but easier to use. E.g.:

- 749 rounded off to the nearest 10 is 750
- 749 rounded off to the nearest 100 is 700
- 749 rounded off to the nearest 1 000 is 1 000
- 749 rounded off to the nearest 5 is 750

The symbol for rounding of is: ≈

Introduction
Ask learners what does it mean to round off to the nearest: 10? 100? 1 000? 5?



Q1 What is the symbol for rounding off?

Answers:
a. ≈

Q2 Round off to the nearest 10.

Example: 789 ≈ 790

Answers:
a. 10 b. 0 c. 80 d. 60 e. 330 f. 450

Q3 Round off to the nearest 100.

Example: 789 ≈ 800

Answers:
a. 0 b. 100 c. 0 d. 800 e. 900 f. 1 800

Q4 Round off to the nearest 1 000.

Example: 789 ≈ 1000

Answers:
a. 0 b. 0 c. 2 000 d. 9 000 e. 14 000 f. 67 000

Topic: Whole numbers, Rounding **Content links:** R1-R3, R5, 1, 44, 47
Grade 8 links: R1, R4, R6, 72 **Grade 9 links:** R1, R6, R10, 17, 20, 76-80

R4 Rounding off to the nearest 5, 10, 100 and 1 000 *continued*

Q5 Complete the table:

	Round off to the nearest 10	Round off to the nearest 100	Round off to the nearest 1 000
a. 7 632	7 630	7 600	8 000
b. 8 471	8 470	8 500	8 000
c. 9 848	9 850	9 800	10 000
d. 5 737	5 740	5 700	6 000
e. 9 090	9 090	9 100	9 000

Q6 Round off to the nearest five. **Example: 4 ≈ 5**

Answers:

a. 5 b. 5 c. 470 d. 590 e. 2 370 f. 3 470

Q7 Complete the table:

	Round off to the nearest 10	Round off to the nearest 100	Round off to the nearest 1 000
a. 2	0	0	0
b. 7	10	0	0
c. 48	50	100	0
d. 781	780	800	1 000
e. 345	350	300	0
f. 2 897	2 900	2 900	2 900

Q8 Why do we round off? Give five examples from everyday life where we round off.

Answers:

a. Change
b. Quick adding
c. Weighing
d. Ages
e. Adding large numbers (estimates)

xi

Problem solving

a. You have a five-digit number. After you round it off to the nearest thousand, you get a six-digit number. What number could your first number have been?

b. You have a four-digit number. After you round it off to the nearest five you get 3 895. What was your original number?

Answers:

a. A number between 99 500 and 99 999.

b. The 4-digit number would be either 3 893, 3894, 3896 or 3897.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

R5 Calculating whole numbers

Topic: Whole numbers **Content links:** R1-R4, 1
Grade 8 links: R1, R4, **Grade 9 links:** R1, R10, 76-80

Objectives

Revise the following done in Grade 6, without use of calculators:

- Add and subtract whole numbers to at least 6-digit numbers
- Multiply whole numbers to at least 4-digit by 2-digit numbers
- Divide to at least 4-digit whole numbers by 2-digit whole numbers
- Perform calculations using all four operations on whole numbers, estimate and round off

Dictionary

Add and subtract large numbers: To add or subtract large numbers, list them in columns and then add or subtract only those digits that have the same place value.

Multiply two large numbers: To multiply two large numbers, write the numbers vertically with the larger number being multiplied by the smaller number which is called the multiplier.

xii

Introduction

Ask the learners to open their workbooks on page xii. Ask learners to name the four basic operations in mathematics. Ask learners how will they:

- Add and subtract large numbers.
- Multiply large numbers.
- Divide large numbers.

Before they start to do the questions, ask them to look through the questions and estimate the answers. After they do the actual calculations they can check their answers using calculators.

Q1

Solve the sums. You can use the method of your choice.

Example 1:

278 467 + 197 539

= 200 000 + 100 000 + 70 000 + 90 000 + 8 000 + 7 000 + 400 + 500 + 60 + 30 + 7 + 9

= 300 000 + 160 000 + 15 000 + 900 + 90 + 16

= 300 000 + 100 000 + 60 000 + 10 000 + 5 000 + 900 + 90 + 10 + 6

= 400 000+ 70 000 + 5 000 + 900 + 100 + 6

= 400 000 + 70 000 + 5 000 + 1 000 + 6

= 400 000 + 70 000 + 6 000 + 6

= 476 006

Example 2:

	2 7 8 4 6 7	
+	1 9 7 5 3 9	
	1 6	(7 + 9)
	9 0	(60 + 30)
	9 0 0	(400 + 500)
	1 5 0 0 0	(8 000 + 7 000)
	1 6 0 0 0 0	(70 000 + 90 000)
	3 0 0 0 0 0	(200 000 + 100 000)
	4 7 6 0 0 6	

Example 3:

	1	1	1	1	1	
	2	7	8	4	6	7
+	1	9	7	5	3	9
	4	7	6	0	0	6

Topic: Whole numbers **Content links:** R1-R4, 1
Grade 8 links: R1, R4, **Grade 9 links:** R1, R10, 76-80

Answers

a.

	80000 + 10000
+	7000 + 2000
	300 + 200
	80 + 10
	2 + 3
=	99595

b.

	65479
+	32599
	18
	160
	900
	7000
	90000
=	98078

c.

	178673
+	145568
	11
	130
	13000
	11000
	200000
=	324241

d.

	237634
+	199999
	13
	120
	1500
	16000
	120000
	300000
=	437633

Q2

Solve the subtraction sums. You can use a method of your choice.

Example 1:

	476006	
–	197539	
	7	(16 – 9)
	60	(90 – 30)
	400	(900 – 500)
	8000	(15 000 – 7 000)
	70000	(16 000 – 9 000)
	200000	(300 000 – 100 000)
	278467	

Example 2:

	3	16	15	9	9	1
	~~4~~	~~7~~	~~6~~	~~0~~	~~0~~	6
–	1	9	7	5	3	9
	2	7	8	4	6	7

Answers

a.

	68763
–	29552
	01
	10
	200
	9000
	30000
	39211

b.

	83251
–	25368
	06
	80
	800
	7000
	50000
	57886

c.

	142637
–	231528
	03
	40
	800
	4000
	90000
	100000
	194843

d.

	532764
–	299999
	05
	60
	700
	2000
	30000
	200000
	232765

R5 Calculating whole numbers *continued*

Topic: Whole numbers **Content links:** R1-R4, 1
Grade 8 links: R1, R4, **Grade 9 links:** R1, R10, 76-80

Q3

Solve the multiplication sums. You can use the method of your choice.

Answers

a.
$$\begin{array}{r} 243 \\ \times\ 89 \\ \hline 2\,187 \\ 19\,440 \\ \hline 21\,627 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 579 \\ \times\ 73 \\ \hline 1\,737 \\ 40\,530 \\ \hline 42\,267 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 241 \\ \times\ 137 \\ \hline 1\,687 \\ 7\,230 \\ 24\,100 \\ \hline 33\,017 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 896 \\ \times\ 476 \\ \hline 5\,376 \\ 62\,720 \\ 358\,400 \\ \hline 426\,496 \\ \hline \end{array}$$

Q4

Solve the division sums. You can use the method of your choice. Answers:

a.
$$\begin{array}{r|l} & 1\,1\,2\,7 \\ 2 & 2\,2\,5\,4 \\ & -\ 2\,0\,0\,0 \\ \hline & 2\,5\,4 \\ & -\ 2\,0\,0 \\ \hline & 5\,4 \\ & -\ 4\,0 \\ \hline & 1\,4 \\ & -\ 1\,4 \\ \hline & 0 \end{array}$$

b.
$$\begin{array}{r|l} & 1\,1\,7 \text{ rem } 3 \\ 12 & 1\,4\,0\,7 \\ & -\ 1\,2\,0\,0 \\ \hline & 2\,0\,7 \\ & -\ 1\,2\,0 \\ \hline & 8\,7 \\ & -\ 8\,4 \\ \hline & 3 \end{array}$$

c.
$$\begin{array}{r|l} & 1\,1\,5 \text{ rem } 15 \\ 25 & 2\,8\,9\,0 \\ & -\ 2\,5\,0\,0 \\ \hline & 3\,9\,0 \\ & -\ 2\,5\,0 \\ \hline & 1\,4\,0 \\ & -\ 1\,2\,5 \\ \hline & 15 \end{array}$$

xv

Problem solving

1. We cycled 2 455 m on the first day and 3 650 m on the second day. How many kilometres did we travel?

   Answer: 455 m + 3 650 m = 6 105 m

2. I jogged 1 550 m and my friend jogged 2 275 m. How much further did my friend jog than I did?

   Answer: 2 275 m – 1 550 m = 725 m

3. The bakery bakes 2 450 biscuits on one day. How many did they bake during March? Note that they only bake six days of the week.

   Answer: 2 450 x 6 x 4 = 58 800

4. My mother bought 3 850 m of string. She has to divide it into 25 pieces. How long is each piece?

   Answer: 3 850 m ÷ 25 = 154 m

R6 Factors and multiples

Topic: Factors and multiples **Content links:** 5-6
Grade 8 links: R2, 3-4 **Grade 9 links:** R2, 2

Objectives

Revise the following done in Grade 6:

- Multiples of 2-digit whole numbers
- Factors of 2-digit whole numbers
- Prime numbers
- Composite numbers
- Find the LCM and HCF of numbers to at least 2-digit whole numbers

Dictionary

Multiples: The products of natural numbers (1, 2, 3, 4, 5, ...) are called the multiples of the number. Multiples are the results of multiplying by an integer, e.g. 3 x 2 = 6 so 6 is a multiple of 2 and 3. The multiples of 6 are 6, 12, 18, 24, ...
Factors: Factors are the whole numbers you multiply together to get another whole number, in other words a whole number that divides exactly into another whole number is called a factor of that number, e.g. 3 and 4 are factors of 12, because 3 x 4 = 12
Prime number: A prime number has only two different factors. The one factor is 1 and the other factor is the prime number itself.
Composite numbers have more than two different factors. E.g. 21 is composite.1 x 21 = 21, 3 x 7 = 21. There are only 4 factors: 1, 21, 3 and 7.
Lowest common multiple: E.g. the lowest common multiple of 3 and 5 is 15, because 15 is a multiple of 3 and also a multiple of 5. We use the abbreviation LCM for lowest common multiple.

HCF: Highest common factor. E.g. The highest common factor of 2, 3 and 4 is 12.

xvi

Introduction

Ask the learners to open their workbooks on page xvi. Go through the definitions with your learners and ask them to give you examples. Practice with finding multiples and factors of whole numbers is especially important when learners do calculations with fractions. They use this knowledge to find the LCM when one denominator is a multiple of another, and also when they simplify fractions or have to find equivalent fractions.

Q1

Write down at least six multiples of the following numbers, and circle the common multiples shared by the two numbers. Answers:

a 2: 2, 4, 6, 8, 10, 12
6: 6, 12, 18, 24, 36, 42

b. 3: 3, 6, 9, 12, 15, 18
9: 9, 18, 27, 36, 45, 54

c. 4: 4, 8, 12, 16, 20, 24, 28
7: 7, 14, 21, 28, 35, 42

d. 5: 5, 10, 15, 20, 25, 30, 35, 40
8: 8, 16, 24, 32, 40, 48

e. 4: 4, 8, 12, 16, 20, 24
5: 5, 10, 15, 20, 25, 30

Q2

Look at the examples above. What is the LCM factor for each:
Answers: a. 6 b. 9 c. 28 d. 40 e. 20

R6 Factors and multiples *continued*

Topic: Factors and multiples **Content links:** 5-6
Grade 8 links: R2, 3-4 **Grade 9 links:** R2, 2

Q3

Write down the factors for the following, and circle the common factors for the two numbers. Answers:

a. 12: 1, 2, 3, 4, 6, 12
   24: 1, 2, 3, 4, 6, 8, 12, 24
b. 28: 1, 2, 4, 7, 14, 28
   21: 1, 3, 7, 21
c. 15: 1, 3, 5, 15
   18: 1, 2, 3, 6, 9, 18
d. 24: 1, 2, 3, 4, 6, 8, 12, 24
   60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
e. 18: 1, 2, 3, 6, 9, 18
   81: 1, 3, 9, 27, 81

Q4

Look at the examples above. What is the highest common factor for each: Answers: a. 12; b. 7; c. 3; d. 12; e. 9

Q5

Complete the following:

Number	Factors	How many factors?	Prime or composite
a. 12	1, 2, 3, 4, 6, 12	6	Composite
b. 41	1, 41	2	Prime
c. 63	1, 3, 7, 9, 21, 63	6	Composite
d. 77	1, 7, 11, 77	4	Composite
e. 33	1, 3, 11, 33	4	Composite
f. 121	1, 11, 121	3	Composite

Q6

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Express each of the following odd numbers as the sum of 3 prime numbers.

Answers: These are some of the possible answers.

a. 29 = 3 + 7 + 19 or 5 + 11 + 13
b. 83 = 5 + 37 + 41 or 13 + 17 + 53
c. 55 = 5 + 23 + 27 or 11 + 13 + 29
d. 53 = 11 + 19 + 23 or 11 + 13 + 29 or 13 + 17 + 23
e. 99 = 17 + 29 + 53 or 13 +19 + 67

Problem solving

Which number or numbers between 1 and 100 has the most factors?

Answer: These numbers all have 12 factors.
Factors of 60: 1 2 3 4 5 6 10 12 15 20 30 60
Factors of 72: 1 2 3 4 6 8 9 12 18 24 36 72
Factors of 84: 1 2 3 4 6 7 12 24 21 28 42 84
Factors of 90: 1 2 3 5 6 9 10 15 18 30 45 90
Factors of 96: 1 2 3 4 6 8 12 16 24 32 48 96

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

R7 Fractions

Topic: Fractions **Content links:** R8, 30-46
Grade 8 links: R5, 65-75 **Grade 9 links:** R5, 11-18

Objectives

Revise the following done in grade 6:

- common fractions
- equivalent fractions with denominators that are multiples of each other.
- addition and subtraction of common fractions with the same and different denominators.
- addition and subtraction of mixed numbers.
- percentage of a whole.

Dictionary

The **numerator** is the top number in a fraction. Shows how many parts we have.

The **denominator** is the bottom number in a fraction. Shows how many equal parts the whole is divided into.

Equivalent fractions are fractions which have the same value, even though they may look different, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

xviii

Introduction

Fractions are used everyday by people who don't even realise that they are using fractions. Name ten examples.
Ask the learners to read the definitions and give five examples of each. Ask the learners how are fractions, decimals and percentages linked. Give an example.

Q1 Complete the fractions to make them equal. Answers: Note that to do this you need to multiply (or divide) both the denominator and the nominator by the same number to get the equivalent fraction.

b. $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$ c. $\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$ d. $\frac{6}{7} \times \frac{3}{3} = \frac{18}{21}$

e. $\frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$ f. $\frac{9}{15} \div \frac{3}{3} = \frac{3}{5}$ g. $\frac{5}{6} \times \frac{3}{3} = \frac{15}{18}$

h. $\frac{7}{9} \times \frac{2}{2} = \frac{14}{18}$ i. $\frac{6}{22} \div \frac{2}{2} = \frac{3}{11}$ j. $\frac{20}{25} \times \frac{4}{4} = \frac{80}{100}$

Q2 What happens to the numerator and denominator? Extend the pattern by writing down three more equivalent fractions.
Answers:

a. $\frac{16}{48} = \frac{32}{96} = \frac{64}{192}$ b. $\frac{81}{405} = \frac{243}{1\,215} = \frac{729}{3\,645}$

Q3 Complete the pattern. Answers:

a. $\frac{40}{48} = \frac{80}{96} = \frac{160}{192}$ b. $\frac{81}{108} = \frac{243}{324} = \frac{729}{972}$

c. $\frac{72}{88} = \frac{144}{176} = \frac{288}{381}$ d. $\frac{125}{875} = \frac{625}{4\,375} = \frac{3\,125}{21\,875}$

Q4 Fill in the empty boxes. Answers:

a. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{\boxed{3}}{4}$ b. $\frac{2}{6} + \frac{1}{2} = \frac{\boxed{4}}{12} + \frac{6}{12}$

Q5 Complete the fraction sums using the diagrams on the right.
Answers:

a. $\frac{3}{4} = \frac{1}{8} + \frac{5}{8} = \frac{6}{8}$

b. $\frac{4}{6} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Q6 Complete the sums: Answers:

a. $\frac{1}{2} = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

b. $\frac{1}{2} = \frac{1}{14} + \frac{6}{14} = \frac{7}{14}$

Q7 Add and then subtract to test your answer. Answers:

a. $\frac{5 \times 2}{7 \times 2} + \frac{2}{14}$ Test $\frac{12}{14} - \frac{2}{14}$

$= \frac{10}{14} + \frac{2}{14}$ $\quad = \frac{10 \div 2}{14 \div 2}$

$= \frac{12}{14}$ $\quad = \frac{5}{7}$

b. $\frac{7 \times 3}{9 \times 3} + \frac{1}{27}$ Test $\frac{22}{27} - \frac{1}{27}$

$= \frac{22}{27} + \frac{1}{27}$ $\quad = \frac{21 \div 3}{27 \div 3}$

$= \frac{22}{27}$ $\quad = \frac{7}{9}$

Q8 Calculate the following. Answers:

a. 3, 6, 9, 12, 15, 18, 21, 24, ...
4, 8, 12, 16, 20, 24, 28, ...
LCM: 12

$\frac{1^{\times 4}}{3_{\times 4}} + \frac{3^{\times 3}}{4_{\times 3}} = \frac{4}{12} + \frac{9}{12} = \frac{13}{12}$

$= 1\frac{1}{12}$

b. 5, 10, 15, 20, 25, 30, ...
6, 12, 18, 24, 30, 36, ...
LCM: 30

$\frac{4^{\times 6}}{5_{\times 6}} + \frac{1^{\times 5}}{6_{\times 5}} = \frac{24}{30} + \frac{5}{30} = \frac{29}{30}$

$= \frac{29}{30}$

Q9 Calculate the following: Answers:

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a. $5\frac{1}{3} + 1\frac{2}{4}$

$= (5 + 1) + (\frac{1}{3} + \frac{2}{4})$

$= 6 + (\frac{1^{\times 4}}{3_{\times 4}} + \frac{2^{\times 3}}{4_{\times 3}})$

$= 6 + \frac{4}{12} + \frac{6}{12}$

$= 6 + \frac{10}{12}$

$= 6\,\frac{10^{\div 4}}{12_{\div 4}} = 6\frac{5}{6}$

or $\frac{16^{\times 4}}{3_{\times 4}} + \frac{6^{\times 3}}{4_{\times 3}}$

$= \frac{64}{12} + \frac{18}{12}$

$= \frac{82}{12}$

$= 6\,\frac{10^{\times 2}}{12_{\times 2}}$

$= 6\,\frac{5}{6}$

b. $4\frac{3}{8} - 3\frac{4}{6}$

$(4 - 3) + (\frac{3}{8} - \frac{4}{6})$

$= 1 + \frac{3^{\times 3}}{8_{\times 3}} - \frac{4^{\times 4}}{6_{\times 4}}$

$= 1 + \frac{9}{24} - \frac{16}{24}$

$= 1\frac{9}{24} - \frac{16}{24}$

$= \frac{33}{24} - \frac{33}{24} - \frac{17}{24}$

or $\frac{35^{\times 4}}{8_{\times 4}} - \frac{6^{\times 3}}{4_{\times 3}}$

$= \frac{105}{24} - \frac{88}{24}$

$= \frac{17}{24}$

Q10 Calculate the following. Answers:

a. $5\frac{1}{3} + 1\frac{2}{4}$

$= \frac{16}{3} + \frac{6}{4} = \frac{64}{12} + \frac{18}{12} = \frac{82}{12}$

$= 6\,\frac{10}{12} = 6\,\frac{5}{6}$

b. $4\frac{3}{8} - 3\frac{4}{6}$

$= \frac{35}{8} - \frac{22}{6} = \frac{105 - 88}{24}$

$= \frac{17}{24}$

Q11 1,2 million goods are sold per annum.
Answers: a. 1,2 million/1 200 000; b. 200 000; c. 600 000; d. 900 000; e. 1 100 000

Q12 What percentage of the circle is red?
Answers: a. 25%; b. 50%; c. 75%; d. 0%

Problem solving

I had $\frac{1}{12}$ of the cake.

My friend had $\frac{1}{4}$ of the cake.

How much cake did we have altogether?

Answer:
$\frac{1}{12} + \frac{1}{4} = \frac{1}{12} + \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$

R8 Decimals

Topic: Fractions (decimal) **Content links:** R8, 40-46
Grade 8 links: R6, 71-75 **Grade 9 links:** R6, 16-18

Objectives

Revise:

- Count forwards and backwards in decimal fractions to at least two decimal places
- Add and subtract decimal fractions with at least two decimal places

Dictionary

Decimal fraction: A decimal fraction is a fraction where the denominator (the bottom number in a common fraction) is a power of ten (such as 10, 100, 1000, etc). Decimal fractions are written with a decimal comma (or point) and no denominator. This makes it a lot easier to do calculations like addition and multiplication with fractions. e.g. 2,45 = 2 + 0,4 + 0,05

Ordering decimal fractions: Ordering decimal fractions can be in ascending order and descending order. e.g. 0,8; 0.73; 0,823.
Ascending order: 0,73; 0,8; 0,823
Descending order: 0,823, 0,8; 0,73

Rounding (decimals): Rounding means to shorten a number and to increase or decrease the value of the last digit of the shortened number so that its value is similar to that of the original number, but easier to use. E.g.:

- 3,6 rounded off to the nearest unit is 4
- 2,32 rounded off to the nearest tenth is 2,3
- 1,738 rounded off to the nearest hundredth is 1,74

Equivalence between common fractions, decimal fractions and percentages: Common fractions, decimal fractions and percentage which have the same value but look different. E.g. = $\frac{25}{100}$ = 0,25 = 25 %

Percent / Percentage: A value expressed as a fraction of 100. Symbol for percentage: %. Percent means 'per hundred'.

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Introduction

Ask the learners where in everyday life do we use:

- Common fractions? Decimal fractions? Percentages?

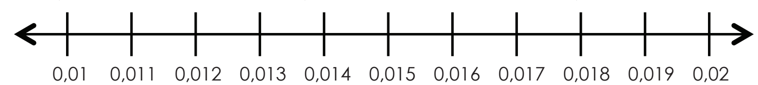
Answers: Examples might be older fruit and vegetables sold at half price (50% off), sale discounts on clothes (20% off, 25% off, half price, etc.)

Q1

Complete the number lines below, using decimal fractions.
Answers:

a. i. 0,1 ii. 0,9 iii. 0,5

b. i. 0,21 ii. 0,09 iii. 0,15



c. i. 0,021 ii. 0,009 iii. 0,015

R8 Decimals *continued*

Topic: Fractions (decimal) **Content links:** R8, 40-46
Grade 8 links: R6, 71-75 **Grade 9 links:** R6, 16-18

Q2 Complete the table below by adding to or subtracting from the number given in the first column. Answers:

Number	Add 0,1	Add 0,01	Add 0,001	Subtract 0,1	Subtract 0,01	Subtract 0,001
a. 0,657	0,757	0,667	0,658	0,557	0,647	0,656
b. 232,232	232,332	232,242	232,233	232,132	232,222	232,231

Q3 Fill in the missing number:
Answers: a. $32,4 + 0,5 = 32,9$ b. $8,452 + 0,04 = 8,492$

Q4 Add and then subtract to test your answer.
Answers:
a. $15,342 = 10 + 5 + 0,3 + 0,04 + 0,002$
b. $456,321 = 400 + 50 + 6 + 0,3 + 0,02 + 0,001$

Q5 Calculate the following using any method

a.
$$\begin{array}{r} 5,326 \\ +\ 4,542 \\ \hline 9,868 \end{array}$$

b.
$$\begin{array}{r} 4,349 \\ +\ 1,874 \\ \hline 6,223 \end{array}$$

c.
$$\begin{array}{r} 32,24 \\ +\ 19,387 \\ \hline 51,627 \end{array}$$

d.
$$\begin{array}{r} 7,630 \\ -\ 4,476 \\ \hline 3,154 \end{array}$$

Q6 Complete the table:
Answers:

Decimal fraction	Common fraction
a. 5,879	$\frac{5\,879}{1\,000}$
b. 18,005	$\frac{18\,005}{1\,000}$

Q7 Answers:
a. R0,50 or 50c; b. R0,50 or 50c; c. R0,50 or 50c; d. R0,25 or 25c;
e. R0,25 or 25c; f. R0,25 or 25c

Q8 Look at the diagram and answer the following: What is 40% of 200? Answer: 80

XXV

Problem solving

I bought trousers for R150 and then got 25% discount. What did I pay for my trousers?
Answer: R112,50

Reflection questions
Did learners meet the objectives??

R9 Patterns

Topic: Numeric and geometric patterns **Content links:** 65-69
Grade 8 links: R7, 27-28 **Grade 9 links:** R7, 27-28

Objectives

Revise the following done in grade 6:

- recognize and use the commutative; associative; distributive property (identify element for addition)

Dictionary

Properties of number: Properties of number include the commutative, associative and distributive properties. Note that these words were not introduced to learners in grade 6. These words will only be introduced in worksheets 1, 2, and 3.

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Learners should give five example for each statement. Discuss these examples in groups and write five examples of each on the board.

Q1

Complete the following:

Answers:

a. $4 - 4 = 0$
b. $0 + 15 = 15$
c. $100\,000 \times 1 = 100\,000$
d. $299\,999 - 299\,999 = 0$
e. $84\,934 \times 1 = 84\,934$

Q2

Replace the shape.

Answers: These are examples of possible answers.

a. $5 - 5 = 0$
b. $30 \times 1 = 30$
c. $8 + 0 = 8$
d. $13 - 13 = 0$
e. $17 \times 1 = 17$

Q3

Complete the flow diagram:

Answers:

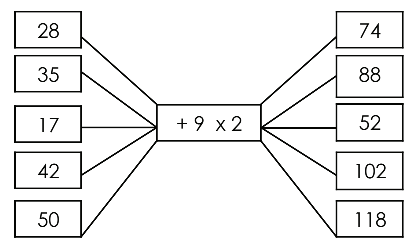
a. 0; 0; 0; 0; 0 [When a number is subtracted from itself the result is 0.]
b. 8; 99; 387 342; $\frac{1}{8}$; 0,75 [When 0 is added to or subtracted from a number, the number remains the same.]

Q4

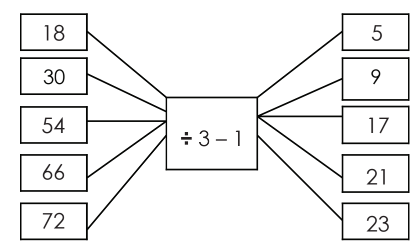
Create your own flow diagrams using the rules:

Answers: Here are some example answers:

a.



b.



R9 Patterns *continued*

Topic: Numeric and geometric patterns **Content links:** 65-69
Grade 8 links: R7, 27-28 **Grade 9 links:** R7, 27-28

Q5

What is the value of x?
Answers:
a. 5 b. 2,5 c. 0,2 d. 0 e. 4,5

Q6

If, a = 2, b = 3, and c = 10, complete and calculate the sums.
Answers:
a. 2 + 3, 3 + 2, Yes
b. 2 x 3, 3 x 2, Yes
c. (2 x 3) x 10, 2 x (3 x 10), Yes
d. (2 + 3) x 10, (2 x 10) + (3 x 10), Yes
e. 10 x 1, 1 x 10, Yes

Q7

Follow the order of operation to calculate each of the following:
Answers:
a. 7 – 3 + 6 = 10
b. 16 + 29 – 87 = –42
c. (96 ÷ 16) x 2 = 12
d. 35 ÷ 5 + (18 – 16) = 9
e. 14 ÷ (36 – 29) + 11= 13

Q8

Follow the order of operation to calculate each of the following:
Answers:
a. 2a + 2b
(2 x 5) + (2 x 2,5)
10 cm + 5 cm
= 15 cm

b. 2a + 2b
(6,1 x 2) + (3 x 2)
12,2 cm + 6 cm
= 18,2 cm

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Problem solving

		2			7			9
	8		2	4	9		3	
	3	1			5	7	2	
		9			8			1
	6	5				8	4	
4			7			2		
	9	3	1			6	5	
	5		8	6	2		7	
8			6				3	2

Sudoku fun

There are 9 rows and 9 columns in a Sudoku puzzle. Every row and column must contain the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. There may not be any duplicate numbers in any row or column.

A region is a 3 x 3 box like the green one shown to the left. There are 9 regions in a traditional puzzle. Like the Sudoku rules for rows and columns, every region must also contain the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9. Duplicate numbers are not allowed in any region.

Answer:

6	4	2	3	1	7	5	8	9
5	8	7	2	4	9	1	3	6
9	3	1	6	8	5	7	2	4
7	2	9	4	5	8	3	6	1
3	6	5	9	2	1	8	4	7
4	1	8	7	3	6	2	9	5
2	9	3	1	7	4	6	5	8
1	5	4	8	6	2	9	7	3
8	7	6	6	9	3	4	1	2

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

R10 2-D shapes and 3-D objects

Topic: 2-D shapes and 3-D objects **Content links:** 95-104
Grade 8 links: R11, R13, 49, 52-58, 60, 127-131
Grade 9 links: R11, R13, 43, 47-52, 114-119

Objectives

Revision:

- Name, describe and identify 2-D shapes and 3-D objects
- Identify faces, vertices and edges in 3-D objects
- Draw 2-D shapes

Dictionary

Geometric figure: A geometric figure is any set of points on a plane or in space, though most frequently used for a 2-D shape that has length and width.

Geometric object: A geometric object is a 3-D object that has length, width and height.

Polygon: A closed two-dimensional figure formed by three or more line segments that do not cross over each other. It is a plane shape with straight sides.

Quadrilateral: A quadrilateral is a polygon with four sides, e.g. rectangle, square, rhombus, parallelogram, kite and trapezium.

Triangle: A polygon with three sides and three angles. The three angles will always add up to 180°. These are names of three special types of triangle: Equilateral, Isosceles and Scalene. Other names tell you about the sides or the angles inside the triangle.

xxx

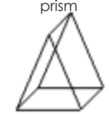
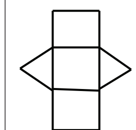
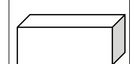
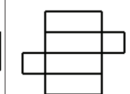
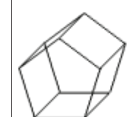
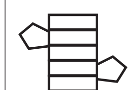
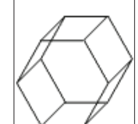
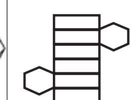
Ask learners what is a:

- 2-D shape
- 3-D object
- 1-D object

Give an example of each.

Q1

Complete the following table:

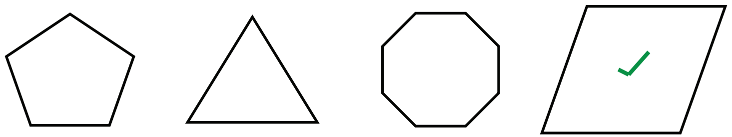
2–D shape within the 3–D object	Name the 3–D object	Draw the net	Number of faces	Number of vertices	Number of edges
2 triangles 3 rectangles	Triangular prism 		5	6	9
2 squares 4 rectangles			6	8	12
5 rectangles 2 pentagons			7	10	15
6 rectangles 2 hectagons			8	12	18

R10 2-D shapes and 3-D objects *continued*

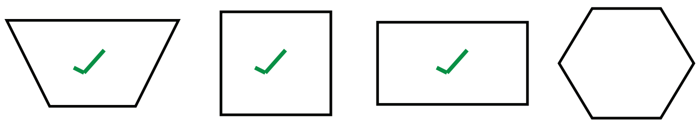
Topic: 2-D shapes and 3-D objects **Content links:** 95-104
Grade 8 links: R11, R13, 49, 52-58, 60, 127-131
Grade 9 links: R11, R13, 43, 47-52, 114-119

Q2

Name the polygons below. Tick all the quadrilaterals.



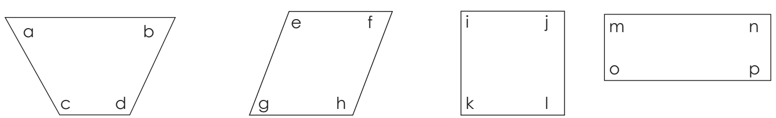
pentagon triangle octagon rhombus



trapezium square rectangle hexagon

Q3

Name the quadrilateral and say if the size of the angles equal 90°, is less than 90° or more than 90°.



trapezium rhombus square rectangle

Answers:

a. less than
b. less than
c. more than
d. more than
e. more than
f. less than
g. less than
h. more than
i. equal square
j. equal
k. equal
l. equal
m. equal
n. equal
o. equal
p. equal

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Q4

Make a tick in the correct answer column. [Or learners can use the < (less than) or > (more than) symbols to give their answers.]

Answers:

This shape can have:	1 right angle	2 right angles	3 or more right angles	No right angles
Square			✔	
Rhombus				✔
Triangle	(✔)			
Hexagon				(✔)
Trapezium		(✔)		(✔)
Quadrilateral	(✔)	(✔)	(✔)	(✔)
Rectangle			✔	
Octagon				✔

[(✔) symbol in brackets indicates that the presence or absence of a right angle or angles depends on the actual shape of the shape.]

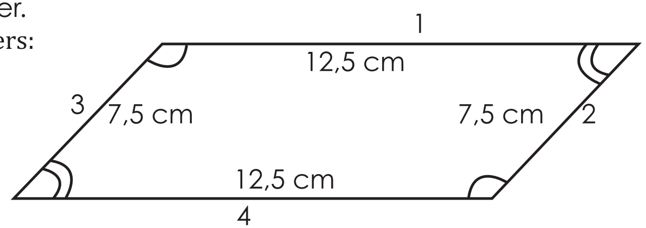
R10 2-D shapes and 3-D objects *continued*

Topic: 2-D shapes and 3-D objects **Content links:** 95-104
Grade 8 links: R11, R13, 49, 52-58, 60, 127-131
Grade 9 links: R11, R13, 43, 47-52, 114-119

Q5

Learners answer the following questions: You know the lengths of 3 sides of a parallelogram: 12,5 cm, 7,5 cm and 7,5 cm. Is that enough information to work out the length of the 4th side? If so, what is it? Make a drawing to support your answer.

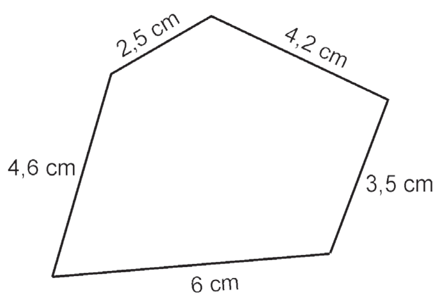
Answers:



Q6

You know the lengths of 4 sides of a pentagon: 2,5 cm, 4,2 cm, 3,5 cm and 6 cm. What will the 5th side be? Make a drawing to support your answer.

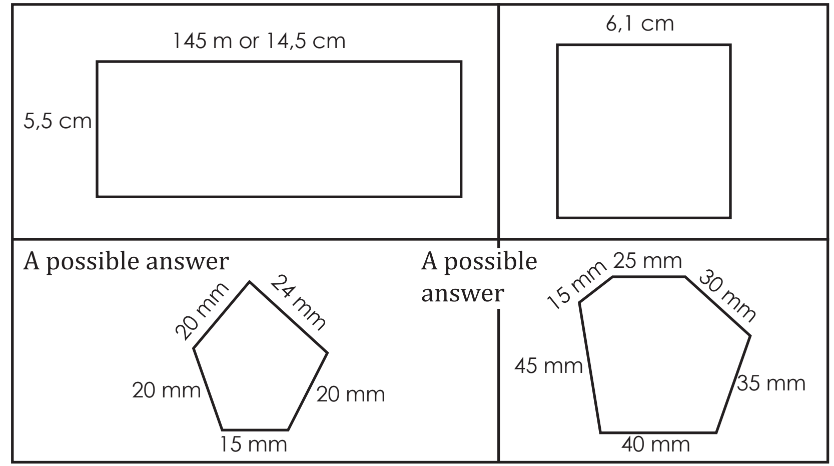
Answers: Answers will differ from learner to learner.



Q7

Draw the following:

Answers:



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Problem solving

Magazine or newspaper search
Find the following shapes in a magazine: quadrilateral, triangle and hexagon. Paste them here and describe their angles and sides.

Quadrilateral	Triangle	Hexagon
• 4 sides • sum of the angles = 360^0	• 3 sides • sum of the angles = 180^0	• 6 sides • sum of the angles = 720^0

R11 Transformations

Topic: Transformations **Content links:** 86-94
Grade 8 links: R12, 121-126 **Grade 9 links:** R12, 105-113

Objectives

Revision:

- Recognise, describe and perform transformations.

Dictionary

Transformation: to change the form or appearance of something. There are many kinds of geometric transformations, including translations, rotations, reflections and enlargements.
Rotation: a transformation that moves points so that they stay the same distance from a fixed point, the centre of rotation.
Rotational symmetry: a figure has rotational symmetry if an outline of the turning figure matches its original shape.
Order of rotational symmetry: how many times an outline matches the original in one full rotation.
Reflection: a transformation that has the same effect as a mirror.
Reflective symmetry: an object is symmetrical when one half is a mirror image of the other half.
Translation: a translation is the movement of an object to a new position without changing its shape, size or orientation.

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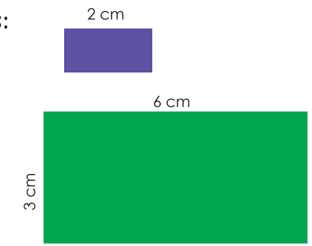
Ask the learners if a reflection is a transformation which has the same effect as a mirror. What effect will the following have? - rotation, - translation, - enlargement

Q1

Answer the following questions. Answers:



Purple rectangle:
a. The length = 2 cm
b. The width = 1 cm
Green rectangle:
c. The length = 6 cm
d. The length = 3 cm
e. The purple rectangle is enlarged 3 times.
[Length: 2 cm x 3 = 6 cm; width: 1 cm x 3 = 3 cm]

Q2

Complete the table. Make drawings if needed.

Rectangle	Perimeter	Area	Enlarge by:	Perimeter	Area
a. Length: 4 cm Width: 2 cm	12 cm	8 cm^2	2 times Length: 8 cm Width: 4 cm	24 cm	32 cm^2
b. Length: 3 cm Width: 2 cm	10 cm	6 cm^2	3 times Length: 9 cm Width: 6 cm	30 cm	54 cm^2
c. Length: 5 cm Width: 4 cm	18 cm	20 cm^2	4 times Length: 20 cm Width: 16 cm	72 cm	320 cm^2
d. Length: 6 cm Width: 3 cm	18 cm	18 cm^2	2 times Length: 12 cm Width: 6 cm	36 cm	72 cm^2
e. Length: 7 cm Width: 6 cm	26 cm	42 cm^2	3 times Length: 21 cm Width: 18 cm	78 cm	378 cm^2

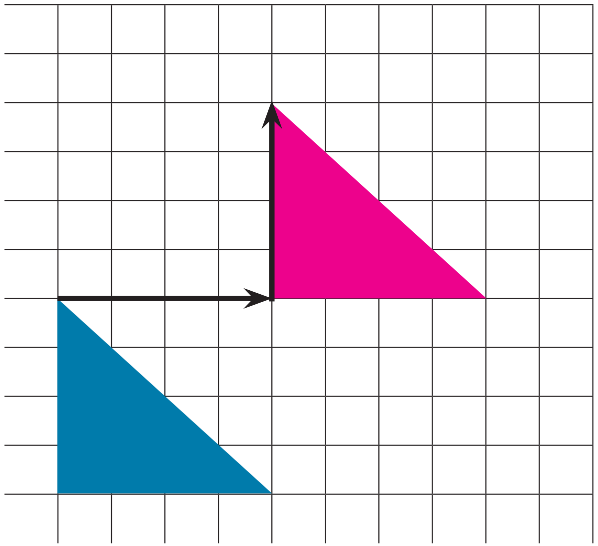
R11 Transformations *continued*

Topic: Transformations **Content links:** 86-94
Grade 8 links: R12, 121-126 **Grade 9 links:** R12, 105-113

To summarise what happens in question 2: When you enlarge a shape by a scale factor the area is enlarged by that scale factor squared. For example. The area of a square 2 cm by 2 cm is 4 cm^2. Enlarge the square by a factor of 2 and you have a square 4 cm by 4 cm and an area of 16 cm^2 = 4^2 cm^2.

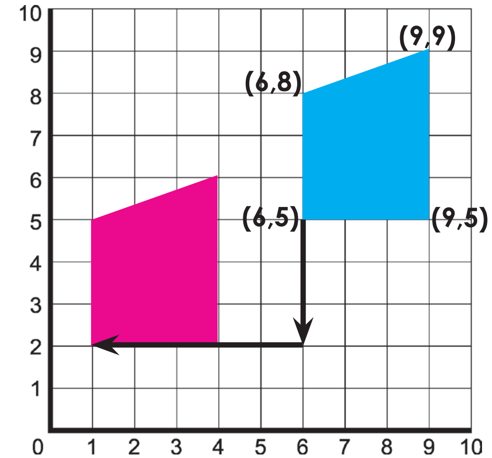
Q3 Slide the figure 4 right, 4 up

Answers:



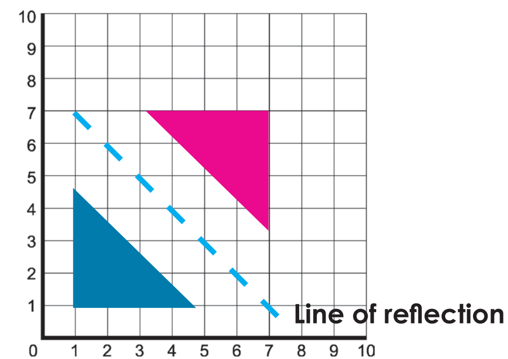
Q4 Plot the coordinates (9,9); (6,8); (6,5); (9,5) and connect the points in order. Then slide 3 down and 5 left and draw the figure at these new coordinates.

Answers:



Q5 Reflect the figure.

Answer:

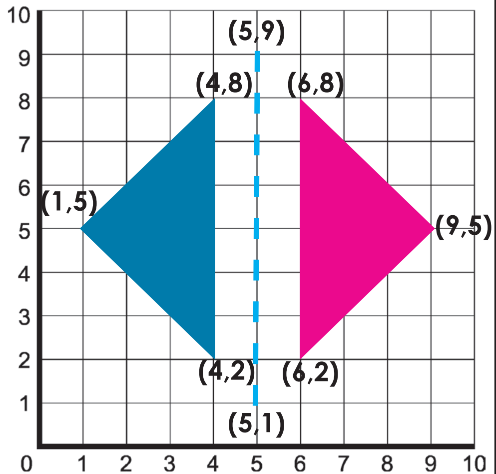


Topic: Transformations **Content links:** 86-94
Grade 8 links: R12, 121-126 **Grade 9 links:** R12, 105-113

Q6

Draw a triangle with coordinates: (4,8); (1,5); (4,2). Then draw its reflection across a reflection line with coordinates (5,9); (5,1). Write the coordinates of the new triangle.

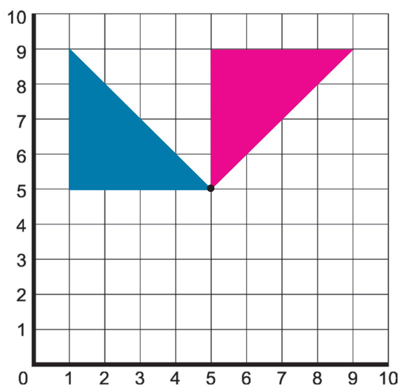
Answers: (6,8);(9,5);(6,2)



Q7

Rotate the figure by a quarter of a revolution around the point (5,5).

Answer:

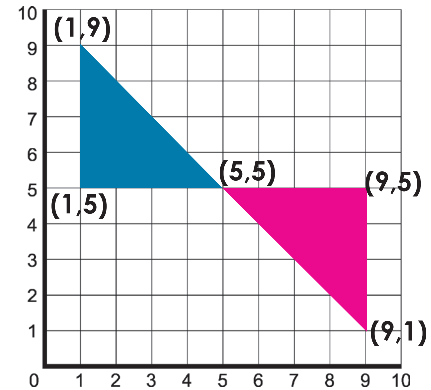


Q8

Draw a half turn image of the figure: Triangle: (5,5); (1,5); (1,9)

Write down the new coordinates.

Answers:
(5,5); (9,5); (9,1)



Q9

Answer: No.

Q10

Answer: Yes.

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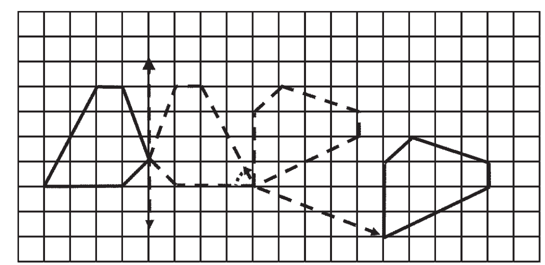
Problem solving

Draw a transformation using reflection, rotation and translation on one graph showing the movement from one figure to the next.

Answer:
(Here is an example of a figure that is: reflected, then rotated – clockwise by 90^0, and then translated 5 blocks to the left and 2 blocks down.)



R12 Area, perimeter and volume

Topic: Size, area, perimeter and volume of 2-D shapes and 3-D objects
Content links: R14, 52-64 **Grade 8 links:** R14-R15, 82-91
Grade 9 links: R14-R15, 60-64, 100-104

Objectives

- Calculate the area and perimeter of 2-D shapes.
- Calculate the volume of a 3-D object.

Dictionary

Perimeter: The distance around a shape
Formula for a square: $4\,l$
Formula for a rectangle: $2\,l + 2\,b$

Area: The amount of surface of a two-dimensional shape.
Formula for a square: l^2
Formula for a rectangle: $l \times b$

Volume: The volume of an object is the amount of space it fills.
Formula for a cube: $l^2 \times h$
Formula for a rectangular prism: $l \times b \times h$

Capacity: Similar in meaning to volume but it refers to the container of that space. So, for example, we speak of a vessel have the **capacity** to hold a certain **volume** of liquid.

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Introduction

Ask the learners to read through the comic strip, and answer the following questions:

- What is breadth? What is width? Are they the same?
- What is perimeter? What is area? What is length?

Q1

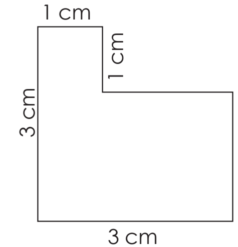
Calculate the perimeter and area of the following polygons.

Answers: Determine the length of the missing sides.

a. Draw a horizontal line from 1 cm down the width so that one has a square 1 cm by 1 cm and a rectangle 2 cm by 3 cm.
Or
Draw a vertical line 1 cm along the length so that one has a rectangle 2 cm by 1 cm and a rectangle 2 cm by 2 cm.

b. 3 cm + 1 cm + 1 cm + 2cm + 2cm + 3 cm = 12 cm

c. 1 cm x 1 cm = 1 cm^2
2 cm x 3 cm = 6 cm^2
Total = 7 cm^2
Or
3 cm x 1 cm = 3 cm^2
2 cm x 2 cm = 4 cm^2
Total = 7 cm^2



Q2

Calculate the perimeter and area of the following rectangles.

Answers:

Perimeter	Area
a. 2 l + 2 b	= l x b
= 2(10 cm) + 2(8 cm)	= 10 cm x 8 cm
= 20 cm + 16 cm	= 80 cm^2
= 36 cm	
b. 2 l x 2 b	l x b
= 2(10 cm) + 2(7,5 cm)	= 10 cm x 7,5 cm
= 20 cm + 15 cm = 35 cm	= 75 cm^2

Topic: Size, area, perimeter and volume of 2-D shapes and 3-D objects
Content links: R14, 52-64 **Grade 8 links:** R14-R15, 82-91
Grade 9 links: R14-R15, 60-64, 100-104

R12 Area, perimeter and volume *cont...*

Q3

If you have a rectangle with the following area, what could its length and breadth be? What is the perimeter? Area = 210 m^2

Answer: These are some possible answers.

Length	Breadth	Perimeter
15 m	14 m	58 m
21 m	10 m	62 m
30 m	7 m	74 m
70 m	3 m	146 m
10,5 m	20 m	61 m
52,5 m	4 m	113 m

Oral questions

After the learners have completed question 3, ask them what dimensions they would choose if they had to build a wall around the area in the most cost effective way.

Answer: the dimensions with the shortest perimetre (61 m)

Q4

Sipho and his father are building a deck because the old one is too small. The old deck was 2,5 m x 3 m. They are going to double the dimensions of the deck. Answers:

a. Original area = 2,5 m x 3 m
New area = (2,5 m x 2) x (3 m x 2)
Area = 5 m x 6 m
Area = 30 m^2

b. Original perimeter = 2,5 m + 3 m + 2,5 m + 3 m
New perimeter = 5 m + 6 m + 5 m + 6 m
Perimeter = 22 m

Q5

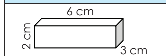
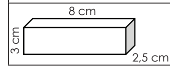
If a rectangular prism has a volume of 36 cubic units, what might be the: a. height; b. width; c. length?

These are some possible answers:

Height	Width	Length
3	3	4
3	2	6
2	2	9
6	1	6

Q6

Complete the table below. Answers:

	Length	Width	Height	Short way to calculate	Volume
	6 cm	3 cm	2 cm	Length x width x height 6 cm x 3 cm x 2 cm	36 cm^3
	8 cm	2,5 cm	3 cm	length x width x height 8 cm x 2,5 cm x 3 cm	60 cm^3

Q7

If you have a rectangular prism with the following volume, what could the length, breadth and height be.

Volume = 2 100 m^3. These are some possible answers:

Length		Breadth		Height		
70 m	x	30 m	x	1 m	=	2 100 m^3
30 m	x	14 m	x	5 m	=	2 100 m^3
25 m	x	21 m	x	4 m	=	2 100 m^3
21 m	x	10 m	x	10 m	=	2 100 m^3
15 m	x	35 m	x	4 m	=	2 100 m^3
12 m	x	25 m	x	7 m	=	2 100 m^3

xxxix

Problem solving

Investigate: How many different ways can you draw a square and rectangles covering 64 square units? Show them.

- Do all of the above shapes have the same area?
- Do they all have the same perimeter?

Now try a similar activity with an object of 64 cubic units.

Answer:	Perimeter in units	Area in square units
8 × 8	32	64
4 × 16	40	64
2 × 32	68	64
1 × 64	130	64

All have same area but different perimeters.

R13 Time

Topic: Time **Content links:** None
Grade 8 links: None **Grade 9 links:** None

Objectives

- Read, tell and write time in 12-hour and 24-hour formats on both analogue and digital instruments
- Calculate elapsed time.

Dictionary

Measure: The use of standard units to find out size or quantity with regard to time, mass, capacity, length, temperature, perimeter, area and volume.

Digital: Refers to the use of digits/numbers. Digital time is shown on a clock or watch that shows numbers that are changed electronically in a little window (as opposed to a number dial with long and short hands of time). This kind of clock is called a digital clock/watch. Example of digital time: 12:45

Watch: A small portable timepiece, especially one worn on the wrist or carried in the pocket. It has moving hands or a digital display.

xl

Introduction
Ask leaners why:
0,5 hours = 30 minutes, not 50 minutes.
Learners give answers back in pairs. Give them enough time to get to an answer.

Q1

This is how long I took to complete my maths homework this week. Help me to complete this table. ***Answers:***

Maths homework	Hours	Minutes	Seconds	hh:mm:ss	I started my homework at:	I finished it at:
Monday	1	30	1	01:30:01	15:00	16:30:01
Tuesday	1	15	25	01:15:25	15:30	16:45:25
Wednesday	1	27	17	01:27:17	16:30	17:57:17
Thursday	0	55	45	00:55:45	17:45	18:40:45
Friday	1	15	9	01:15:09	14:50	16:05:09

Q2

I visited my grandmother over the weekend. On Saturday, I arrived 10:57:02 at her house. I left on Sunday at 13:45:05. How long was my visit to my grandmother?

Answers:
Saturday 10:57:02 to 24:00:00 = 13 hours 2 minutes and 58 seconds
Sunday 00:00:00 to 13:45:05 = 13 hours 45 minutes and 5 seconds
Total = 26 hours 48 minutes and 3 seconds

R13 Time *continued*

Topic: Time **Content links:** None
Grade 8 links: None **Grade 9 links:** None

Q3

Complete the table. ***Answers:***

Weeks	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6
Days	7	10,5	14	17,5	21	24,5	28	31,5	35	38,5	42
Hours	168	252	336	420	504	588	672	756	840	924	1 008
Minutes	10 080	15 120	20 160	25 200	30 240	35 280	40 320	45 360	50 400	55 440	60 480

Q4

Convert years to weeks and days:
Answers:

a. 260 (260,89) weeks and 1 826 (1 826.25) days [Taking into account that there are 365,25 days a year. If a simpler formula of 52 weeks a year (instead of 52.18 weeks) is used then the answers are 260 weeks and 1 820 days.]

b. 1 330 (1330,55) weeks and 9 313 (9 313.875) days [Taking into account that there are 365,25 days a year. If a simpler formula of 52 weeks a year is used then the answers are 1 326 weeks and 9 282 days.]

Q5

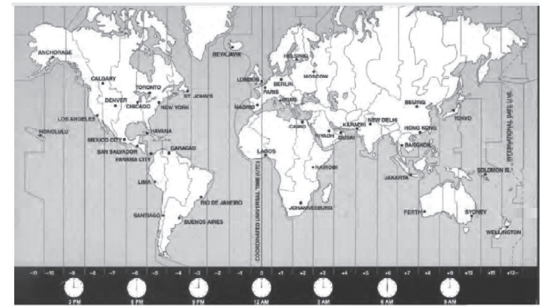
Convert centuries to years:
Answers:

a. $10 \times 100 = 1\ 000$
b. $500 + 25 = 525$

Q6

Time zones:
Answers:

a. 24. There is one time zone per hour therefore hours = 24 time zones.
b. Zimbabwe, Lesotho, Egypt, etc.
c. USA, Canada, etc.



xli

Problem solving

It took Sam 3 hours to travel 100 km. How many kilometres per hour did he travel? How long will it take him to travel 120 km? Give your answer in hours and minutes. What do you think he was travelling on at this speed?

Answer:
100 km divided by 3 hours = 33,33 km per hour.
120 km will take $\frac{120}{100} \times 3$ hours = 3,6 hours = 3 hours and 36 minutes
He was probably on a bicycle.

Reflection questions

Did learners meet the objectives?

Topic: Measurement **Content links:** R12, 52-64
Grade 8 links: R14-R15, 82-91 **Grade 9 links:** R14-R15, 60-64, 100-104

R14 Temperature, length, mass and capacity

Objectives

Convert:

- Length: mm, cm, m, km
- Mass: mg, g, kg, t
- Capacity: ml; l; kl
- Measure temperature

Dictionary

Measure: The use of standard units to find out size or quantity with regard to time, mass, capacity, length, temperature, perimeter, area and volume.

Temperature: A measurement of how hot or cold something is. Measured with a thermometer. Measured in degrees Celsius - temperature scale used in South Africa (°C).

Length: A distance between two points. The measuring units used in the Intermediate Phase were: millimetre (mm), centimetre (cm) and kilometre (km).

Mass: This measures the amount of matter that makes up an object. It is similar to weight, but mass stays constant, while the weight measurement could change according to the gravity in the environment.

An object would be weightless in space, but still have the same mass as on earth. The measuring units used in the Intermediate Phase for mass are: gram (g), and kilogram (kg).

Capacity: The amount a container can hold. The measuring units used in the Intermediate Phase are: millilitre (ml) and litre (l).

xlii

Introduction

Ask the learners what the following is, give an example of each and with what do we measure it.

- Temperature
- Length
- Mass
- Capacity

Q1

Write down each temperature.

Answers:

a. -5^0C	b. 0^0C	c. 9^0C
d. 2^0C	e. -8^0C	f. -8^0C
g. 9^0C	h. -3^0C	i. 1^0C less than -8^0C

Q2

What is the difference in temperature shown in Question 1 between:

Answers:

a. 5^0C	b. 9^0C	c. 2^0C
d. 10^0C	e. 3^0C	

R14 Temperature, length, mass and capacity *continued*

Q3 Answer the following questions about length.

Answers: a. 10 mm b. 100 cm c. 1 000 mm d. 1000 m e.

		mm	cm	m	km
i.	9 cm	90	9	0,09	0,00009
ii.	3 m	3 000	300	3	0,003
iii.	2 km	2 000 000	200 000	2 000	2
iv.	10,5 m	10 500	1 050	10,5	0,0105
v.	3 600 mm	3 600	360	3,6	0,0036

f. 2 500 – (450 km + 565 km + 900 km) = 2 500 – 1 915 = 585 km

Q4 Answer the following questions on mass.

Answers: a. 1 000 g b.1 000 kg c. 1 000 mg d. 1 000 000 mg

e.

		mg	g	kg	t
i.	3 500 g	3 500 000	3 500	3,5	0,0035
ii.	2 kg	2 000 000	2 000	2	0,002
iii.	2,5 kg	2 500 000	2 500	2,5	0,0025
iv.	3 t	3 000 000 000	3 000 000	3 000	3
v.	5 000 000 mg	5 000 000	5 000	5	0,005

f. {[(0,250 kg + 0,500 kg – 0,200 kg) x 2] + 1 000 kg} ÷ 2
= {[(0,550 kg) x 2] + 1 000 kg} ÷ 2
= {[1,1 kg] + 1 000 kg} ÷ 2
= {1 001,1 kg} ÷ 2
= 500,55 kg (or 500 550 g)

Q5 Answer the following questions on capacity.

Answers: a. 1 000 ml b. 1 000 l c. 1 000 000 ml

d.

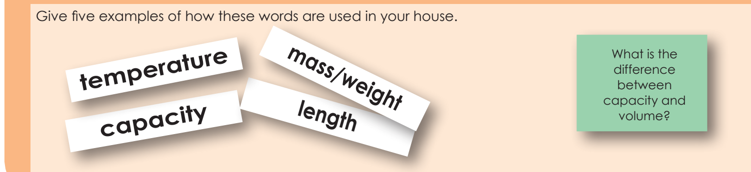
		ml	l	kl
i.	5 250 ml	5 250	5,25	0,00525
ii.	4,5 l	4 500	4,5	0,0045
iii.	3 kl	3 000 000	3 000	3
iv.	9 999 ml	9 999	9,999	0,009999
v.	1,75 l	1 750	1,75	0,00175

e. 375 000 l; 375 kl

xlv

Problem solving

Give five examples of how these words are used in your house.



Answer: Temperature: Oven/microwave; Capacity: Milk/water; Mass/weight: Food; Length: Garden, swimming pool

R15 Probability

Topic: Probability **Content links:** N137-140 [13–136]
Grade 8 links: 135-138 **Grade 9 links:** 138-143

Objectives

- Perform simple experiments where the possible outcomes are equally likely and:
- List the possible outcomes based on the conditions of the activity

Dictionary

Probability: Probability refers to the chance or likelihood of something happening.
Outcome: An outcome (in this context) is the result of a single trial of an experiment.

xlvi

Introduction

Look at the following pictures and ask yourself, "What is the probability that this will happen today?"



Q1 Draw and make these two nets on cardboard, cut, fold and stick them to make two dice.

		c	
e	a	b	f
		d	

		c	
e	a	b	f
		d	

Q2 Roll the two dice a 100 times and write down each time the same two letters occur. Use tallies to record your answers in the table below.

Q3 Compare your answers with those of a friend. Are they the same? Why?
Answer: In 100 throws of the two dice, same letter combinations should appear about 16 times, and each individual same letter combination about 3 times each.

Q4 You need to prepare. You need an empty bag.
You need to make a set of 10 cards using old card board or paper. Each card should be 4 cm by 4 cm.

x	y	z	m	n
a	a	a	b	k

R15 Probability *continued*

Topic: Probability **Content links:** N137-140 [13–136]
Grade 8 links: 135-138 **Grade 9 links:** 138-143

Q5 Draw a card from the bag and record it below. Place the card back into the bag. Do this 100 times.

Letter on the card	Times landed on the letter
x	
y	
z	
m	
a	
b	
k	

Q6 Compare your answers with your friend. Are they the same? Why?

Q7 Drawing a number x card from the bag has a probability of 1 out of 10. We can write it as

Answers:

$y = \frac{1}{10}$ $z = \frac{1}{10}$ $m = \frac{2}{10}$ $a = \frac{3}{10}$ $b = \frac{1}{10}$ $k = \frac{1}{10}$

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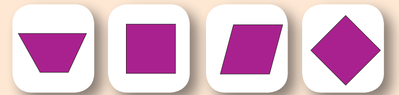
Problem solving

Card fun: Do a similar activity but use the following quadrilateral cards.



Answer: Learner's own answer. It should be about 1 in 4 probability (25%) of drawing a particular card.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

R16 Data

Topic: Data handling **Content links:** 126-136
Grade 8 links: R16, 92-104 **Grade 9 links:** R16, 123-137

Objectives

- Revise the data handling process.

Dictionary

Data: A complete set of individual pieces of information or facts that have been collected, but have not yet been interpreted.

Data handling: This is a process. It begins with a question. The purpose of collecting data is to find the answer to this question.
Learners should be given the opportunity to collect data to answer 'real' questions related to the learners' experience.

It is possible to use data from first-hand (primary) sources when the data is collected by the learners directly. Data can also come from second-hand (secondary) sources such as prepared databases, reference books, newspapers, registers, weather statistics and so on.

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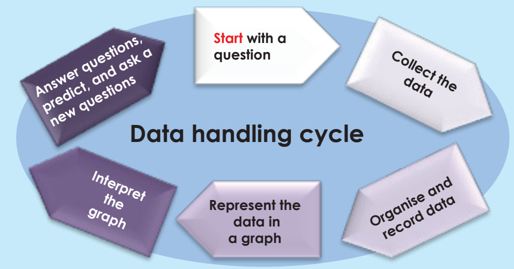
Learners look at the data handling diagram and explain it with the support of the teacher.

Look at this data-handling cycle and describe it.



Q1

Answer the question on the pictograph.
Answers:
a.

1st quarter Jan – March	2nd quarter April – June	3rd quarter July – September	4th quarter October – December
350 000	550 000	450 000	600 000

b. People saved money during the year to replace their car at the end of the year.
c. The data was collected. Then it was organised and represented in the pictograph. Then it will be interpreted (as Question 1b. asks the learner to do).

Q2

Sort the data using the table below.

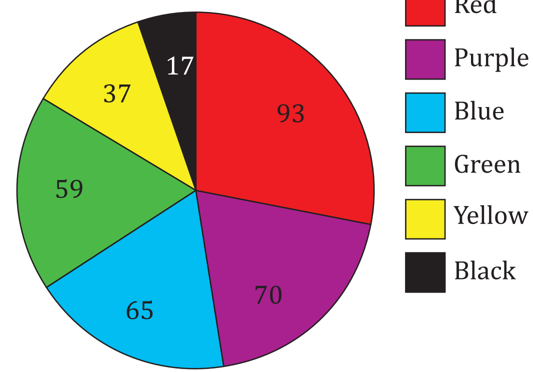
Answers:

Colour	Tally	Frequency
Red		93
Purple		70
Blue		65
Green		59
Yellow		37
Black		17
Total		341

Q3

Use the information from the frequency table to draw and label the pie chart.

Favourite colours



xlix

Problem solving

Collect data about cell phones usage in your class and draw a bar chart of your results. Explain what you need to do.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Properties of numbers **Content links:** R9, 1-4, 111
Grade 8 links: 2 **Grade 9 links:** 10

1 Commutative property of addition and multiplication

Objectives
- Recognise and use the commutative property of whole numbers

Dictionary
Commutative property: The commutative property of addition and multiplication says that you can swap numbers around and still get the same answer when you add or multiply.

Equation: An equation says that two things are the same, using an equal sign (=). E.g. : 7 + 4 = 11 – 1

2 **Introduction**

Commutative property of addition and multiplication

Are the following true or false?
3 + 4 = 4 + 3
3 x 4 = 4 x 3
20 + 5 = 5 + 20
20 x 5 = 5 x 20
What do you notice?

The **commutative property of addition and multiplication** says that you can swap numbers around and still get the same answer when you add or multiply. The order in which you combine the numbers does not matter.

An **equation** says that two things are the same using an equal sign (=), e.g. 7 + 4 = 12 – 1

Q1 Use the commutative property of addition or multiplication to make the equations true.

Example: 5 + 1 = 1 + 5 (addition) and 5 x 1 = 1 x 5 (multiplication)

Answers:
a. 13 + 2 = 2 + 13
b. 62 + 31 = 31 + 62
c. 4 x 5 = 5 x 4
d. 7 x 9 = 9 x 7
e. 9 x 8 = 8 x 9
f. 12 x 15 = 15 x 12
g. Learner's own answers

Q2 Use the commutative property of addition or multiplication to make the equations true.

Example: $f + e = e + f$ (addition) and $f \times e = e \times f$ (multiplication)

Answers:
a. $a + b = b + a$
b. $c \times d = d \times c$
c. $m \times n = n \times m$
d. $h + g = g + h$
e. $l \times p = p \times l$
f. $s \times t = t \times s$
g. Learner's own answers

1 Commutative property of addition and multiplication *continued*

Q3

Show that the given equations are equal when you substitute, $a = 2$, $b = 5$ and $c = 3$

Example: $a + b = b + a$ (addition)
$a + b = 2 + 5$ and $b + a = 5 + 2$
$= 7$ $= 7$
$a + b = b + a$

$a \times b = b \times a$ (addition)
$a \times b = 2 \times 5$ and $b \times a = 5 \times 2$
$= 10$ $= 10$
$a \times b = b \times a$

Answers:

a. $c + a = c + a$
$3 + 2 = 2 + 3$
$5 = 5$

b. $c \times a = a \times c$
$3 \times 2 = 2 \times 3$
$6 = 6$

c. $b \times a = a \times b$
$5 \times 2 = 2 \times 5$
$10 = 10$

d. $b + a = a + b$
$5 + 2 = 2 + 5$
$7 = 7$

e. $b \times c = c \times b$
$5 \times 3 = 3 \times 5$
$15 = 15$

f. $b + c = c + b$
$5 + 3 = 3 + 5$
$8 = 8$

Q4

Write an equation to show how each diagram illustrates the commutative property of multiplication.

Answers:

a. $4 \times 3 = 3 \times 4$
b. $5 \times 6 = 6 \times 5$
c. $6 \times 2 = 2 \times 6$
d. $4 \times 1 = 1 \times 4$

3

Problem solving

If $a = 25$ and $b = 30$, show that the commutative properties of addition and multiplication apply.

Answer: Learner's own answer. It should be something like this:

$a + b = b + a$
$25 + 30 = 30 + 25$
$55 = 55$

$a \times b = b \times a$
$25 \times 30 = 30 \times 25$
$750 = 750$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

2 Associative property of addition and multiplication

Objectives

- Recognise and use the associative property of number.

Dictionary

Associative property: The associative property of addition and multiplication says that it doesn't matter how you group numbers when you add or multiply.

4

Introduction

Are the following true or false?

5 + (3 + 2) = (5 + 3) + 2

9 x (2 x 3) = (2 x 3) x 9

(12 + 14) + 13 = 12 + (14 + 13)

(11 x 2) x 4 = 11 x (2 x 4)

What do you notice?

The **associative property of addition and multiplication** says that it doesn't matter how you group numbers when you add or multiply.

What will happen if you subtract and divide these equations? Will this still be true?

Q1

Use the associative property of addition or multiplication to make the statements true.

Example: (5 + 1) + 3 = 5 + (1 + 3) (addition)

(5 x 1) x 3 = 5 x (1 x 3) (multiplication)

Answers:

a. (6 + 2) + 4 = 6 + (2 + 4)   12 = 12
b. (7 + 3) + 1 = 7 + (3 + 1)   11 = 11
c. 8 x (10 x 4) = (8 x 10) x 4   320 = 320
d. 4 x (5 x 2) = (4 x 5) x 2   40 = 40
e. (11 x 3) x 2 = 11 x (3 x 2)   66 = 66
f. (12 x 2) x 4 = 12 x (2 x 4)   96 = 96

Q2

Use the associative property of addition to make the statements true.

Example: $f + (g + h) = (f + g) + h$ (addition)

f x $(g$ x $h) = (f$ x $g)$ x h (multiplication)

Answers:

a. $a + (b + c)$
b. $m + (n + c)$
c. $g \times (h \times i)$
d. $c \times (d \times f)$
e. $k \times (z \times d)$
f. $a + (d + v)$
g. $a \times (c \times d)$
h. $k \times (l \times m)$
i. $v + (c + r)$

2 Associative property of addition and multiplication *continued*

Q3

Solve if $a = 2$, $b = 4$ and $c = 3$

Examples:

$a + (b + c) = (a + b) + c$	$a \times (b \times c) = (a \times b) \times c$
$2 + (4 + 3) = (2 + 4) + 3$	$2 \times (4 \times 3) = (2 \times 4) \times 3$
$2 + 7 = 6 + 3$	$2 \times 12 = 8 \times 3$
$9 = 9$	$24 = 24$
$\therefore a + (b + c) = (a + b) + c$	$\therefore a \times (b \times c) = (a \times b) \times c$

Answers:

a. $(c + a) + b = c + (a + b)$
$(3 + 2) + 4 = 3 + (2 + 4)$
$5 + 4 = 3 + 6$
$9 = 9$
$\therefore (c + a) + b = c + (a + b)$

b. $(b \times a) \times c = a \times (b \times c)$
$(4 \times 2) \times 3 = 2 \times (4 \times 3)$
$8 \times 3 = 2 \times 12$
$24 = 24$
$\therefore (b \times a) \times c = a \times (b \times c)$

c. $b \times (c \times a) = c \times (a \times b)$
$4 \times (3 \times 2) = 3 \times (4 \times 2)$
$4 \times 6 = 3 \times 8$
$24 = 24$
$\therefore b \times (c \times a) = c \times (a \times b)$

d. $b + (c + a) = (b + c) + a$
$4 + (3 + 2) = (4 + 3) + 2$
$4 + 5 = 7 + 2$
$9 = 9$
$\therefore b + (c + a) = (b + c) + a$

Q4

If $m = 1$, $n = 7$ and $q = 2$, show that the equations are equal.

Answers:

a. $(q + m) + n = q + (m + n)$
$(2 + 1) + 7 = 2 + (1 + 7)$
$3 + 7 = 2 + 8$
$10 = 10$
$\therefore (q + m) + n = q + (m + n)$

b. $(n \times m) \times q = m \times (n \times q)$
$(7 \times 1) \times 2 = 1 \times (7 \times 2)$
$7 \times 2 = 1 \times 14$
$14 = 14$
$\therefore (q + m) + n = q + (m + n)$

c. $n \times (q \times m) = q \times (n \times m)$
$7 \times (2 \times 1) = 2 \times (7 \times 1)$
$7 \times 2 = 2 \times 7$
$14 = 14$
$\therefore n \times (q \times m) = q \times (n \times m)$

d. $n + (q + m) = (n + q) + m$
$7 + (2 + 1) = (7 + 2) + 1$
$7 + 3 = 9 + 1$
$10 = 10$
$\therefore n + (q + m) = (n + q) + m$

5

Problem solving

If $a = 25$, $b = 30$ and $c = 10$, write an associative property of addition and multiplication statement and solve it.

Answer: Example answers are:

$a + (b + c) = (a + b) + c$
$25 + (30 + 10) = (25 + 30) + 10$
$25 + 40 = 55 + 10$
$65 = 65$

$a \times (b \times c) = (a \times b) \times c$
$25 \times (30 \times 10) = (25 \times 30) \times 10$
$25 \times 40 = 750 \times 10$
$7\,500 = 7\,500$

Reflection questions

Did learners meet the objectives?

Topic: Properties of numbers **Content links:** R9, 1-4, 113
Grade 8 links: 2 **Grade 9 links:** 10

3 Distributive property of multiplication over addition

Objectives

- Recognise and use the distributive property of numbers.

Dictionary

Distributive property: You will get the same answer when you multiply a group of numbers added together as when you do each multiplication separately and then add them together.
e.g. 2 x (3 + 4) = (2 x 3) + (3 x 4)
$a(b + c) = (a \times b) + (a \times c)$

6

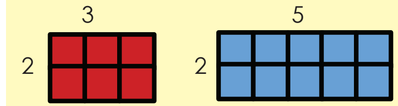
Introduction

2(3) 3(9) 4(100)
4(6) 7(8)

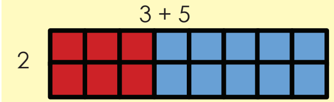
What do the brackets mean?
Look at this statement:
2(3 + 2).
How do you think I will calculate this?

The distributive property lets you multiply a single number and each of two or more numbers between brackets (the products of which you then add together).
You will get the same answer when you multiply a group of numbers added together as when you do each multiplication separately and then add them together.
2(3 + 2) = 2 (5) = 10
2(3 + 2) = (2 × 3) + (2 × 2) = 6 + 4 = 10
Usually we follow the rule that anything in brackets must be done first. In this example it would have been very easy to do this, 2(3+2) = 2(5) = 10. But the distributive property becomes very useful when what is inside the brackets is more complicated.

Q1 Use the distributive property to write a sum for each diagram so that you can calculate the total number of blocks in each drawing.
Answers:

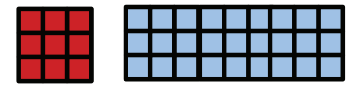


$(2 \times 3) + (2 \times 5)$

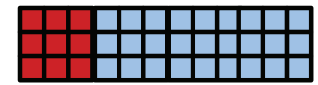


$2(3 + 5)$

a.

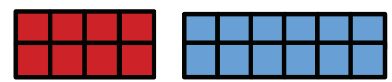


$(3 \times 3) + (3 \times 9)$



$3(3 + 9)$

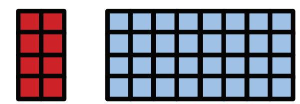
b.



$(2 \times 4) + (2 \times 6)$

$2(4 + 6)$

c.



$(4 \times 2) + (4 \times 8)$

$4(2 + 8)$

Topic: Properties of numbers **Content links:** R9, 1-4, 113
Grade 8 links: 2 **Grade 9 links:** 10

3 Associative property of multiplication over addition *continued*

Q2

Use the distributive property of multiplication to make these statements true.

Example: 4(5 + 9) = (4 x 5) + (4 x 9)

Answers:

a. 3(4 + 2) = (3 x 4) + (3 x 2)

3 | 4 | 2
12 + 6 = 18

b. 10(2 + 3) = (10 x 2) + (10 x 3)

10 | 2 | 3
20 + 30 = 50

c. 5(3 + 1) = (5 x 3) + (5 x 1)

5 | 3 | 1
15 + 5 = 20

Q3

Use the distributive property of multiplication to make these statements true.

Example: 4 x 5 + 4 x 3 = (4 x 5) + (4 x 3) = 4(5 + 3)

a. (3 x 2) + (3 x 5)
= 3(2 + 5)
= 3 x 7 = 21

3 | 2 | 5
6 + 15 = 21

b. (6 x 1) + (6 x 4)
= 6(1 + 4)
= 6 x 5 = 30

6 | 1 | 4
6 + 24 = 30

c. (3 x 2) – (3 x 1)
= 3(2 – 1)
= 3 x 1 = 3

3 | 2 | –1
6 – 3 = 3

Q4

If a = 3, b = 2 and c = 4, calculate the following:

Example: $a(b + c) = a \times b + a \times c$
3(2 + 4) = 3 x 2 + 3 x 4
3(6) = 6 + 12
18 = 18

a. $b(a + c) = (b \times a) + (b \times c)$
$2(3 + 4) = (2 \times 3) + (2 \times 4)$
$2(7) = 6 + 8$
$14 = 14$

b. $c(b + a) = (c \times b) + (c \times a)$
$4(2 + 3) = (4 \times 2) + (4 \times 3)$
$4(5) = 8 + 12$
$20 = 20$

c. $a(c + b) = (a \times c) + (a \times b)$
$3(4 + 2) = (3 \times 4) + (3 \times 2)$
$3(6) = 12 + 6$
$18 = 18$

7

Problem solving

If $a = 5$, $b = 9$ and $c = 11$, write a distributive property statement and calculate the answer.

Answer:

$a(b + c)$
$= 5(6 + 11)$
$= (5 \times 9) + (5 \times 11)$
$= 45 + 55$
$= 100$

or

$b(a + c)$
$= 9(5 + 11)$
$= (9 \times 5) + (9 \times 11)$
$= 45 + 99$
$= 144$

Topic: Properties of numbers **Content links:** R9, 1-4
Grade 8 links: 2 **Grade 9 links:** None

4 Zero as the identity of addition, one as the identity of multiplication, and other properties of numbers *continued*

Objectives

- Recognise and use 0 in terms of additive property (identify element for addition).
- Recognise and use 1 in terms of its multiplicative property (identify element for multiplication).

Dictionary

Zero the identify of addition:
The answer will always be the number that zero is added to,
e.g. 4 + 0 = 4; 0 + 9 = 9

One as the identify of multiplication:
The answer will be the number that one is multiplied by,
e.g. 4 x 1 = 4; x x 1 = x

8

Introduction

What do you notice?

3 + 0 = 5 + 0 = 100 + 0 =
0 + 16 = 0 + 250 = 72 + 0 =

3 x 1 = 5 x 1 = 100 x 1 =
1 x 16 = 1 x 250 = 1 x 72 =

DEFINITION

Zero as the identity of addition:
The sum of zero and any number is the number itself. The answer will always be the number that **zero** is **added** to.

DEFINITION

One as the identity of multiplication:
The product of 1 and any number is always the number itself. The answer will always be the number that **one** is multiplied **by**.

Q1

Use zero as the identity of addition, or one as the identity of multiplication to write a sum for the following:
Answers:

		Zero as the identity of addition	One as the identity of multiplication
a.	5	5 + 0 = 5	5 x 1 = 5
b.	7	$7 + 0 = 7$	7 x 1 = 7
c.	9	$9 + 0 = 9$	9 x 1 = 9
d.	100	$100 + 0 = 100$	100 x 1 = 100
e.	34	$34 + 0 = 34$	34 x 1 = 34
f.	2,5	$2{,}5 + 0 = 2{,}5$	2,5 x 1 = 2,5
g.	0,1	$0{,}1 + 0 = 0{,}1$	0,1 x 1 = 0,1

Q2

Use zero as the identity of addition, or one as the identity of multiplication to solve the following:
Answers:

a. $b + 0 = b$
$b \times 1 = b$

b. $d \times 1 = d$
$d + 0 = d$

c. $e \times 1 = e$
$e + 0 = e$

Q3

Choose the correct property of number to write an equivalent statement to complete the equation.
Answers:

b. 2(3 + 9) = (2 x 3) + (2 x 9)
c. 3 + (4 + 8) = 4 + (8 + 3)
d. 5(9 − 8) = (5 x 9) − (5 x 8)
e. 9 + 12 = 12 + 9
f. (2 x 5) x 11 = 2 x (5 x 11)

v

4 Zero as the identity of addition, one as the identity of multiplication, and other properties of numbers *continued*

Q4

Say if the following is true or false. If it is false, explain why it is false.

Answers:

a. True
b. False (commutative property does not apply to subtraction)
c. True
d. True
e. False (associative property does not apply to subtraction)
f. True

Q5

If $a = 2$, $b = 5$, $c = 8$, solve the following: Answers:

a. $a + c = c + a$
$2 + 8 = 8 + 2$
$10 = 10$

b. $b + (c + a) = (b + c) + a$
$5 + (8 + 2) = (5 + 8) + 2$
$5 + 10 = 13 + 2$
$15 = 15$

c. $a + 0 = 2 + 0$
$= 2$

d. $b(a + c)$ or $b(a + c)$
$= 5(2 + 8)$ $= 5(2 + 8)$
$= 5(10)$ $= (5 \times 2) + (5 \times 8)$
$= 50$ $= 10 + 40$
$= 50$

e. $a(c - b)$ or $a(c - b)$
$= 2(8 - 5)$ $= 2(8 - 5)$
$= 2(3)$ $= (2 \times 8) - (2 \times 5)$
$= 6$ $= 60 - 10$
$= 6$

f. $b \times 1$
$= 5 \times 1$
$= 5$

Q6

Match column A with column B

Answers:

Column A	Column B
Associative property of numbers	$a \times 1 = a$
Commutative property of numbers	$(a + b) + c = a + (b + c)$
Distributive property of numbers	$a + 0 = a$
Zero as the identity of addition	$a + b = b + a$
One as the identity of multiplication	$a(b + c) = (a \times b) + (a \times c)$

9

Problem solving

- What should I add to a number so that the answer will be the same as the number?
- By what should I multiply a number so that the answer will be the same as the number?
- Write five statements that are true using the properties of number.
- Write five statements that are false using the properties of number. Explain your answer.

Answers:

a. 0
b. 1

True (Possible answers)

c. i. $a = a + 0$
ii. $a + 1 = 1 + a$
iii. $a + b = b + a$
iv. $a \times c = c \times a$
v. $b = b \times 1$

False (Possible answers)

i. $0 = a + 0$
ii. $a \times 1 = 1$
iii. $a - b = b - a$
iv. $a \div c = c \div a$
v. $a - 1 = 1 - a$

5 Multiples

Topic: Multiples and factors **Content links:** R6, 6
Grade 8 links: R2, 3-5 **Grade 9 links:** R2, 2

Objectives

- Multipless of 2-digit and 3-digit whole numbers
- Find the LCM of numbers to at least 3-digit whole numbers

Dictionary

Multiples: The products of natural numbers (1, 2, 3, 4, 5, ...) are called the multiples of the number. Multiples are the results of multiplying by an integer, e.g. 3 x 2 = 6. 6 is a multiple of 2 and 3. The multiples of 6 are 6, 12, 18, 24…

LCM: Lowest Common Multiple
E.g. The Lowest Common Multiple of 3 and 5 is 15, because 15 is a multiple of 3 and also a multiple of 5.

10

Introduction
Ask the learners to look at the number board. Ask, "How fast can you give me the first 12 multiples of 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s and 12s?" Then later ask, "How did the number board help you?"

Remind the learners about the concept of the Lowest Common Multiple (LCM). Ask them what the LCM of 50 and 60 is. Ask them what the LCM of 200 and 900 is.

How fast can you give me the first 12 multiples of 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s and 12s?

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Q1

Introduction
Use the number board to complete the following:

Example: The multiples of 6 are 6, 12, 18, ... 72, or

We can write it as multiples of 6: {6,12,18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

Answers:
a. Multiples of 4: {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}
b. Multiples of 7: { 7, 14, 21, 28, 35, 42, 49, 56, 63, 70}
c. Multiples of 5: { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50}
d. Multiples of 8: {8, 16, 24, 32, 40, 48, 56, 64, 72, 80}
e. Multiples of 2: {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
f. Multiples of 9: {9, 18, 27, 36, 45, 54, 63, 72, 81, 90}

5 Multiples *continued*

Topic: Multiples and factors **Content links:** R6, 6
Grade 8 links: R2, 3-5 **Grade 9 links:** R2, 2

Q2

Write down the first 12 multiples of the numbers below. Circle all the common multiples and identify the lowest common multiple (LCM).

Example: Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

The LCM is 4.

Answers

a. Multiples of 5: {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}
Multiples of 10: {10, 20, 30, 40, 50, 20, 24, 28, 32, 36}
LCM = 10

b. Multiples of 5: {5, 10, 15, 20, 25, 30, 35}
Multiples of 6: {6, 12, 24, 30, 36}
LCM = 30

c. Multiples of 90: {90, 180, 270, 360, 450, 540, 630, 720, 810, 900, 990, 1 080}
Multiples of 20: {20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240}
LCM = 72

Q3

What is the LCM for the following?

Answers

a. Multiples of 2 and Multiples of 8
Multiples of 2: {2, 4, 6, 8, 10, 12, 14, 16, ...}
Multiples of 8: {8, 16, 24, 32, ...} LCM = 8

b. Multiples of 3 and Multiples of 6:
Multiples of 3: {3, 6, 9, 12, 15, 18, ...}
Multiples of 6: {6, 12, 18, 24, 30, 36, ...} LCM = 6

c. Multiples of 5 and Multiples of 3:
Multiples of 3: {3, 6, 9, 12, 15, 18, 21, 24, ...}
Multiples of 5: {5, 10, 15, 20, 25, 30, 35, 40, ...} LCM = 15

d. Multiples of 4 and Multiples of 8:
Multiples of 4: {4, 8, 12, 16, 20, 24, ...}
Multiples of 8: { 8, 16, 24, 32, 40, ...} LCM = 8

e. Multiples of 70 and Multiples of 60:
Multiples of 70: {70, 140, 210, 280, 350, 420, 490, ...}
Multiples of 60: {60, 120, 180, 240, 300, 360, 420, 480, ...} LCM =

f. Multiples of 100 and Multiples of 125:
Multiples of 100: {100, 200, 300, 400, 500, 600, ...}
Multiples of 125: {125, 250, 375, 500, 625, ...} LCM = 500

11

Problem solving

In our homes there are various things that come in multiples. Give five examples of multiples from your home.

Answers:
a. knives and forks; b. cups; c. chairs; d. glasses; e. windows; f. eggs

6 Divisibility and factors

Topic: Multiples and factors **Content links:** R6, 5
Grade 8 links: R2, 3-5 **Grade 9 links:** R2, 2

Objectives

- Revise factors of 2-digit whole numbers
- Prime factors
- List prime factors of numbers to at least 3-digit whole numbers.
- Find the HCF of numbers to at least 2-digit whole numbers

Dictionary

Factors: Factors are the whole numbers you multiply together to get another whole number, in other words a whole number that divides exactly into another whole number is called a factor of that number. : e.g. 3 and 4 are factors of 12, because 3 x 4 = 12

HCF: Highest common factor E.g. The highest common factor of 2, 3 and 4 is 12.

12

Introduction

Learners need to find the missing information.

Your little brother messed up your notes. Find the missing information.
A number is divisible by 8 if the number formed by the last three digits is divisible by 8.
A number is divisible by 3 if the sum of the digits is divisible by 3.
A number is divisible by 10 if the last digit is 0.
A number is divisible by 5 if the last digit is either 0 or 5.
A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
A number is divisible by 9 if the sum of the digits is divisible by 9.
A number is divisible by 2 if the last digit is 0, 2, 4, 6 or 8.
A number is divisible by 6 if it is divisible by 2 and it is divisible by 3.

Q1

Tick if the numbers are divisible by 2, 3, 4, 5 or 10. You can have more than one answer.

	2	3	4	5	10
a. 376	✔		✔		
b. 7 232	✔		✔		
c. 9 050	✔			✔	✔
d. 6 312	✔	✔	✔		
e. 2 355		✔		✔	

Q2

The following numbers are divisible by?

Example: 6 is divisible by 1, 2, 3 and 6.

Answers:
a. 12: 1; 2; 3; 4; 6; 12
b. 36: 1; 2; 3; 6; 9; 12; 18; 36
c. 42: 1; 2; 3; 6; 7; 14; 21; 42
d. 24: 1; 2; 3; 4; 6; 8; 12; 24
e. 64: 1; 2; 4; 8; 16; 32; 64

Q3

Which two numbers, when multiplied, give you this number?

Example: 6 = 2 x 3, 6 = 1 x 6

Answers:

a.	b.	c.	d.	e.
3 x 4	1 x 36	1 x 42	1 x 24	1 x 64
2 x 6	3 x 12	2 x 21	2 x 12	2 x 32
1 x 12	4 x 9	3 x 14	3 x 8	4 x 16
	6 x 6	6 x 7	4 x 6	8 x 8

Q4

What do you notice if you compare question 2 and 3's answers? Answer: Answers are the same.

6 Divisibility and factors *continued*

Topic: Multiples and factors **Content links:** R6, 5
Grade 8 links: R2, 3-5 **Grade 9 links:** R2, 2

Q5

For each of the numbers given below, write down:
(i) All the possible multiplication sums using only two numbers that will give you this number.
(ii) All the numbers used in these multiplication sums, in ascending order (but do not repeat a number).
(iii) Complete the sentence: "These are the factors of ___."
(iv) Complete the sentence: "Factors of ___ = {_____}."

Answers

a. i. 18: 1 x 18; 2 x 9; 3 x 6
   ii. 1; 2; 3; 6; 9; 18
   iii. These are the factors of 18
   iv. Factors of 18 = {1; 2; 3; 6; 9; 18}
b. i. 25: 1 x 25; 5 x 5
   ii. 1; 5; 25
   iii. These are the factors of 25
   iv. Factors of 25 ={1; 5; 25}
c. i. 36: 1 x 36; 2 x 18; 3 x 12; 4 x 9; 6 x 6
   ii. 1; 2; 3; 4; 6; 9; 18; 36
   iii. These are the factors of 36
   iv. Factors of 36 = {1; 3; 6; 9; 12; 18; 36}

Q6

Complete the following, using the example to guide you.

Example: i. Factors of 12 are 1, 2, 3, 4, 6 and 12
Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30
ii. The common factors are: 1, 2, 3, 6
iii. The highest common factor is 6.

a. i. Factors of 8: {1, 2, 4, 8} Factors of 16: {1, 2, 4, 8, 16}
   ii. 1, 2, 4, 8
   iii. 8
b. i. Factors of 3: {1, 3} Factors of 12: {1, 2, 3, 4, 6, 12}
   ii. 1,3
   iii. 3
c. i. Factors of 3: {1, 3} Factors of 9: {1, 3, 9}
   ii. 1,3
   iii. 3

Q7

Complete the table.

	Words	Factors	Common factors	HFC
a. 4 and 8	Factors of 4 and Factors of 8	1, 2, 4, 1, 2, 4, 8	1, 2, 4,	4
b. 9 and 12	Factors of 9 and Factors of 12	1, 3, 9 1, 2, 3, 4, 6, 12	1, 3	3
c. 4 and 28	Factors of 4 and Factors of 28	1, 2, 4 1, 2, 4, 7, 14, 28	1, 2, 4	4
d. 12 and 36	Factors of 12 and Factors of 36	1, 2, 3, 4, 6, 12 1, 2, 3, 4, 6, 12, 18, 36	1, 2, 3, 4, 6, 12	12

13

Find out!

When in everyday life do we use HCF?
Answer: When we use the HCF to simplify fractions. For example, hot dog viennas sometimes come in packs of 10, while bread rolls come in either 6 or 12. What is the smallest number of each you have to buy to have the same number of each.

7 Ratio

Topic: Ratio and rate **Content links:** 8, 66, 114
Grade 8 links: None **Grade 9 links:** 3

Objectives

- Compare two or more quantities of the same kind (ratio)
- Solve problems by sharing in a given ratio where the whole is given
- Solve problems to find a percentage of a whole

Dictionary

Ratio: A ratio compares the size, or magnitude, of two or more numbers of the same kind.

Part-to-part ratio: E.g. In a group of 4 children (the whole) there is a part-to-part ratio of 1 boy to 3 girls, written as 1:3

Part-to-whole ratio: E.g, In a group of 4 children (the whole) there is 1 boy in the 4 children, written as 1:4, and 3 girls in the 4 children, written as 3:4.

Part-to-whole ratios can also be written as fractions or percentages, e.g. The ratio of boys to all children in the class is 1:4 or $\frac{1}{4}$ or 25 %.

The ratio of girls to all children in the class is 3:4 or $\frac{3}{4}$ or 75 % 75.

14

Introduction
Remember that a ratio is a comparison between two numbers. Discuss the diagram at the top of page 14 with the learners.

Q1

Write the following ratios as fractions of the whole.

Example: 2 boys : 3 girls is the same as = $\frac{2}{5}$ are boys and $\frac{3}{5}$ are girls

Answers:

a. $3:4 = \frac{3}{7}$ and $\frac{4}{7}$

b. $5:7 = \frac{5}{12}$ and $\frac{7}{12}$

c. $6:8 = \frac{6}{14}$ and $\frac{8}{14}$

d. $3:9 = \frac{3}{12}$ and $\frac{9}{12}$

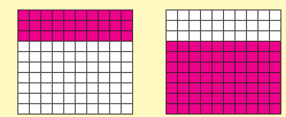
e. $1:2 = \frac{1}{3}$ and $\frac{2}{3}$

f. $7:9 = \frac{7}{16}$ and $\frac{9}{16}$

Q2

Write the following ratios as percentages.

Example: 3:7
is the same as $\frac{3}{10}$ and $\frac{7}{10}$
= 0,3 and 0,7
= 30% and 70%



Answers:

a. 4:6
$= \frac{4}{10}$ and $\frac{6}{10}$
= 0,4 and 0,6
= 40% and 60%

b. 2:8
$= \frac{2}{10}$ and $\frac{8}{10}$
= 0,2 and 0,8
= 20% and 80%

c. 5:5
$= \frac{5}{10}$ and $\frac{5}{10}$
= 0,5 and 0,5
= 50% and 50%

d. 12:13
$= \frac{12}{25} \times \frac{4}{4}$ and $\frac{13}{25} \times \frac{4}{4}$
$= \frac{48}{100}$ and $\frac{52}{100}$
= 0,48 and 0,52
= 48% and 52%

Multiply the denominator and numerator (the top and bottom respectively) with the same number (in this case 4 because 25 × 4 = 100) to get the fraction out of 100).

e. 20:30
$= \frac{20}{50} \times \frac{2}{2}$ and $\frac{30}{50} \times \frac{2}{2}$
$= \frac{40}{100}$ and $\frac{60}{100}$
= 0,40 and 0,60
= 40% and 60%

f. 1:3
$= \frac{1}{4} \times \frac{25}{25}$ and $\frac{3}{4} \times \frac{25}{25}$
$= \frac{25}{100}$ and $\frac{75}{100}$
= 0,25 and 0,75
= 25% and 75%

Q3

Solve the problems.

Answers:

a. Red : Green = 6:4 $\frac{6}{10}$ are red; $\frac{4}{10}$ are green.
0,6 are red and 0,4 are green.
60% red and 40% green.

b. If side = 2 units then the area will be 2^2 units = 4 square units.
Double the side will be 4 units and then the area will be 16 square units.
∴ when side of square is doubled the area will increase = 4 times.
The ratio of the Area of the original: Area of new will be:
4:16 or 1:4 = $\frac{1}{5}$ and $\frac{4}{5}$ = 0,2 and 0,8 = 20% and 80%

15

Problem solving

There are 600 pupils in a school. The ratio of boys to girls in this school is 9:11. How many girls and how many boys are in this school?

Answers:
Boys: $\frac{9}{20} \times 600 = 270$
Girls: $\frac{11}{20} \times 600 = 330$

Reflection questions

Did learners meet the objectives?

8 Rate

Topic: Ratio and rate **Content links:** 7, 12-13
Grade 8 links: 8-10 **Grade 9 links:** 3

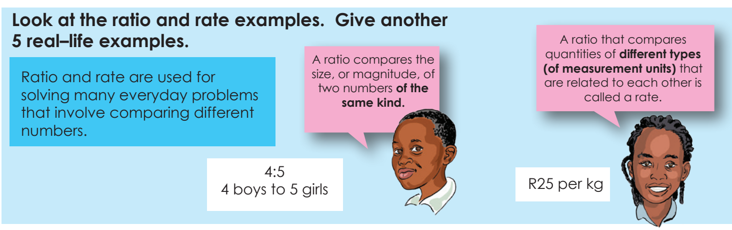
Objectives
- Compare two quantities of different types (that are related to each other in some other way) (Rate)
- Solve rate problems

Dictionary
Ratio: A ratio compares the size, or magnitude, of two or more numbers of the same kind.

Rate: A ratio that compares quantities of different types (that are related to each other in some way) is called a rate.
Examples: R4/kg, R30/l (litres symbol), R15/km, R100/hour

16

Introduction
Tell the learners that ratio and rate are used for solving many real-world problems that involve comparing different quantities. Ask them to look at the two examples in their workbooks and then give another five.



Ratio examples:
- ***Juice concentrate and water***
- ***Two gears***
- ***Left handed people to right handed people***
- ***Recipes: sugar to margarine***
- ***Ratio of two different sized paintings***

Rate examples:
- ***Rand per kilogram***
- ***Rand per volume***
- ***Rand per mass***
- ***Rand per length***
- ***Kilometres per hour***

Q1

Find the unit rate.

Example: 50 hamburgers in 10 days = **5** hamburgers **per** day.

Answers:
a. 8 b. 12 c. 6 d. 8 e. 40

Q2

Find the unit rate for each.

Example: $\frac{600 \text{ kilometres}}{60 \text{ litres}} = \frac{10 \text{ kilometres}}{1 \text{ litre}}$ = 10 kilometres/litre

Answers:
a. R2/kg b. 10 m/s c. R25/l d. 30 km/h

8 Rate *continued*

Topic: Ratio and rate **Content links:** 7, 12-13
Grade 8 links: 8-10 **Grade 9 links:** 3

Q3

Solve the following. Show all calculations.

Answers:

a. (120 leaves)/(4 hours)
$= 120 \div 4$
= 30 leaves/hour

b. (1 000 km)/(100 litres)
$= 1\,000 \div 100$
= 10 km/litre

c. (9 goals)/(5 matches)
$= 9 \div 5$
= 1,8 goals/match

d. Rate = 120 m/h × 4
$= 120 \times 4$
= 480 metres after 4 hours

Q4

We use rate on a daily basis. Give five examples and then write it as an unit rate.

Answer: Here are some examples:

Rate daily example	Unit rate
a. We travelled 5 km to school, and it took us 10 minutes.	30 km/h or 2 m/km
b. R100 for 2 kg meat $= 100 \div 2 =$ R50/kg	R50/kg
c. R100 for 10 litres petrol $= 100 \div 10 =$ R10/litre	R10/litre
d. We buy 4 cartons of milk and get 2 for free. $4 \div 2 = 2$	Buy 2 and get 2 free
e. We buy 15 loaves of bread for 10 people. $15 \div 10 = 1,5$	1,5 loaves per person

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Problem solving

A water tank that holds 100 litres is leaking at a rate of 2 litres/min. How long will it take to waste 24 litres at this rate?

Answer: 12 minutes

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

9 Money in South Africa

Topic: Financial maths **Content links:** 10-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Objectives

Solve problems that involve whole numbers, percentages and decimal fractions in financial context

Dictionary

Money in South Africa: The rand, sign: R; code: ZAR, is the currency of South Africa. It takes its name from the Witwatersrand the ridge upon which Johannesburg is built and where most of South Africa's gold deposits were found. The rand has the symbol "R" and is equal to100 cents, symbol "c".

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Introduction

Ask the learners what the currency was before Rand and cents.

1652 – 1800: Reals, Rix dollars, VOC
1800 to 1923: Coin of the British realm
1874: Burgerspond
1874 – 1932: Strachan and Co token
1892 to 1901 Kruger coinage
1902: Veld pond
1823 – 1964 South African pounds, shillings and pence
1932: Gold standard dropped and token coins outlawed

The **rand**, sign: **R**; code: **ZAR**, is the currency of South Africa. It takes its name from the Witwatersrand the ridge upon which Johannesburg is built and where most of South Africa's gold deposits were found. The rand has the symbol "R" and is equal to100 cents, symbol "c".

Find out what was the currency before Rand and cents.

You sell some goods. Move one row up and earn R100.

You buy some goods. Move one row down and pay R100.

Direction of play:
Start on A10 and play in the direction of J10.
Move then one row up.
Move in the direction of A9.
Move one row up.
Move in the direction of J8. Continue this pattern.

9 Money in South Africa *continued*

Topic: Financial maths **Content links:** 10-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Q1

If these were the results of the numbers your dice landed on, how much money do you have at the end of the game. After each result use a number sentence or word sum to describe what happened. Answers: Here is an example of how things might go.

Number on dice	Number sentence or word sum
6	Earns R20
6	R20 + R100 = R120
3	R120 + R200 = R320
6	R320 + R100 = R420
2	R420 + R100 = R520
6	R520 + R0 = R520
3	R520 + R0 = R520
2	R520 + R0 = R520
5	R520 + R0 = R520
5	R520 + R0 = R520
6	R520 + R50 = R570
2	R570+ R0 = R570
4	R570 + R20 = R590
2	R590 + R200 = R790
6	R790 – R100 = R690

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Problem solving

Make your own dice and use two stones as tokens. Play this game with a family member.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

10 Finances – profit, loss and discount

Topic: Financial maths **Content links:** 9, 11-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Objectives

- Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as profit, loss and discount
- Use rounding off in calculations involving money

Dictionary

Profit is the surplus remaining after total costs are deducted from total revenue.
Loss is the excess of expenditure over income.
Discount is the amount deducted from the asking price before payment.

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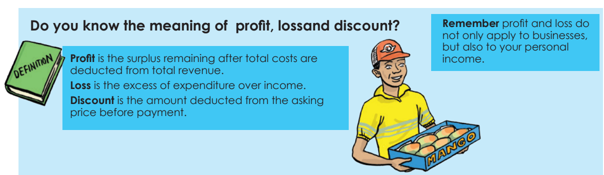
Ask the learners the following:
Do you know the meaning of profit, loss and discount? Ask them or give the learners some examples.
Tell learners that: Remember profit and loss do not only apply to businesses, but also to your personal income.



Q1

Learners must if they are making a profit or a loss in the examples and how much.
Answers:

a. Profit = Income – Expenses
Profit = 65c – 45c = 20c

b. Profit = Income – Expenses
Profit per pencil= R2,40 – R2,00 = 40c
Profit on pencils = 40c × 40 = R16,00

c. Profit = Income – Expenses
Profit on juices = (R2,50 × 40) – (R1,50 × 40) = R100 – R60 = R40
Stall expenses = R50
Profit = R40 – R50 = – R10
Loss = R10

d. Profit = Income – Expenses
Sales of sweets = 30c × 75 = R22,50
Cost of packet of sweets = R10,45
Profit = R22,50 – R10,45 = R12,05

[If the cost of only 75 sweets, not the whole packet, is considered, then the cost per sweet is less, 7,8375c as against 10,45c and the profit is R14,6625 (rounded to R14.66)]

10 Finances – profit, loss and discount *cont...*

Topic: Financial maths **Content links:** 9, 11-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

e. Profit = Income – Expenses
Number of bananas = $3 \times 12 \times 12 = 432$
Number sold = $432 \times 80 = 345{,}6 = 346$ (work with whole bananas)
Sales of bananas = $346 \times 65c = R224{,}90$
Cost of bananas = $3 \times R75 = R225$
Profit = $R224{,}90 - R225 = -10c$
Loss = 10c

[Loss = 36c if the tally of banana is not rounded off.
Cost of bananas = $3 \times R75 = R225$
Number of bananas = $3 \times 12 \times 12 = 432$

Number sold = $432 \times \frac{80}{100} = 345{,}6$

Sales of bananas = $345{,}6 \times 65c = R224{,}64$
Cost of bananas = $3 \times R75 = R225$
Loss = $R225 - R224{,}64 = 36c$]

Profit can be calculated in different ways. Normally when we talk about 10 % profit we calculate it on the cost price. We sometimes also refer to a 10 % mark–up.
The formula for the percentage profit is:
$\frac{\text{Profit (= Selling Price – Cost Price)}}{\text{Cost Price}} \times \frac{100}{1}$
For example, if I sold a football which cost me R200 for R220 I make a 10% profit.
$\frac{\text{R20 (= R220 – R200)}}{\text{R200}} \times 100 = 10\ \%$

Are you making a profit or a loss? How much?
Profit can be calculated in different ways. Normally, when we talk about 10% profit (or "mark-up"), we calculated it on the cost price. To get the selling price we use this formula:
Selling price = Cost price + (Cost price x profit %)
E.g. R200 + (R200 x 10%) = R200 + R20 = R220
To get the percentage profit we use this formula:
Percentage profit = $\frac{\text{(Selling Price – Cost Price)}}{\text{Cost price}} \times 100$
E.g. $\frac{(R220 - R200)}{200} \times \frac{100}{1} = 10\%$

Answers:
a. Selling price = Cost price + (Cost price × profit %)
$= 45c + (45c \times 25\%)$
$= 45c + (45c \times \frac{80}{100})$
$= 45c + 11{,}25c$
$= 56{,}25c$
$= 56c$ [rounded off]

b. Selling price = $127c + (127c \times \frac{17}{100})$
$= 127c + 21{,}59c$
$= 148{,}59c$
$= R1{,}49$ [rounded off]

c. Cost per juice $= R1{,}50 + \frac{R50}{200} = R1{,}75$

Selling price $= R1{,}75 + (R1.75 \times \frac{35}{100})$
$= R2{,}3625$
$= R2{,}36$ [rounded off]

Q3

Will I still make a profit if I sell it with discount? Answers:

a. Cost per sweet $= R12{,}45 \div 100 = R0{,}1245 = 12{,}45c$
Sale price for loose sweets $= 20c$
Discount price for 10 or more sweets $= 20c \times \frac{75}{100} = 15c$

Sweets sold $= (35 \times 20c) + (25 \times 15c) = 700c + 375c = 1\ 075c$
Cost of sweets sold $= 60 \times 12{,}45c = 747c$
Profit amount $= 1\ 075c - 747c = 328c$
$= R3{,}28$

b. Number bananas $= 3 \times 12 \times 12 = 432$

Number sold at 65c $= 432 \times \frac{80}{100} = 345{,}6 = 346$ [only whole bananas are sold]

Sale amount of 65c bananas $= 346 \times 65c = R224{,}90$
Number sold at 80% discount $= 432 - 346 = 86$
Sale amount of discounted bananas $= 86 \times (65c \times \frac{20}{100}) = R11{,}18$

[80% discount means they are sold at 20% of original price]
Total sales amount $= R224{,}90 + R11{,}18 = 236{,}08$

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Problem solving

If you bought your bicycle for R1 300 and you are selling it for R1 500, what percentage discount, on selling price, can you give your friend who wants to buy your bicycle and still make R50 profit?

Answer:
Selling price $= R1\ 500$
Discounted sale price $= R\ 1\ 300 + R50 = R1350$
Discount amount $= R1\ 500 - R\ 1350 = R\ 150$

Discount $= (150 \div 1\ 500) \times \frac{100}{1} = 10\%$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

11 Finances – budget

Topic: Financial maths **Content links:** 9-10, 12-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Objectives

- Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as budgets

Dictionary

Budget: Budget is the estimate of cost and revenues over a specified period.

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Introduction

Ask the learners the following questions:

- Do you know what a budget is?
- Can I have my own budget or is it only for adults?

Tell the learners that:

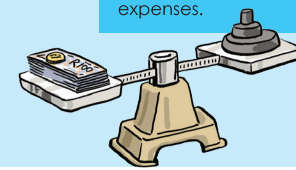
- Budget is like a scale where you try to balance your income and your expenses.

Important: Your income should always outweigh your expenses.

Do you know what a budget is?
Can I have my own budget or is it only for adults?

Budget is the estimate of cost and revenues over a specified period.

Budget is like a scale where you try to balance your income and your expenses.
Important: Your income should always outweigh your expenses.



Q1

Determine your income

Creating a budget is the most important step in controlling your money.The first rule of budgeting: **spend less than you earn!**
Example: If you received R50 allowance (pocket money) per month and another R30 for your birthday, you cannot spend more than R80 for the entire month.

Answers:

Income	Estimated amount
Pocket money	R100
Birthday money	R50
Wash car	R20
Sell CD to a friend	R45
Estimated total income	**R215**

11 Finances – budget

Topic: Financial maths **Content links:** 9-10, 12-13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Q2 Estimate expenses. Example:

Expenses	Estimated amount
Airtime	R50
Tuck shop	R25
New t-shirt	R100
Gift for mother	R25
Estimated total expenses	**R200**

Q3 Am I making a surplus?

	Estimated amount
Total income	R215
Total expenses	R195
Net income	**R20**

Q4 What can I do with my surplus?
Answers:
Example: I can save up for a movie in the holiday.

Q5 Savings
Answers: R499,95 ÷ R80 = 6.249375
Therefore I must save for 7 months to buy the computer game.

Q6 Track your budget. Answer: Learners' own answers. Example

	Actual amount	Estimated amount	Difference
Income			
Pocket money	50	50	0
Car wash	20	10	10
Mow lawn	15	30	5
Birthday gift	50	0	50
Total income	**135**	**90**	**45**
Expenses			
Sweets	15	15	0
Movie	40	50	10
Stationery	60	40	20
Total expenses	**115**	**105**	**10**
Net income	**20**	**15**	**35**

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Problem solving

Describe in your own words what you think of this saying: "A budget tells us what we can't afford, but it doesn't keep us from buying it."

Learners' own answers. The statement suggests that you can find ways of buying what you cannot afford -- for example by getting a loan. But is this a sensible option?

12 Finances – loans and interest

Topic: Financial maths **Content links:** 9-11,13
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Objectives

Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as loans and simple interest

Dictionary

Loan: A loan is sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

Interest: Interest is the fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, also called interest rate.

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Ask the learners if the know what a loan and what interest is. Tell leaners that it is never a good idea to borrow money. Rather save until you can afford to buy something.

What is a loan? What is interest?

A **loan** is a sum of money that an individual or a company lends to an individual or company with the objective of gaining profits from interest when the money is paid back.

Interest is the fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, also called interest rate.

It is never a good idea to borrow money. Rather save until you can afford to buy something.

Q1

Calculating interest amount

When someone lends money to someone else, the borrower usually pays a fee to the lender. This fee is called 'interest'. There are two kinds of interest: 'simple' and **'compound'**. **'Simple'** or **'flat rate'** interest is usually paid each year as a fixed percentage of the amount borrowed or lent at the start. With 'compound' interest you also pay interest on the interest!
The **simple interest** formula is as follows:

Interest = Principal × Rate × Time

where:
'Interest' is the total amount of interest paid,
'Principal' is the amount lent or borrowed,
'Rate' is the percentage of the principal charged as interest each year.
'Time' is the time in years to pay back the loan.

Answers:

a. $R1\,500 \times 10\% = R150$

b. $R1\,500 \times 110\% = R1\,650$

c. $R1\,650 \times 52 = R31{,}73$ per week

d. $R1\,500 \times 112\% = R1\,680$
$R1\,680 - R1\,650 = R30$ more

Q2

Calculating interest rate. Answers:

a. $\dfrac{(R3\,900 - R3\,000)}{2} = R450$ per year

b. $\dfrac{R450}{R3\,000} \times \dfrac{100}{1} = 15\%$

c. 52 weeks/year $\therefore$ 104 in 2 years
$R3\,900 \div 104 = R37{,}50$ per week

d. $\dfrac{(R3\,360 - R3\,000)}{R3\,000} \times \dfrac{100}{1} = 12\%$

Q3

Calculating repayment period

Answers:

~~a. Interest = Principal × Rate × Time = Interest~~
~~Principal × Rate Principal × Rate Interest for one year~~

b. Total interest: R6 750 – R5 000 = R1 750
Interest for one year: R5 000 × 10% = R500
Period: R1 750 ÷ R500 = 3,5 years

c. Total interest: R8 360 – R5 000 = R3 360
Interest for one year: R5 000 × 12% = R600
Period: R3 360 ÷ R600 = 5,6 years

d. Total interest: R14 700 – R7 500 = R7 200
Interest for one year: R7 500 × 12% = R900
Period: R7 200 ÷ R900 = 8 years

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Problem solving

I am repaying R452 per month on my loan. The interest rate the bank charged me was 15 % simple interest. I have to repay my loan over 48 months. I calculated that the total amount of interest I am paying over the 48 months is: R8 136. What was the original amount I borrowed at the bank?

Answers:

Total repayment = R452 x 48 = R21 696
Interest = R8 136

Principal = Total repayment – Interest
= R21 696 – R8 136 = R13 560

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

13 Finances

Topic: Financial maths **Content links:** 9-12
Grade 8 links: R10, 6-10 **Grade 9 links:** R10, 6-9

Objectives

Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as:

- loans
- simple interest
- accounts
- profit, loss and discount
- budgets

Dictionary

Profit: Profit is the surplus remaining after total costs are deducted from total revenue.
Loss: Loss is the excess of expenditure over income.
Discount: Discount is the amount deducted from the asking price before payment.
Budget: Budget is the estimate of cost and revenues over a specified period
Loan: A loan is sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.
Interest: Interest is the fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, also called interest rate.

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Let us review these financial terms.

Profit is the surplus remaining after total costs are deducted from total revenue.
Loss is the excess of expenditure over income.
Discount is the amount deducted from the asking price before payment.

Budget is the estimate of costs and revenues over a specified period

A **loan** is sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

Interest is the fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, also called interest rate.

Q1

You are starting your own lemonade stall. Answers: a.

Income	Estimated amount
Lemonade sold	R75
Estimated total income	R75
Expenses	
Lemons	$200 \times 10c = R20,00$
Sugar	$1 \times R10 = R10,00$
Cups	$30 \times 10c = R3,00$
Commission (brother)	$30 \times 15c = R4,50$
Estimated total expenses	R37,50
Net Income	$R75,00 - R37.50 = R37,50$

Let us work it out for 30 cups

Q2

It is going very well with your lemonade stall and you are still making a 100% profit on the cost of 30 cups a week sold at R2,50 a cup and your brother continues to help you. You decide to buy a lemonade maker. The lemonade maker will cost you R1 750 and you asked your family to lend you the money. They agree to lend you the money at 15 % simple interest per year. You have to repay them in one year. With the lemonade maker you will be able to sell 150 cups per month. Will you still be profitable? What percentage profit or loss will you make?

Cost of lemonade maker plus interest
= R1 750 + (R1 750 × 15%) = R2 012,50
Cost of 30 cups of lemonade = R37,50
Sale of 30 cups at R2,50 a cup = R75,00
Cost per 150 cups = R37,50 × 5 = R187,50
Cost of lemonade per year = R187,50 × 12 = R2 250
Sales of 150 cups = R75,00 × 5 = R375,00
Sales per year (12 months) = R375,00 × 12 = R4 500

Profit = R4 500 – (R2 012 + R2 250) = R238

Percentage profit on cost = $\frac{\text{Profit}}{\text{Cost}} \times \frac{100}{1} = \frac{\text{R238}}{\text{R4 262}} \times \frac{100}{1} = 5,58\%$

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b. Profit
c. 100% $\frac{\text{R37,50 profit}}{\text{R37,50 cost}} \times \frac{100}{1}$
d. Profit = Principal + (Principal × Rate × Time)
= R37,50 + (R37,50 × 100
New selling price = R2,50 +

e. Cost will decrease by 15c commission per cup = 15c × 30 = R4,50
Total cost is now R37,50 – R4,50 = R33,00
Profit will therefore increase by 15c per cup = 15c × 30 = R4,50
Total profit now = R37,50 + R4,50 = R42,00

New % profit = $\frac{\text{R42}}{\text{R33}} \times \frac{100}{1}$ = 127,27 % [rounded off]

Problem solving

You are buying dried fruit in big bags and repacking them into smaller bags. A big bag of mixed dried fruit cost you R476 and you can repack it into 50 small bags. The trip to the market cost you R50 and the small bags 50c each. For how much must you sell the small bags of dried fruit to make a 33 % profit?

Answer:
Total costs = R476 + R50 + (50 × 50c) = R551
Cost per bag = R551 ÷ 50 = R11,02
Price including 33,33% profit = R11,02 × 1,3333%
= R14,69 [rounded off]

14 Square and cube numbers

Topic: Exponents and roots **Content links:** 15-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Objectives

- Perform calculations involving square and cube numbers.
- Compare and order square and cube roots
- Determine squares to at least 12^2 and their square roots.
- Determining cubes to at least 6^3 and their cube roots.

Dictionary

Square number: A number mutiplied by itself. E.g. $4^2 = 4 \times 4 = 16$
Emphasize that: $12^2 = 12 \times 12$ and not 12×2
Cube number: A number multiplied by itself and then that result multiplied by the original number again. E.g. $4^3 = 4 \times 4 \times 4 = 64$, so 64 is a cubed number. Emphasize that: 1^3 means $1 \times 1 \times 1$ and not 1×3.

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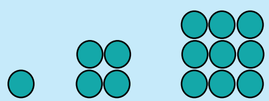
Introduction

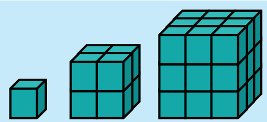
Ask the learners to look at these patterns and answer questions.

Look at the following pattern:



If we have one circle in the first pattern, four circles in the second pattern and nine circles in the third pattern, how many circles will we have in the tenth pattern? How did you work out your answer?

Answer: $10 \times 10 = 100$



If we have one cube in the first pattern, eight cubes in the second pattern and twenty seven cubes in the third pattern. How many cubes will we have in the fourth pattern? How did you work out your answer?

$4 \times 4 \times 4 = 64$

Q1

The numbers above are called square and cube numbers.

Q2

Write the following as square numbers:

Example: $13 \times 13 = 13^2$

This 2 is the **exponent**. We say 13 squared or 13 to the power of 2.

Answers:
a. $2 \times 2 = 2^2$ b. $7 \times 7 = 7^2$ c. $5 \times 5 = 5^2$
d. $10 \times 10 = 10^2$ e. $3 \times 3 = 3^2$ f. $11 \times 11 = 11^2$

Q3

Write the following as multiplication sentences:

Example: $15^2 = 15 \times 15$

Answers:
a. $5^2 = 5 \times 5$ b. $9^2 = 9 \times 9$ c. $4^2 = 4 \times 4$
d. $2^2 = 2 \times 2$ e. $7^2 = 7 \times 7$ f. $12^2 = 12 \times 12$

Q4

For 3^2, identify: a. the base number. b. the exponent.
Answers:
a. 3 is the base number and 2 is the exponent.
Remind learners that a number to the power of 1 stays the same, e.g. $4^1 = 4$.

14 Square and cube numbers *continued*

Topic: Exponents and roots **Content links:** 15-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Q5 Colour all the square numbers on the multiplication board.
Answers:

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Q6 Arrange these numbers in ascending order:
Answers: $1^2, 2, 2^2, 5, 3^2, 10, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2$

Q7 Arrange the above numbers in descending order:
Answers: $12^2, 11^2, 10^2, 9^2, 8^2, 7^2, 6^2, 5^2, 4^2, 10, 3^2, 5, 2^2, 2, 1^2$

Q8 Fill in <, > or =
Answers:
a. $2^2 = 2 \times 2$ b. $5^2 > 5 \times 2$ c. $9^2 = 9 \times 9$
d. $8^2 > 2 \times 8$ e. $11^2 > 10 \times 11$ f. $3 \times 3 = 3^2$

Q9 Numbers which have an exponent of 2 are called square numbers.

Q10 Write the following as cube numbers:

Example: $6 \times 6 \times 6 = 6^3$

Answers:
a. $3 \times 3 \times 3 = 3^3$ b. $2 \times 2 \times 2 = 2^3$ c. $5 \times 5 \times 5 = 5^3$

Q11 Write the following as multiplication sums.
Answers:
a. $2^3 = 2 \times 2 \times 2$ b. $4^3 = 4 \times 4 \times 4$ c. $1^3 = 1 \times 1 \times 1$

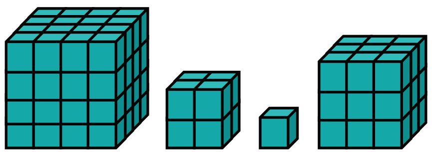
Q12 Explain in your own words what a cube number is.
Answer: A number multiplied by itself and then that result multiplied by the original number again.

Q13 Identify: 4^3 a. the base number b. the exponent
Answer:
a. Base number is 4, exponent is 3

14 Square and cube numbers *continued*

Topic: Exponents and roots **Content links:** 15-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Q14 State the number of cubes in each of the diagrams below using exponents. Then arrange these numbers written in exponential form in ascending order.



Answers:
$C = 1^3, B = 2^3, D = 3^3, A = 4^3$

Q15 Put these numbers in ascending order:
Answers:
$1^3, 2^3, 3^3, 4^3, 5^3$

Q16 Fill in <, > or =
Answers:

a. $2^3 > 2 \times 2$
b. $125 = 5^3$
c. $1 \times 1 = 1^2$
d. $27 = 3^3$
e. $6 < 3^3$
f. $5^3 > 8$

Q17 First estimate and then calculate the answers:

Example: $5^2 + 3^2 = 25 + 9 = 34$

Answers:
a. $4 + 100 = 104$
b. $36 - 9 = 27$
c. $64 + 100 = 164$

Q18 First estimate and then calculate the answers:

Example: $5^2 + 3^2 = 25 + 27 = 52$

Answers:
a. $216 - 25 = 191$
b. $4 + 27 = 31$
c. $729 - 16 = 713$

Q19 First estimate and then calculate the answers:
Answers:
a. $4 + 27 - 1 = 30$
b. $125 - 64 + 27 = 88$
c. $16 + 64 + 4 = 84$

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Problem solving

Add the smallest square number and the largest square number that is smaller than 100. Do the same with cube numbers.

Answers:
$1^2 + 9^2 = 1 + 81 = 82$
$1^3 + 4^3 = 1 + 64 = 65$

15 Square and cube roots

Topic: Exponents and roots **Content links:** 14, 16-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Objectives

- Perform calculations using square and cube numbers
- Compare and order square and cube roots
- Determine squares to at least 12^2 and there square roots.
- Determining cubes to at least 6^3 and cube roots.

Dictionary

Square roots: One of the two identical factors of a number that is the product of those factors. The symbol is $\sqrt{}$
e.g. $\sqrt{4} = 2$ because $2^2 = 4$
A square root is the converse operation of squaring.

Cube roots: One of the three identical factors of a number that is the product of those factors.
The symbol is $\sqrt{}$
E.g. = $\sqrt[3]{27} = 3$ because $3^3 = 27$
A cube root is the converse operation of cubing.

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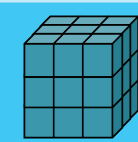
Introduction
Ask learners to look at these diagrams and explain them.

What do you think these diagrams represent?

1	2	3
4	5	6
7	8	9

$\sqrt{9}$

$3 \times 3 = 9$,
so the square root of 9 is 3.



$3 \times 3 \times 3 = 27$
$\sqrt[3]{27}$
so the cube root of 27 is 3.

Q1

What square number and root does the diagram represent? Use the example to guide you.

Example: a. $3 \times 3 = 9$, so the square number is 9 and the square root ($\sqrt{}$)of 9 is 3

Answers:
b. 16
4
$4 \times 4 = 16$ so the square root of 16 ($\sqrt{}$) is 4.

c. 25
5
$5 \times 5 = 25$ so the square root of 25 ($\sqrt{}$) is 5.

Q2

Write the following using the symbol for square root:

Answers: a. $\sqrt{9} = 3$ b. $\sqrt{25} = 5$

Square and cube roots *continued*

Topic: Exponents and roots **Content links:** 14, 16-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Q3 Calculate the square root:

Answers:

a. $\sqrt{81} = \sqrt{9 \times 9} = 9$ b. $\sqrt{1} = \sqrt{1 \times 1} = 1$

c. $\sqrt{121} = \sqrt{11 \times 11} = 11$ d. $\sqrt{64} = \sqrt{8 \times 8} = 8$

e. $\sqrt{36} = \sqrt{6 \times 6} = 6$ f. $\sqrt{169} = \sqrt{13 \times 13} = 13$

Q4 Write the following in ascending order.

Answer: $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$

Q5 Write the following in ascending order.

Answer: $\sqrt{2.2}$, $\sqrt{3.3}$, $\sqrt{4.4}$

Q6 Write the following in descending order.

Answers:

a. $\sqrt{9^2}$, $\sqrt{100}$, $\sqrt{25}$, $\sqrt{16}$, $\sqrt{2^2}$

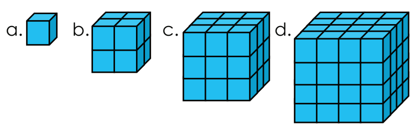
Q7 Fill in <, >, or =

Answers:

a. $\sqrt{36} > \sqrt{25}$ b. $\sqrt{81} > \sqrt{27}$ c. $\sqrt{9} < \sqrt{16}$

d. $\sqrt{81} = 3^2$ e. $3^2 > \sqrt{36}$ f. $4^2 > \sqrt{25}$

Q8 What is a cube root of the cubes below?



Answers:

a. so the cube root of 1 is 1

b. so the cube root of 8 is 2

c. so the cube root of 27 is 3

d. so the cube root of 64 is 4

Q9 Write the following using the symbol for cube root:

Answers:

a. $\sqrt[3]{27}$ b. $\sqrt[3]{8}$

Q10 Calculate the cube root.

Example: $\sqrt{27}$ Since 27 = 3 x 3 x 3

$= \sqrt[3]{3 \times 3 \times 3}$

$= 3$

Answers:

a. 2 b. 4 c. 1

Q11 Write the following in ascending order:

Answers:

$\sqrt[3]{1}$; $\sqrt[3]{8}$; $\sqrt[3]{27}$; $\sqrt[3]{125}$

Q12 Write the following in descending order:

Answers:

$\sqrt{4 \cdot 4 \cdot 4}$; $\sqrt{3\ 3\ 3}$; $\sqrt{2 \cdot 2 \cdot 2}$

Q13 Write the following in ascending order:

Answers:

1^3; $\sqrt[3]{27}$; 2^3; 4^3

Topic: Exponents and roots **Content links:** 14, 16-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-20

Q14 Fill in <, >,or =

Answers:

a. > b. > c. > d. < e. < f. >

Q15 Write the following in ascending order.

Answers:

$\sqrt[3]{1}$, $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{125}$

Q16 Calculate:

Example: $\sqrt{16} + \sqrt{25}$
$= 4 + 5$
$= 9$

Answers:

a. $3 + 4 = 7$ b. $5 - 4 = 1$ c. $10 + 9 = 19$ d. $5 + 8 = 13$

Q17 Calculate:

Example: $\sqrt[3]{64} - \sqrt[3]{27}$
$= 4 - 3$
$= 1$

Answers:

a. $6 + 3 = 9$ b. $3 - 2 = 1$ c. $4 + 6 = 10$ d. $3 + 4 = 7$

Q18 Calculate:

Example: $\sqrt[3]{125} - 16$
$= 5 + 4$
$= 9$

Answers:

a. $6 - 5 = 1$ b. $4 + 2 = 6$ c. $5 + 2 = 7$ d. $5 - 3 = 2$

Q19 Calculate:

Example: $\sqrt[3]{27} + 3^2 - 25$
$= 3 + 9 - 5$
$= 7$

Answers:

a. $6 + 16 - 4 = 18$ b. $81 - 3 + 2 = 80$
c. $27 + 64 + 5 = 96$ d. $12 - 4 + 2 = 10$

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Problem solving

Square and cube fun

a. Write down all the two–digit square numbers.
b. Write down all the three–digit cube numbers.
c. Write down the square roots of all the two–digit square numbers.
d. Write down the cube roots of all the two–digit and three–digit cube numbers.

Note that these are the perfect square and cube numbers.

Answers:

a. $\sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}, \sqrt{64}, \sqrt{81}$
b. $\sqrt{125}, \sqrt{216}, \sqrt{343}, \sqrt{512}, \sqrt{729}$
c. $\sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$
d. $\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$

16 Exponential notation

Topic: Scientific notation **Content links:** 19
Grade 8 links: 19 **Grade 9 links:** 21

Objectives

- Solve problems involving numbers in exponential form.
- Recognise exponential notation

Dictionary

Exponential notation: Exponential notation is a way to write (and read) mathematical content in a short and clear way. E.g. 4 x 4 x 4 is written 4^3 in exponential form. 4^3 is read "4 to the power of 3" where 4 is called the base of the power and 3 is called the exponent or index.

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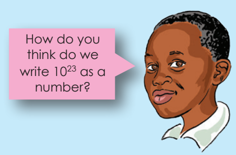
Introduction

Tell the learners that in science we deal with numbers that are sometimes extremely large or small.

In science, we deal with numbers that are sometimes extremely large or extremely small.

There are 602 000 000 000 000 000 000 000 water molecules in 18 grams of water. A shorter way of writing the same number is exponential notation to show all those zeros as a number to the power of ten:

$6{,}02 \times 10^{23}$ is the shorter way of representing the number of all those molecules. Such a number can be read as "Six comma zero two to the power of twenty-three."



Q1

How fast can you calculate the following?

Example: 10 x 10 x 10 x 10 = 10 000

Answers:
a. 100 b. 100 000 c. 10 000
d. 1 000 e. 10 000 000 f. 1 000 000

Q2

Complete the table:

Sum	Exponential format	Answer
a. 10 x 10	10^2	100
b. 10 x 10 x 10	10^3	1 000
c. 10 x 10 x 10 x 10	10^4	10 000
d. 10 x 10 x 10 x 10 x 10	10^5	100 000
e. 10 x 10 x 10 x 10 x 10 x 10	10^6	1 000 000

Q3

Identify the base number and the exponent: 10^8
Answer: 10 base number, 8 exponent

Q4

Match column B with column A. Answers

A	B
10^7	a. ten to the power of nine
10^5	b. ten to the power of seven
10^8	c. ten to the power of five
10^3	d. ten to the power of eight
10^9	e. ten to the power of three

16 Exponential notation *continued*

Topic: Scientific notation **Content links:** 19
Grade 8 links: 19 **Grade 9 links:** 21

Q5

Write the following in exponential form.

Example: $10 \times 10 \times 10 \times 10 = 10^4$

Answers:

a. 10^9 b. 10^5 c. 10^6

Q6

Expand the following statements:

Example: $10^3 = 10 \times 10 \times 10$

Answers:

a. 10 x 10
b. 10 x 10 x 10 x 10
c. 10 x 10 x 10 x 10 x 10
d. 10 x 10 x 10 x 10 x 10 x 10
e. 10 x 10 x 10 x 10 x 10 x 10 x 10
f. 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10

Q7

Your cousin wrote this in his maths book 10^5. What does this mean?

Answer:

a. Ten to the power of five $(10)^5$

Q8

Give some practical examples of where exponential notation is used.

Answers: Examples are:

a. Scientists use this to calculate large numbers
b. Engineering

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Problem solving

Write one billion in exponential notation.

Answer:

a. 10^{12}

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

17 Estimate and calculate exponents

Topic: Exponents and roots **Content links:** 14-16, 18-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-26

Objectives

- Recognize and use the appropriate laws of operations with numbers involving exponents and square and cube roots
- Perform calculations involving all four operations using numbers in exponential form, limited to exponents up to 5, and square and cube roots

Dictionary

Exponent: The exponent of a number shows you how many times to use the number in a multiplication.
E.g. $3^2 = 3 \times 3 = 9$.
3^2 could be read as:
• 3 to the second power, or
• 3 to the power of two, or
• 3 squared
Exponents are also called powers or indices (Singular: index)

BODMAS: The order in which we carry out a calculation is important.
BODMAS stands for such an order:
B brackets
O other (power and square roots)
D division and
M multiplication (left–to–right)
A addition and
S subtraction (left–to–right)

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Introduction

Make this a fun activity for learners to see who can first identify the which number(s) will give you an answer of 10^4.

Which multiplication sums will give you an answer of 10^4?

10 × 1 000	1 × 10 × 1 000	10 × 100	10 × 100 × 10	100 × 1 000
1 × 1 000	100 × 10 × 1	10 × 10 × 10 × 10	1 × 1 × 1 × 1	1 000 × 10
1 × 1 000 × 10	10 × 10 × 100	100 × 10 × 1 × 1	1 × 10 000	100 × 10 + 10
10 000 × 1	100 × 10 × 10 × 1	1 000 × 1 000	100 × 10	10 + 10 + 10 + 10
100 × 10 × 10	10 × 10	10 × 1 × 1 000	10 × 10 × 10	100 × 100

Q1 Write in expanded form.

Example: $10 \times 10 \times 10 \times 10 = 10^4$

Answers:
a. 10^5 b. 10^3 c. 10^7 d. 10^4 e. 10^8

Q2 Write in expanded form.

Example: $10^4 = 10 \times 10 \times 10 \times 10$

Answers:
a. 10 x 10 x 10
b. 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10
c. 10 x 10 x 10 x 10 x 10 x 10 x 10
d. 10 x 10 x 10 x 10 x 10
e. 10 x 10 x 10 x 10
f. 10 x 10 x 10 x 10 x 10 x 10

Q3 Calculate.

Answers:
a. 1 000 + 100 = 1 100
b. 10 000 + 1000 000 = 1 010 000
c. 100 000 + 1 000 = 101 000

Remember the BODMAS order. In this example, first calculate the exponent, then do the addition.

Example: $10^4 + 10^3$
= 10 000 + 1 000
= 11 000

Topic: Exponents and roots **Content links:** 14-16, 18-19
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-26

17 Estimate and calculate exponents *cont...*

Q4

Calculate.

Example: $4 + 10^3$
$= 4 + 1\,000$
$= 1\,004$

Remember BODMAS

Answers:
a. $5 + 10\,000 = 10\,005$
b. $100\,000 \times 9 = 900\,000$
c. $10\,000 \times 7 = 70\,000$

Q5

Calculate.

Example: $2 \times 10^4 + 3 \times 10^5$
$= 2 \times 10\,000 + 3 \times 100\,000$
$= (2 \times 10\,000) + (3 \times 100\,000)$
$= 20\,000 + 300\,000$
$= 320\,000$

Remember BODMAS

Answers:
a. $(3 \times 1\,000) + (4 \times 10\,000)$
$3\,000 + 40\,000$
$43\,000$

b. $(8 \times 10\,000) + (3 \times 100)$
$80\,000 + 300$
$80\,300$

c. $(5 \times 100) + (8 \times 1\,000\,000)$
$500 + 8\,000\,000$
$8\,000\,500$

Q6

Calculate:

Example: $2 \times 10^4 + 3 \times 10^3 + 4 \times 10^5$
$= 2 \times 10\,000 + 3 \times 1\,000 + 4 \times 100\,000$
$= (2 \times 10\,000) + (3 \times 1\,000) + (4 \times 100\,000)$
$= 20\,000 + 3\,000 + 400\,000$
$= 423\,000$

Answers:
a. $(1 \times 100) + (8 \times 100\,000) + (3 \times 1\,000\,000)$
$100 + 800\,000 + 3\,000\,000$
$3\,800\,100$
b. $(3 \times 1\,000) + (8 \times 1\,000) + (7 \times 10\,000\,000)$
$3\,000 + 8\,000 + 70\,000\,000$
$70\,011\,000$
c. $(5 \times 1\,000) + (6 \times 100) + (2 \times 10\,000)$
$5\,000 + 600 + 20\,000$
$25\,600$
d. Learner's own answer

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Problem solving

Calculate ten to the power of three plus ten to the power of two plus three times ten to the power of one.

Answer: $10^3 + 10^2 + (3 \times 10^1)$
$1\,000 + 100 + (3 \times 10) = 1\,130$

Remind the learners that any number to the power of 1 is that same number.

Topic: Exponents and roots **Content links:** 14-17, 19, 76
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-26

18 Estimate and calculate more exponents

Objectives
- Solve problems involving numbers in exponential form

Dictionary
Square number: a number multiplied by itself, e.g. $4^2 = 4 \times 4 = 16$
Emphasize that: $12^2 = 12 \times 12$ and not 12×2

Cube number: a number multiplied by itself and then that result multiplied by the original number again, e.g. $4^3 = 4 \times 4 \times 4 = 64$, so 64 is a cubed number. Emphasize that: 1^3 means $1 \times 1 \times 1$ and not 1×3

Power of ten: any of the integer powers of the number ten; in other words, ten multiplied by itself a certain number of times

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Introduction
Ask the learners to match the words with the pictures:
- Square numbers – A picture of a tiled floor.
- Cube numbers – A picture of a wooden box.

Ask the learner what it means if we have a number to the power of 0. It always equals one. Ask the learners what it means if we have a number to the power of 1. It always equals the number itself.

Q1 Calculate.

Example: $2^2 + 2^3 = 4 + 8 = 12$ Remember BODMAS

Answers:

a. $4 + 144 = 148$	b. $16 + 100 = 116$
c. $8 + 121 = 129$	d. $216 + 1 = 217$
e. $9 + 8 = 17$	f. $25 + 8 = 33$

Q2 Calculate.

Example: $2^2 + 3^3 + 4^2 = 4 + 27 + 16 = 47$

Answers:

a. $4 + 64 + 9 = 77$	b. $125 + 36 + 81 = 242$
c. $49 + 8 + 8 = 66$	d. $25 + 100 + 144 = 269$
e. $121 + 16 + 27 = 164$	f. $125 + 81 - 36 = 170$

Q3 How fast can you calculate the following?

Answers:

a. 9	b. 27
c. 25	d. 121
e. 16	f. 4
g. 125	h. 16
i. 36	

18 Estimate and calculate more exponents *cont...*

Q4 Calculate.

Example: $(12 - 9)^3$
$= (3)^3$
$= 27$

Answers:

a. $(8 - 4)^3$
$= (4)^3$
$= 64$

b. $(7 + 1)^2$
$= (8)^2$
$= 64$

c. $(9 + 2)^2$
$= (11)^2$
$= 121$

d. $(18 - 9)^2$
$= (9)^2$
$= 81$

e. $(11 - 6)^3$
$= (5)^3$
$= 125$

f. $(16 - 11)^3$
$= (5)^3$
$= 125$

Q5 Create your own number sentences and calculate the answers.
Answers: These are examples

a. $2^3 + 3^3 + 4^3$
$= 8 + 27 + 64$
$= 99$

b. $2^2 + 4^2 + 6^2$
$= 4 + 16 + 36$
$= 56$

c. $3^3 + 4^3 + 2^2$
$= 27 + 64 + 4$
$= 95$

d. $4^3 - 2^2$
$= 64 - 4$
$= 60$

e. $(2^2 + 3^2) + (4^3 + 3^3)$
$= (4 + 9) + (64 + 27)$
$= 13 + 91$
$= 104$

f. $3^2 + 2^3 + 5^3 + 4^3$
$= 9 + 8 + 125 + 64$
$= 206$

g. $2^3 + 4^3 - 3^3$
$= 8 + 64 - 27$
$= 72 - 27$
$= 45$

h. $(3^2 + 7^2) - (5^2 \times 1^2)$
$= (9 + 49) - (25 \times 1)$
$= 58 - 25$
$= 33$

i. $6^3 + 12^2$
$= 216 + 144$
$= 360$

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Problem solving

What is four to the power of three minus one to the power of one plus one hundred to the power of one. Check your answer using a calculator.

Answer:
$4^3 - 1^1 + 100^1$
$= 64 - 1 + 100$
$= 163$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

19 Numbers in exponential form

Topic: Exponents and roots **Content links:** 14-18
Grade 8 links: R3, 14-18 **Grade 9 links:** R3, 12, 19-26

Objectives

- Compare and represent whole numbers in exponential form:
- $a^b = a \times a \times a \times a \ldots$ for number of b factors.

Dictionary

Exponential notation: Exponential notation is a way to write (and read) mathematical content in a short and clear way. E.g. 10 x 10 x 10 is written 10^3 in exponential form. 10^3 is read "10 to the power of 3" where 10 is called the base of the power and 3 is called the exponent or index.

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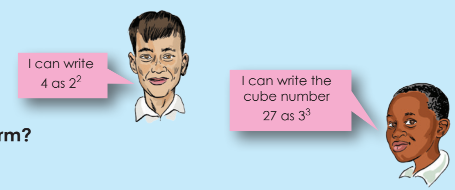
Introduction

If:

- square numbers are 1, 4, 9,16, 25, …
- Cube numbers are 1, 8, 27, 64, 81, …



How can I write it in exponential form?

How will we write it in exponential form: 1^2, 2^2, 3^2, 4^2, 5^2, … 1^3, 2^3, 3^3, 4^3, 5^3, …

Q1

Extend the pattern another three times (up to the power of 5). Use your calculator, where necessary, to calculate the answers. Answers:

a. $20 \times 20 \times 20 = 20^3$
$20 \times 20 \times 20 \times 20 = 20^4$
$20 \times 20 \times 20 \times 20 \times 20 = 20^5$

b. $10 \times 10 \times 10 = 10^3$
$10 \times 10 \times 10 \times 10 = 10^4$
$10 \times 10 \times 10 \times 10 \times 10 = 10^5$

c. $17 \times 17 \times 17 = 17^3$
$17 \times 17 \times 17 \times 17 = 17^4$
$17 \times 17 \times 17 \times 17 \times 17 = 17^5$

d. $38 \times 38 \times 38 = 38^3$
$38 \times 38 \times 38 \times 38 = 38^4$
$38 \times 38 \times 38 \times 38 \times 38 = 38^5$

e. $59 \times 59 \times 59 = 59^3$
$59 \times 59 \times 59 \times 59 = 59^4$
$59 \times 59 \times 59 \times 59 \times 59 = 59^5$

f. $15 \times 15 \times 15 = 15^3$
$15 \times 15 \times 15 \times 15 = 15^4$
$15 \times 15 \times 15 \times 15 \times 15 = 15^5$

Q2

Expand the exponential notation and use your calculator to calculate the answer.

Example: 18^4
$= 18 \times 18 \times 18 \times 18$
$= 104\ 976$

Answers:

a. 22^3
$= 22 \times 22 \times 22$
$= 10\ 648$

b. 81^2
$= 81 \times 81$
$= 6\ 561$

c. 74^4
$= 74 \times 74 \times 74 \times 74$
$= 29\ 986\ 576$

d. 39^1
$= 39 \times 1$
$= 39$

e. 97^7
$= 97 \times 97 \times 97 \times 97 \times 97 \times 97 \times 97$
$= 80\ 798\ 284\ 478\ 113$

f. 32^8
$=32 \times 32 \times 32 \times 32 \times 32 \times 32 \times 32 \times 32$
$= 1\ 099\ 511\ 627\ 776$

Q3

Extend the pattern one more time.

Answers:

a. $a \times a \times a = a^3$
$a \times a \times a \times a = a^4$

b. $b \times b \times b = b^3$
$b \times b \times b \times b = b^4$

c. $m \times m \times m = m^3$
$m \times m \times m \times m = m^4$

d. $r \times r \times r = r^3$
$r \times r \times r \times r = r^4$

e. $k \times k \times k = k^3$
$k \times k \times k \times k = k^4$

f. $n \times n \times n = n^3$
$n \times n \times n \times n = n^4$

Q4

Expand.

Example: m^4
$= m \times m \times m \times m$

Answers:

a. $a^3 = a \times a \times a$
b. $b^2 = b \times b$
c. $r^4 = r \times r \times r \times r$
d. $m^1 = m \times 1$
e. $p^7 = p \times p \times p \times p \times p \times p \times p$
f. $p^8 = p \times p \times p \times p \times p \times p \times p \times p$

Q5

Calculate question 3 and 4 if: $a = 10$, $b = 3$, $m = 100$, $r = 5$, $k = 1$, $n = 20$, $p = 2$

Answers:

a. a^3
$= 10^3$
$= 1\ 000$

b. b^2
$= 3^2$
$= 27$

c. m^3
$= 100^3$
$= 1\ 000\ 000$

d. r^3
$= 5^3$
$= 125$

e. k^3
$= 1^3$
$= 1$

f. n^3
20^3
$= 8\ 000$

43

Problem solving

I have fifty-four to the power of one, and seventy-nine to the power of one. What will the total be if I add these two numbers?

Answer:
$54^1 + 79^1 = 133$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

20 Constructing geometric objects

Objectives

- Accurately use a protractor to measure and classify acute, right, obtuse and straight angles
- Use a protractor to measure and draw angles

Dictionary

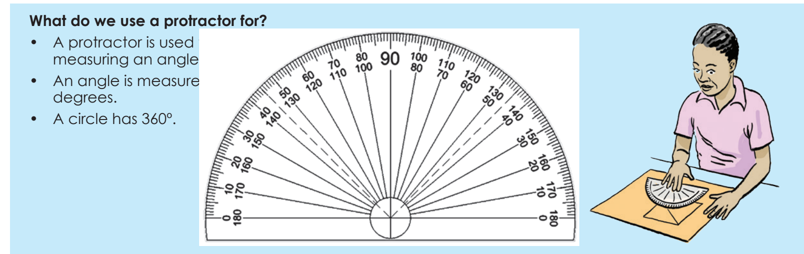
Angle: An angle is made when the two straight lines meet or cross each other at a fixed point. The size of the angle is measured by the amount one line has turned in relation to the other.
Acute angle: an angle between 0° and 90°
Obtuse angle: an angle between 90° and 180°
Reflex angle: an angle between 180° and 360°
Right-angle triangle: a right angled triangle is a triangle which has a right angle (90°) in it.
Straight angle: It is a straight line. It measures 180°.

Construct: To construct is to draw a shape, line or angle accurately using a compass, straight edge or protractor.

Note: Some construction exercises may forbid the use of a protractor or straight edge to make measurements.

44

Introduction



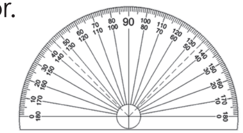
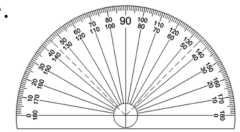
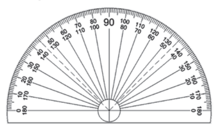
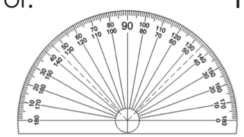
Ask the learners to look at the introduction and answer the following questions:

- What is a protractor? (Show them an example or ask them to take out their protractors.)
- What do we use a protractor for?
- This is a 180° protractor. Do we get any other types of protractors?
- How can you use your 180° protractor to draw a 360° circle?

Q1

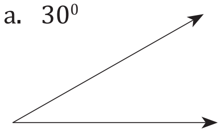
How will you measure angles using a protractor. Fill in the missing words. These can help you (you can use a word more than once): angle, sides, curved, centre, zero
Answers: on the next page

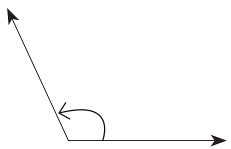
20 Constructing geometric objects *continued*

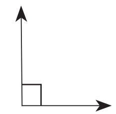
Find the centre hole on the straight edge of the protractor. 	Place the hole over the vertex of the angle you wish to measure. 
Line up the zero on the straight edge of the protractor with one of the sides of the angle. 	Find the point where the second side of the angle intersects the zero edge of the protractor. 

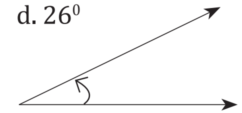
Q2 Name four professions where people will use protractors.
Answers:
a. Mathematicians
b. Builders
c. Architects
d. Carpenters

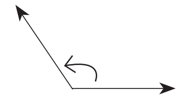
Q3 Measure each angle. Answers:

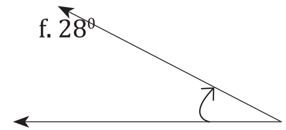
a. 30^0


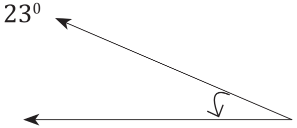
b. 112^0


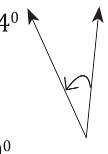
c. 90^0


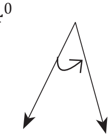
d. 26^0


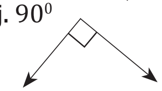
e. 120^0


f. 28^0


g. 23^0


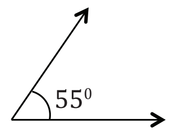
h. 34^0


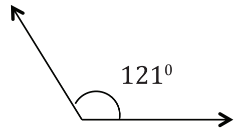
i. 34^0


j. 90^0


As a practical activity, when the learners have completed question 3, ask them to classify each of the angles as either acute, right or obtuse.
Answer:
acute (a, d, f, g, h, i),
right (c, j),
obtuse (b, e)

Q4 Estimate and draw an angle. Possible answers:


55^0


121^0

Problem solving

If you measure an angle that is between 0° and 45°, how big could the angle be? Where in nature do we find an angle of that size?

Answer: many answers are possible such as the angle between two branches, between flower stems, the angle flower petals form when rotating, etc.

45

21 Angles and sides

Topic: Angles **Content links:** 22-23
Grade 8 links: 45-46, 49 **Grade 9 links:** 39-40, 47, 53-56

Objectives

- Accurately use a protractor to measure and classify acute, right and obtuse and straight angles
- Use a protractor to measure and draw angles

Dictionary

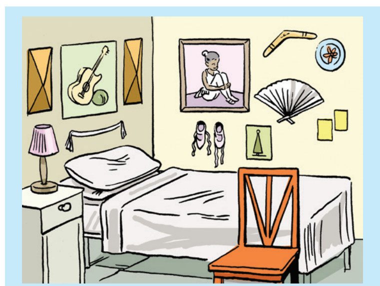
Angle: An angle is a figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle.
Interior angle: An interior angle is an angle inside a shape.
Exterior angle: The exterior angle is the angle between any side of a shape, and a line extended from the next side.

46

Introduction

Ask the learners to look at the picture and identify all the 90° angles, the angles similar than 90° and the angles bigger than 90°. Ask learners the following questions:

- Which type of angle do we get more often in a room?
- Which type of angle do we get less often in a room?



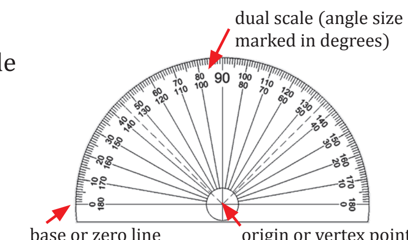
Q1 What is an angle?
Answer: An angle is the measurement in degrees between two lines which start at the point OR An angle is made when the two straight lines meet or cross each other at a fixed point and the size of the angle is measured in degrees by the amount one line has turned in relation to the other.

Q2 Match column B with column A. Answer:

A: Name of angle	B: Degrees
Acute angle	90°
Right angle	360°
Obtuse angle	Less than 90°
Straight angle	Between 180° and 360°
Reflex angle	Between 90° and 180°
Revolution	180°

Q3 What is a protractor? Answer:
A flat half-circle shaped tool, made of transparent plastic, used for measuring angles in degrees (°).

Q4 Label the protractor.



21 Angles and sides *continued*

Topic: Angles **Content links:** 22-23
Grade 8 links: 45-46, 49 **Grade 9 links:** 39-40, 47, 53-56

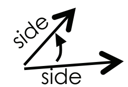
Q5 Measure and name each angle.

Answers:

a. 30^{0} acute angle
b. 112^{0} obtuse angle
c. 251^{0} reflex angle
d. 90^{0} right angle
e. 180^{0} straight angle

Q6 What is a side (or ray)?

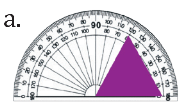
Answer: a. One length of the angle formed.



Q7 Look at the pictures of the protractors.Write down the size of the angle being measured each time and also use your ruler to measure the length of the sides of each shape.

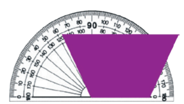
Answers:

a.



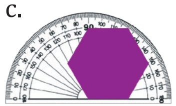
Angle: 60^{0}
Length of sides: 28 mm

b.



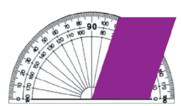
Angle: 115^{0}
Length of sides: 28 mm and 48 mm

c.



Angle: 120^{0}
Length of sides: 16 mm

d.



Angle: 71^{0}
Length of sides: 24 mm (x2), 35 mm (x2)

Q8 Name the angles.

Answers:

Angle size	Name of angle
40°	Acute angle
96°	Obtuse angle
180°	Straight angle/line
172°	Obtuse angle
200°	Reflex angle
145°	Obtuse angle
60°	Acute angle
2°	Acute angle
359°	Reflex angle
240°	Reflex angle

Q9 Identify and name four angles in the picture.

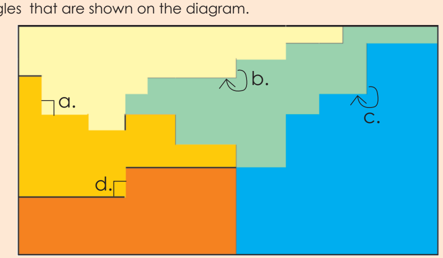
Answers:

a. 180^{0} straight line
b. 90^{0} corner
c. 360^{0} stove plate
d. 120^{0} cupboard top

49

Problem solving

a. Add the angles that are shown on the diagram.



b. If I have an angle that is not an acute angle and is smaller than 180°, what type of angle is it?

Answers:
a. 720^{0}
b. obtuse

22 Size of angles

Topic: Angles **Content links:** 21, 23
Grade 8 links: 45-46, 49 **Grade 9 links:** 39-40, 47, 53-56

Objectives

- Accurately use a protractor to measure and classify acute, right and obtuse and straight angles
- Use a protractor to measure and draw angles

Dictionary

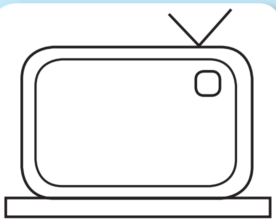
Angle: The amount a line turns from one position to another, around a fixed point.

50

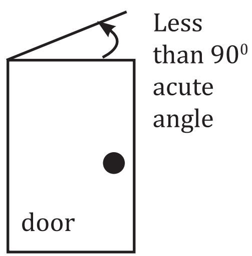
Introduction

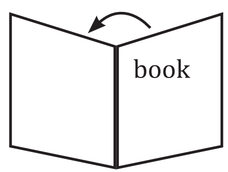
Introduce this lesson by asking learners what is an angle. They don't have to name the angles but just to describe them. Ask learners to make three drawings of angles in everyday life.

What is an angle? Make three drawings of angles that you can see in your home.



TV on a shelf 90^0

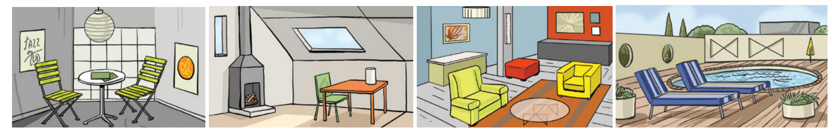




More than 90^0 obtuse angle

Q1

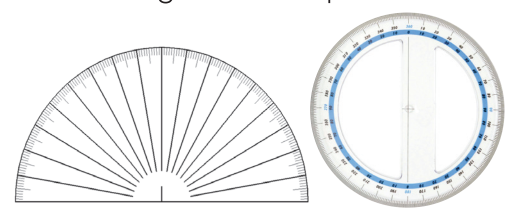
Find angles in these pictures and measure them using your protractor. (Note: the angles in the pictures will not be all the same as they are on real objects because of perspective in the pictures).



Answers:
These are many possible answers, e.g.
a. 360^o revolution in circular table top
b. 90^o right angles in windows, table top
c. 180^o straight angle in tables, carpet edge
d. 145^o obtuse angle in seat cushions

Q2

Fill in the degrees on the protractors.

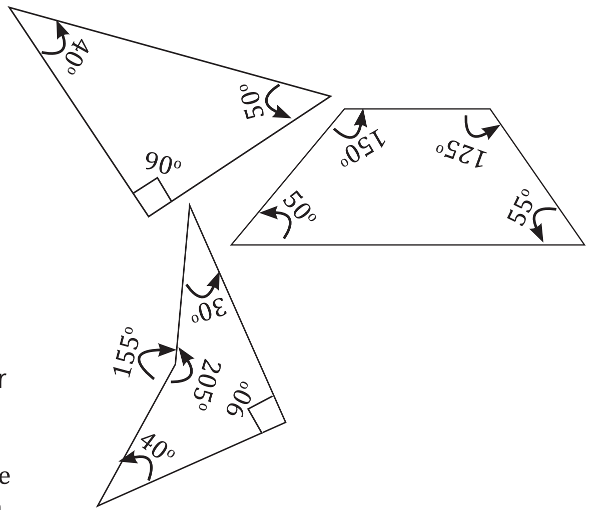


Answers must have at least:
a. 0^0, 90^0 and 180^0
b. 0^0 (360^0), 90^0, 180^0, and 270^0

22 Size of angles *continued*

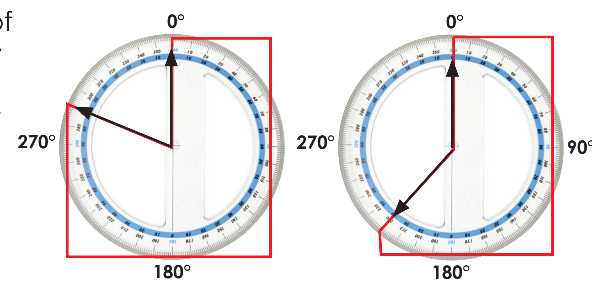
Topic: Angles **Content links:** 21, 23
Grade 8 links: 45-46, 49 **Grade 9 links:** 39-40, 47, 53-56

Q3 What is the angle size of these polygons?
Answers:



Q4 The angle measured below is 290°. Is it possible to get a polygon with an interior angle of 290°? Explain your answer.
Possible answer: a. Yes. The shape of the polygon makes for a large obtuse angle.

What is the size of the angle? Draw a polygon with the same interior angle. Answers: 220^0. This is a possible polygon.



Q5 Measure the angles you marked.

Answers:

a. As marked by learner. Mainly right angles and some smaller than 90^0 angles for the roof.
b. Mainly right angles.

53

Problem solving

What are the most common angles you will find in your home?
What angles are the most common in motor vehicles?

Answers:

- right angles
- acute and obtuse angles

The learners' answers may differ. If so, ask them why they differ.

Reflection questions

Did learners meet the objectives?

23 Using a protractor

Topic: Constructions **Content links:** 24-26, 97, 103-104
Grade 8 links: R11, 45-48, 50-55, 63, 132-133 **Grade 9 links:** R11, 39-42, 44-46, 121-122

Objectives

- Accurately construct geometric figures appropriately using a pair of compass, ruler and protractor, including angles to one degree of accuracy

Dictionary

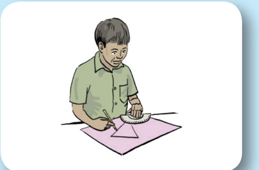
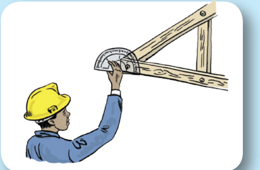
Protractor: An instrument used to measure or draw angles, usually in the shape of a half circle marked out in degrees (⁰) from 0⁰ to 180°. The measuring unit for angles is degree (⁰). To measure an angle, place the centre line in the little half circle (sometimes a little hole) of the protractor on the vertex of the angle. Line up the zero line on the protractor with one side of the angle. Then read the measurement where the other side touches the protractor scale.

54

Introduction

Ask the learners to look at the pictures. Ask them what these people are using their protractors for.

Look at the pictures. What are these people using their protractors for?

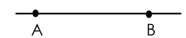




Q1

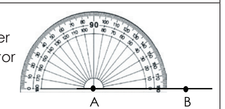
The step-by-step instructions below show how to draw a 45⁰ angle. Follow these instructions to draw the angles given in the questions. Answers:

Use a protractor to draw some angles. Do this by following the step–by–step instruction on the left.

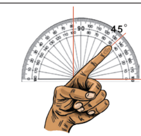
Step 1: Draw a line segment. Label it AB.



Step 2: Place the protractor so that the origin (small hole) is over the point A. Rotate the protractor so that the base line is exactly along the line AB.

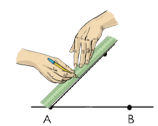


Step 3: Using (in this case) the inner scale, find the angle desired – In 1. it is 45°.

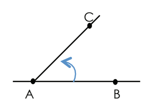


Step 4: Make a mark at this angle, and remove the protractor.

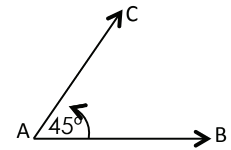
Step 5: With the protractor or a ruler draw a straight line from A to the mark you just made. Label this point C.



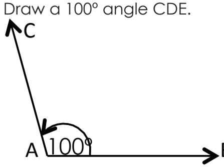
Step 6: The line drawn makes an angle BAC with a measure of 45°.



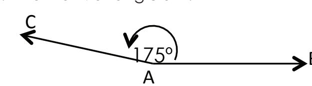
1. Draw a 45° angle ABC.



2. Draw a 100° angle CDE.



3. Draw a 175° angle JKL.



23 Using a protractor *cont...*

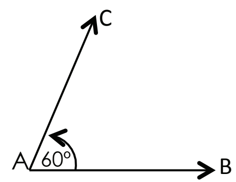
Topic: Constructions **Content links:** 24-26, 97, 103-104
Grade 8 links: R11, 45-48, 50-55, 63, 132-133 **Grade 9 links:** R11, 39-42, 44-46, 121-122

Q2

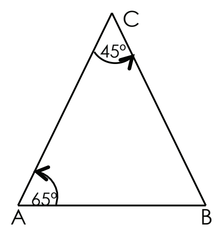
Use a ruler and a protractor to draw and label geometric figures. Write down the steps how you construct it.

Answers:

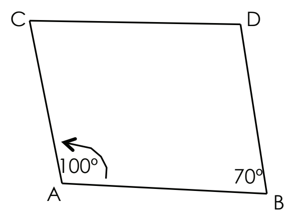
a. Draw a straight line segment and label it A B. Measure 60^0. and connect A to C. Mark the angle.



b. Follow above steps for 65^0. Turn the paper and starting from point C measure 45^0. Connect C to line segment AB.



c. Draw a line segment AB and measure 100^0 from A and 70^0 from B. Connect B with D and A with C. Connect C with D.

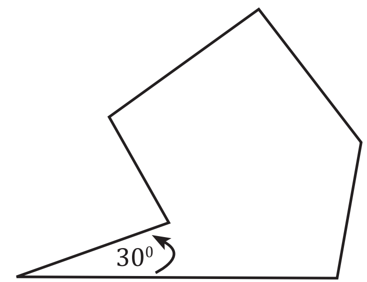


55

Problem solving

Draw a polygon with six sides where one angle is 30°.

Answer: Many shapes of polygons are possible. This example is an irregular hexagon, "irregular" because all its sides are of different lengths.



Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

24 Parallel and perpendicular lines

Topic: Straight lines, Constructions **Content links:** 23-26, 97, 103-104
Grade 8 links: 45-51, 59, 61-63 **Grade 9 links:** 39-46, 53, 55

Objectives

- Accurately construct parallel and perpendicular lines appropriately using a pair of compasses
- Recognise, describe and define perpendicular lines, parallel lines and line segments

Dictionary

Parallel lines: Two or more lines which are equidistant, in other words the distance between one line and another is consistent throughout. The perpendicular height between 2 parallel lines is identical wherever it is measured.

Perpendicular lines: Lines that intersect (meet) at right angles (90°) to each other

Compass (construction): An instrument with two arms, one with a sharp point and one which holds a pencil that can be used to draw circles or arcs

Compass (direction): An instrument that shows us directions by means of a small magnetic needle that points toward magnetic North of the earth.

Make sure learners do not confuse the two.

56

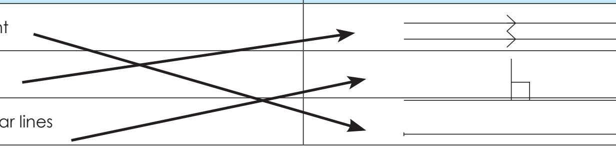
Introduction
Ask the learners to look at these structures and identify the parallel and perpendicular lines. (Note that they will need to ignore the effects of perspective in the pictures and to use what they know about angles and lines in reality.)

Look at the structures. Identify the parallel, perpendicular and line segments.

Q1 What mathematical instrument is a compass? See dictionary: Compass (construction)
Answer: An instrument with two arms, one with a sharp point and one which holds a pencil that can be used to draw circles or arcs.

Q2 Match column B with column A.
Answers:

Column A	Column
Line segment	
Parallel lines	
Perpendicular lines	



24 Parallel and perpendicular lines *cont...*

Q3

Draw the following line segments with a ruler.

Answers:

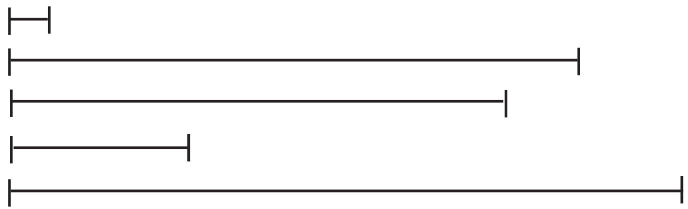
a. 5 mm

b. 7.5 cm

c. 65 mm

d. 23 mm

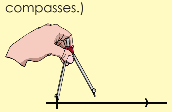
e. 8.9 cm



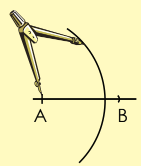
Q4

Revision: Construct a perpendicular line to bisect a given line. Use the guidelines to help you.

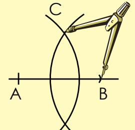
Step 1
Draw a line and mark points A and B on it. Put the compass point on A and open it so that the pencil touches point B. (So you have "measured" the length of AB with the pair of compasses.)



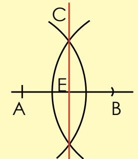
Step 2
Leaving the compass point on A, draw an arc with the compass approximately two thirds of the line length.



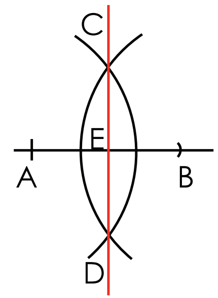
Step 3
With the compasses' width the same, move the compass point to B and draw another arc which crosses the first arc at two points. Label these points C and D.



Step 4
Draw a line through points C and D bisecting the line AB at E.



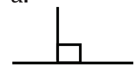
Answer:



Q5

What symbol do we use to show:

Answers:

a. 

b. 

c. >

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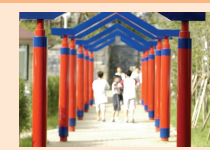
Problem solving

In reality are these lines and pillars parallel or not? Say why or why not.



Answer:

They are parallel because they are the same distance apart. Photographs sometimes give us a perspective that that makes it appear as if the lines are not parallel.

25 Construct angles and a triangle

Topic: Constructions **Content links:** 21-24, 27, 103
Grade 8 links: 45-51 **Grade 9 links:** 39-46

Objectives

- Accurately construct angles and triangles appropriately using a pair of compasses

Dictionary

Construct: To construct is to draw a shape, line or angle accurately using a compass, straight edge, protractor or triangle.

Construction: Construction in geometry means drawing of geometric items such as lines and circles using only a pair of compasses and straight edge. You are not allowed to measure angles with a protractor, or measure lengths with a ruler.

Angle: The amount a line turns from one position to another, around a fixed point.

Triangle: A polygon with three sides and three angles. The three angles will always add up to 180^0. These are names of three special types of triangle: Equilateral, Isosceles and Scalene. Names tell you about the sides or the angles inside the triangle.

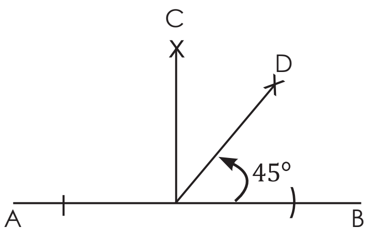
58

Introduction
Ask the learners identify the triangles in the picture and estimate the sizes of their interior angles.

Identify the triangles and estimate the size of the angles.

Q1

Construct a 45^0 angle. Use the guidelines to help you.
Answers:



Q2

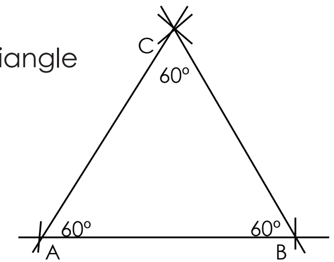
Give five real life examples where we will find 45° angles.
Answers:
a. Joints b. Roofs c. Toys d. Buildings e. Cars

25 Construct angles and a triangle *cont...*

Topic: Constructions **Content links:** 21-24, 27, 103
Grade 8 links: 45-51 **Grade 9 links:** 39-46

Q3

Construct an equilateral triangle.
Follow the steps and construct your triangle below. Answers:



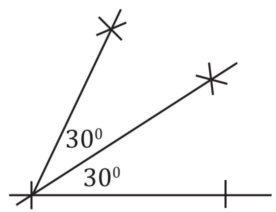
Q4

Construct a triangle with one angle of 90° and one angle of 60° without using a protractor.
Answers: Learner's own triangle. . The learner will need to have constructed a perpendicular line (to get the right angle) and then an equilateral triangle (to get the 60^0 angle).

The learner will need to construct a perpendicular to a line (thereby getting a right angle) and then from the point where the perpendicular meets the line construct an equilateral triangle (thereby getting three 60^0 angles). The one side of the equilateral triangle is extended to meet the perpendicular thus forming a larger triangle with a right angle and a 60^0 angle.

Q5

Construct a 30^0 angle.
Use the guidelines below.



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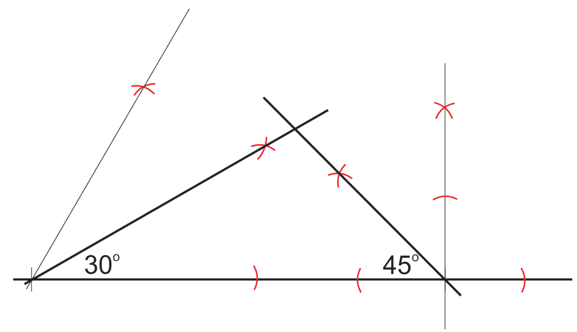
Problem solving

Construct any figure with at least one 30° and one 45° angle.

Answer: Learner's own triangle. The learner will need to have constructed an equilateral triangle (to get a 60^0 angle), divided a 60^0 angle to get a 30^0 angle and then constructed a perpendicular line to get a right angle which is then divided in half (to get a 45^0 angle).



Reflection questions

Did learners meet the objectives?

26 Circles

Topic: Constructions **Content links:** None
Grade 8 links: None **Grade 9 links:** None

Objectives
Accurately construct circles appropriately using a pair of compasses, ruler and protractor

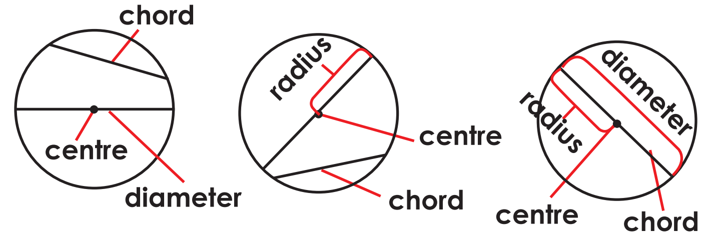
Dictionary
Circle: the set of all points on a plane that are the same fixed distance from a centre point

62

Introduction
Ask the learners what to these pictures have in common.
They are all forming circles.

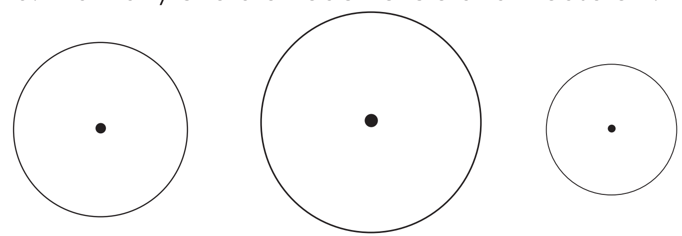
What do all these pictures have in common?

Q1 Label the circle.



Q2 What is a circle?
Answers: A circle is the set of all points on a plane that are the same fixed distance from a centre point.

Q3 Measure the diameter of each circle. What is the radius of the circles?
a. Underneath each circle write its radius.
b. Draw any chord on each circle and measure it.



Answers: [Approximate lengths due to variations in printing]
a. Radius: 12 mm b. Radius: 15 mm c. Radius:9 mm
Chords: Learner's own measurement

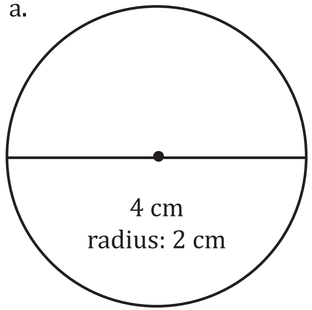
Topic: Constructions **Content links:** None
Grade 8 links: None **Grade 9 links:** None

Q4

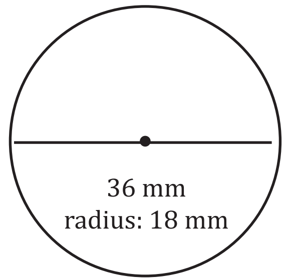
Draw circles with the given diameters.

Answers:

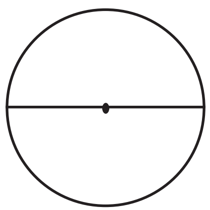
a.



b.

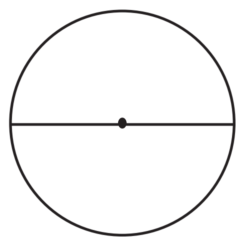


c.



2,6 cm
radius: 1,3 cm

d.



30 mm
radius: 15 mm

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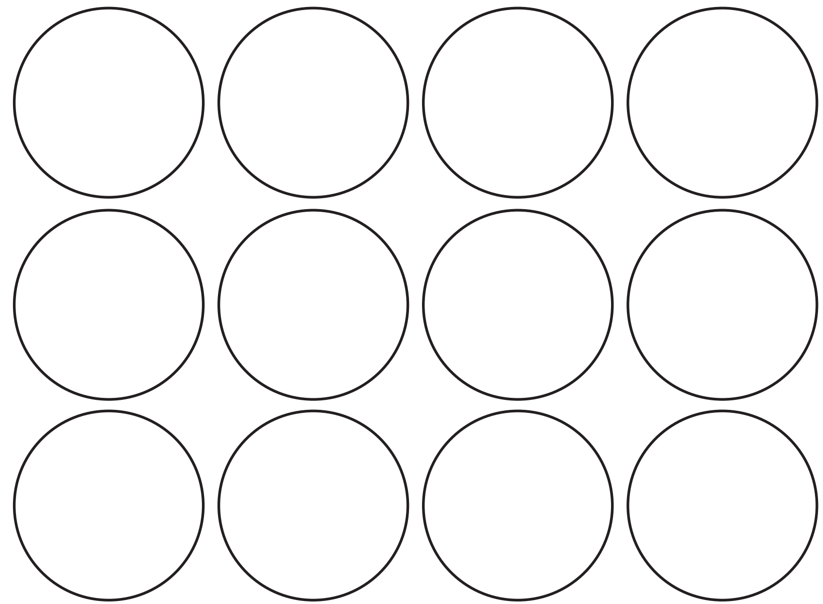
Problem solving

Draw a circle with a radius of 25 mm. Continue drawing circles with 25 cm radii to fill a separate sheet of paper with circle patterns.



27 Triangles

Topic: 2-D shapes **Content links:** R10, 26, 28-29
Grade 8 links: 52-58 **Grade 9 links:** R13, 41, 43, 47-52

Objectives

Describe, sort name and compare triangles according to their sides and angles, focusing on:

- Equilateral triangles
- Isosceles triangles
- Right-angled triangles

Dictionary

Triangle: A polygon with three sides and three angles. The three angles will always add to 180°. These are names of three special types of triangle: Equilateral, Isosceles and Scalene. Names tell you about the sides or the angles inside the triangle.
Equilateral triangle: This is a triangle with three sides of equal length and three equal angles of 60°.
Isosceles triangle: This triangle has two sides of equal length and two angles equal.
Right-angle triangle: A right angled triangle is a triangle which has a right angle (90°) in it.
Scalene triangle: This triangle has no sides or angles that are equal.

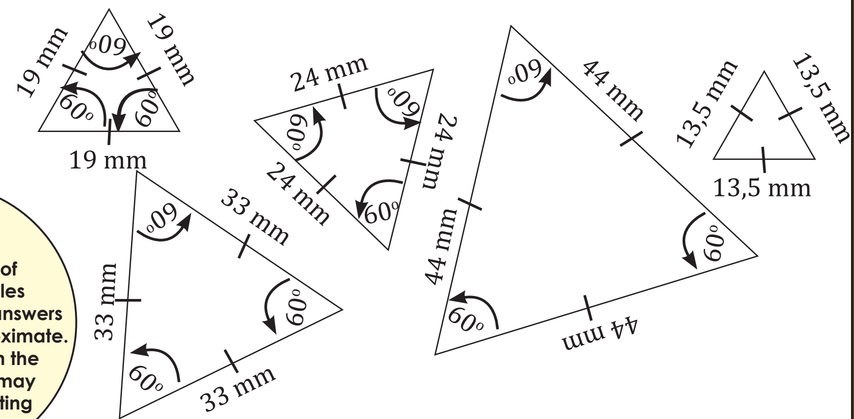
When checking the learners' measurements of lengths and angles remember that the answers given here are approximate. The dimensions on the workbook page may vary due the printing process.

64

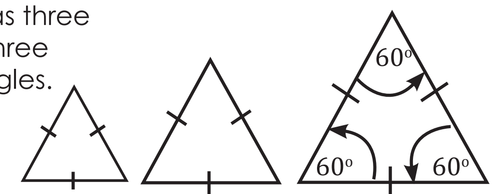
Introduction

What do these triangular road signs mean? Draw another two.

Q1 Measure each of these triangles.
a. Measure the sides Answer: See diagrams below. [Approximate lengths due to variations in printing]
b. What do you notice? Answer: the sides are equal
c. Measure the angles of the triangles. Answer: All are 60°.
d. Label each triangle.



Q2 An equilateral triangle has three sides and angles. Draw three different equilateral triangles. Label each. Answers:



27 Triangles *continued*

Topic: 2-D shapes **Content links:** R10, 26, 28-29
Grade 8 links: 52-58 **Grade 9 links:** R13, 41, 43, 47-52

Q3

Measure each of these triangles.

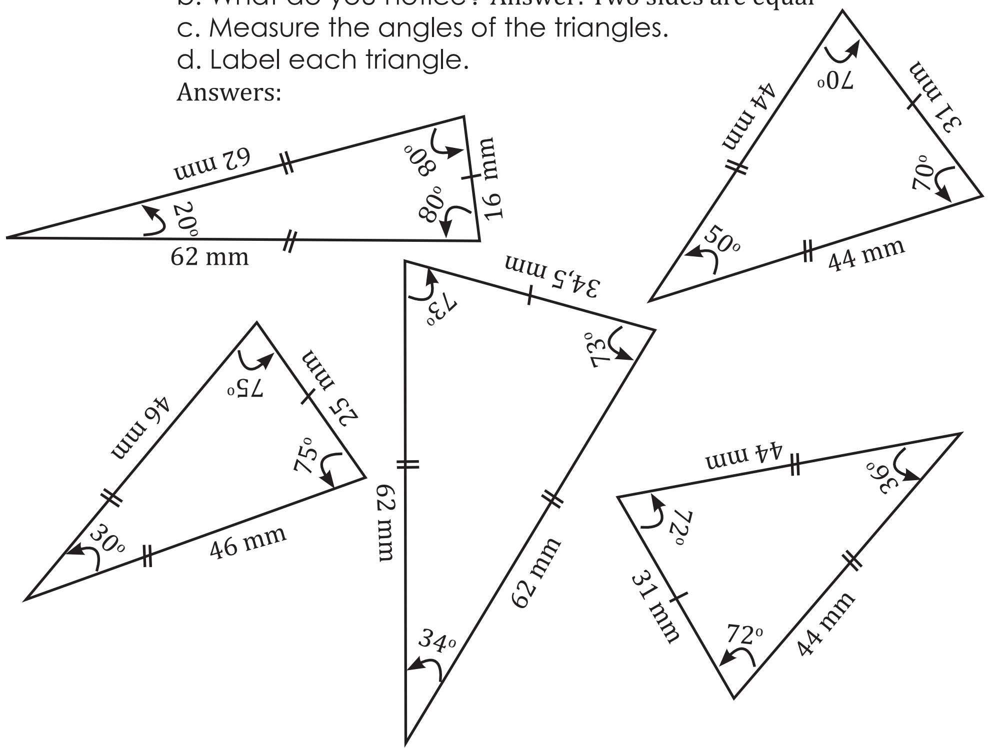
a. Measure the sides. See diagrams below. [Approximate lengths due to variations in printing]

b. What do you notice? Answer: Two sides are equal

c. Measure the angles of the triangles.

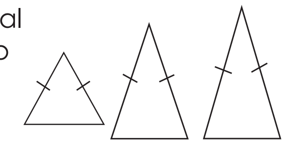
d. Label each triangle.

Answers:



Q4

An isosceles triangle has two sides of equal length and the angles opposite these two sides are also equal. Draw three different isosceles triangles. Answers:



Q5

Measure each of these triangles.

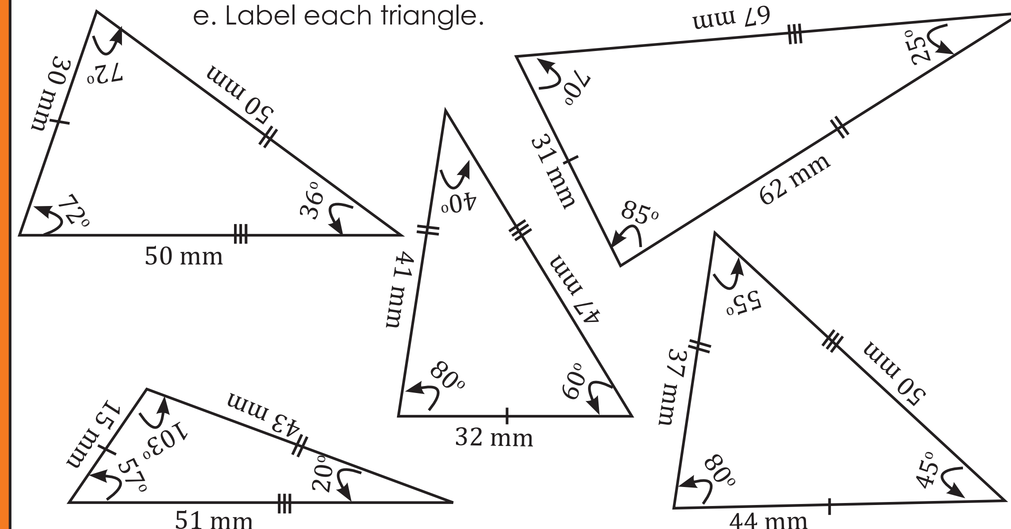
a. Measure the sides. See diagrams below. [Approximate lengths due to variations in printing]

b. What do you notice? Answer: all 3 sides are different

c. Measure the angles of the triangles.

d. What do you notice? Answer: All the angles are different.

e. Label each triangle.



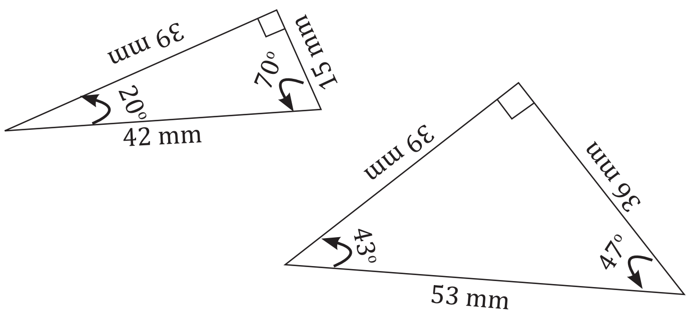
27 Triangles *continued*

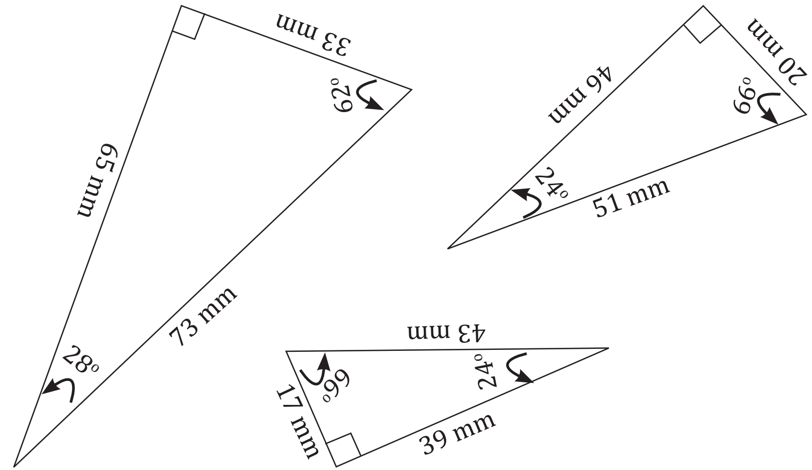
Topic: 2-D shapes **Content links:** R10, 26, 28-29
Grade 8 links: 52-58 **Grade 9 links:** R13, 41, 43, 47-52

Q6 A scalene triangle has three sides of different lengths. Draw three different scalene triangles.
Answers: Learner's drawing of three triangles each with three sides of different lengths.

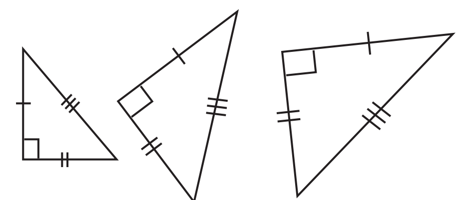
Q7 Measure each of these triangles.

a. Measure the sides. Answer: See diagrams below. [Approximate lengths due to variations in printing]
b. What do you notice? Anwers: All the sides are different lengths
c. Measure the angles of the triangles.
d. What do you notice? Answer: All the angles are different. One of the angles is a right angle.
e. Label each triangle.





Q8 Draw three triangles of different size each with a right angle (90°). Answers:



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Problem solving

Create your own gift wrapping by drawing triangles on a sheet of paper. You should use all the types of triangles you have learned about.

Answer: Learner's own answer

28 Polygons

Topic: 2-D shapes **Content links:** R10, 26, 27, 29
Grade 8 links: 58 **Grade 9 links:** R13, 43, 49-50

Objectives

- Describe, sort, name and compare polygons
- Describe, sort, name and compare quadrilaterals in terms of size of angles (right angles or not, length of sides, and whether sides are parallel or perpendicular to each other)
- Solve simple geometric problems involving unknown sides and angles in triangles and quadrilaterals using known properties and definitions

Dictionary

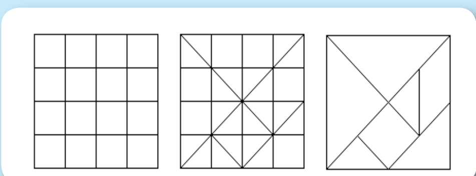
Polygon: A plane shape completely enclosed by three or more straight edges, e.g. triangles, quadrilaterals and pentagons.

68

Introduction

Tell the learners to use Cut-out 1 to make a tangram.
They will use these trangram pieces later on in this worksheet.
Ask the learners why they think a tangram is called a disectional puzzle? A Tangram is a dissection puzzle consisting of seven pieces which fit together to form a shape.

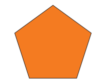
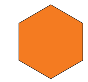
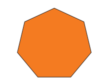
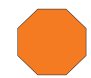
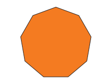
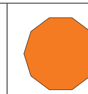
Use the diagrams below to make your own Chinese puzzle, the tangram.



Answer: Because something is 'cut-up into pieces' so that so that one can understand its structure and how it works.

Q1

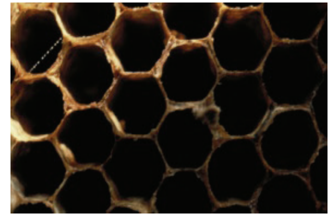
Complete this table. Answers:

Polygon						
Number of sides	5	6	7	8	9	10
Angle size	70°	110°	126°	135°	135°	145°
Total sum of angles	350°	660°	882°	1 080°	1 215°	1 450°

Q2

What is this? Where would you find it? What polygon/s can you identify?

a.



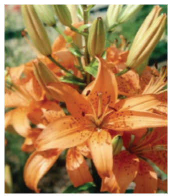
b.

Answers: a. Wasp nest hexagon b. Paving stone hexagon

28 Polygons *continued*

Topic: 2-D shapes **Content links:** R10, 26, 27, 29
Grade 8 links: 58 **Grade 9 links:** R13, 43, 49-50

Q3 What geometric figures do you see?

Answers:
a. Polygon (Octagon)
b. Star

Q4 Identify, name and describe the following polygons in these pictures.

a.

Answers:
Triangles; parallelograms, trapeziums, rombuses

b.

Rectangles; squares

Q5 The tangram in Cut-out 1 is a dissection puzzle. It consists of seven pieces, called tans, which fit together to form a shape of some sort. The objective is to form a specific shape with seven pieces. The shape has to contain all the pieces, which may not overlap.

Answers: These are possible answers.

a.

b.

c.

d.

e.

f.

Q6 Say whether each of the following is a quadrilateral or not. Provide reasons for your answer.

Answers: a., b. and f. are quadrilaterals (they have four sides each). The other shapes have more than four sides.

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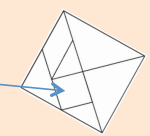
Problem solving

What fraction of the tangram is this square?

Answer: The square is one eighth of the tangram. This could be found by reordering the tangram shapes.



29 Congruent and similar shapes

Topic: 2-D shapes **Content links:** None
Grade 8 links: 55 **Grade 9 links:** 52

Objectives

Recognize and describe similar and congruent figures by comparing:

- Size
- Shapes

Solve simple geometric problems involving unknown sides and angles in triangles and quadrilaterals using known properties and definitions.

Dictionary

Congruent: Having the same shape and size. Congruent shape have all sides and angles equal.

Similar: Having the same shape but different in size. Similar shapes have the corresponding angles in each shape the same.

Hypotenuse: The longest side of a right-angled triangle which is opposite the right angle.

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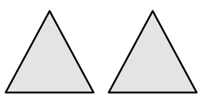
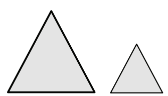
Introduction

Make the following drawing on the board. Ask learners to compare them.



The triangles have the same shape and size.



The triangles have the same shape but differ in size.

Q1

What do you notice about these pictures.

Answers: They all have the same shape and size.

Q2

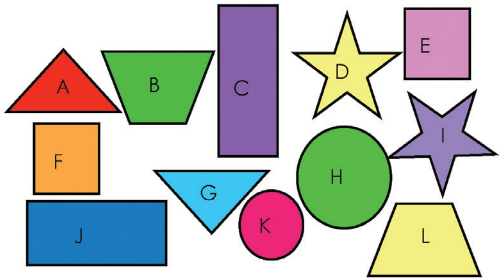
What do you notice about these pictures.

Answers:They all have the same shape but differ in size.

Congruent and similar shapes *cont...*

Topic: 2-D shapes **Content links:** None
Grade 8 links: 55 **Grade 9 links:** 52

Q3 Which of the following shapes are congruent?

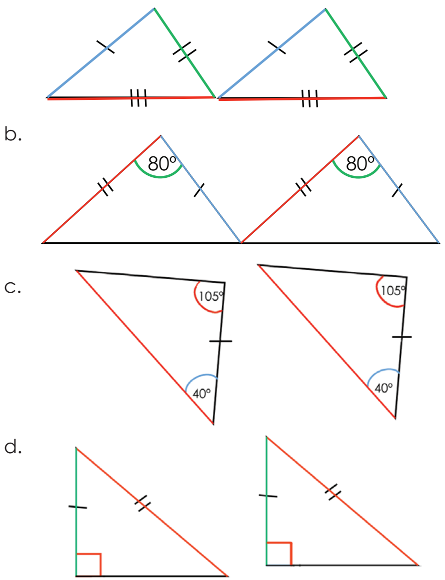


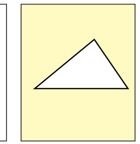
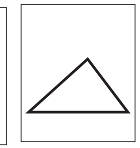
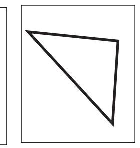
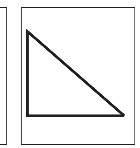
Answers: The following shapes are congruent: A and G; B and L; D and I

Q4 Draw a similar shape for:
Answers: learner's own drawings

Q5 Are these shapes congruent? Give reasons for your answer.
Answers:
a. Yes, they have the same shape and size.
b. Yes, they have the same shape and size.
c. Yes, they have the same shape and size.

Q6 All these triangles are congruent. Write down what is the same in both triangles. We did the first one for you.



SSS side side side	All three corresponding sides are equal.	
SAS side angle side	Two sides enclosing an angle are equal	
SAA side angle angle	One side and two angles are equal.	
SSA side side angle	Two sides and one angle are equal.	

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Problem solving

Where in nature will we see similarity and congruency? Draw a picture to illustrate your answer.

Answer: Learner's own answer

30 Fractions

Topic: Fractions **Content links:** R7, 31-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Objectives

- Count forward and backwards in fractions
- Identify, recognise and name proper, improper and mixed fractions
- Compare and order fractions

Dictionary

Proper Fraction: A proper fraction is a fraction in which the numerator (the top number) is smaller than the denominator (the bottom number). It is less than one. E.g.: $\frac{3}{4}$

Improper Fraction: An improper fraction is where the numerator (the top number) is greater than or equal to the denominator (bottom number). E.g.: $\frac{5}{2}$

Mixed Fraction: A mixed fraction is a whole number and proper fraction combined into one "mixed number". It is larger than one. Also called a mixed number. E.g.: $2\frac{1}{4}$.

A mixed fraction can be changed into an improper fraction and vice versa.

Common Fraction: A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. Also called a **vulgar fraction**.

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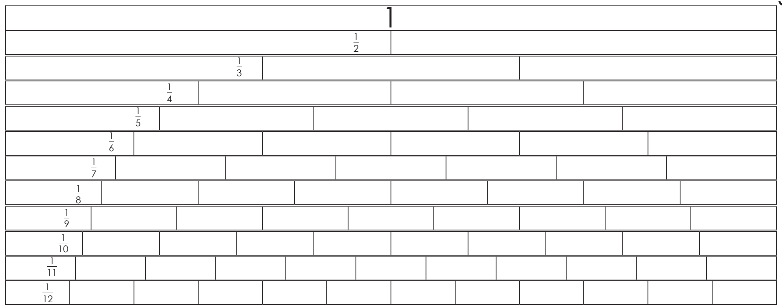
Introduction

Ask the learners to look at the fraction strips. Ask them the following questions:

- What is this?
- How will you use it to determine equivalent fractions.

Use the fraction strips and answer the following:

- Give all the fractions equivalent to: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$
- Give four fractions bigger and one fraction smaller than: $\frac{2}{5}$



Q1

Complete the following: Answers:

a. $\frac{1}{4}; \frac{2}{4}; \frac{3}{4}; 1$

b. $\frac{1}{9}; \frac{2}{9}; \frac{3}{9}; \frac{4}{9}; \frac{5}{9}; \frac{6}{9}; \frac{7}{9}; \frac{8}{9}; 1$

c. $\frac{1}{11}; \frac{2}{11}; \frac{3}{11}; \frac{4}{11}; \frac{5}{11}; \frac{6}{11}; \frac{7}{11}; \frac{8}{11}; \frac{9}{11}; \frac{10}{11}; 1$

d. $\frac{1}{5}; \frac{2}{5}; \frac{3}{5}; \frac{4}{5}; 1$

e. $\frac{1}{6}; \frac{2}{6}; \frac{3}{6}; \frac{4}{6}; \frac{5}{6}; 1$

f. $\frac{1}{8}; \frac{2}{8}; \frac{3}{8}; \frac{4}{8}; \frac{5}{8}; \frac{6}{8}; \frac{7}{8}; 1$

30 Fractions *continued*

Topic: Fractions **Content links:** R7, 31-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Q2 Complete the number lines: Answers:

a. $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, 1

b. $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{7}{7}$, 1

c. $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{9}{11}$, $\frac{10}{11}$, 1

d. $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1

e. $\frac{1}{14}$; $\frac{2}{14}$; $\frac{3}{14}$; $\frac{4}{14}$; $\frac{5}{14}$; $\frac{6}{14}$; $\frac{7}{14}$; $\frac{8}{14}$; $\frac{9}{14}$; $\frac{10}{14}$; $\frac{11}{14}$; $\frac{12}{14}$; $\frac{13}{14}$; 1

Q3 Count from: Answers:

a. $\frac{2}{10}$; $\frac{3}{10}$; $\frac{4}{10}$ b. $\frac{1}{20}$; $\frac{2}{20}$; $\frac{3}{20}$; $\frac{4}{20}$; $\frac{5}{20}$; $\frac{6}{20}$; $\frac{7}{20}$; $\frac{8}{20}$; $\frac{9}{20}$ c. $\frac{4}{15}$; $\frac{5}{15}$; $\frac{6}{15}$; $\frac{7}{15}$; $\frac{8}{15}$; $\frac{9}{15}$; $\frac{10}{15}$

d. $\frac{1}{100}$; $\frac{2}{100}$; $\frac{3}{100}$; $\frac{4}{100}$; $\frac{5}{100}$; $\frac{6}{100}$; $\frac{7}{100}$; $\frac{8}{100}$ e. $\frac{10}{50}$; $\frac{11}{50}$; $\frac{12}{50}$

Q4 Complete the number lines: Answers:

a. 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2

b. 0, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, 1, $1\frac{1}{5}$, $1\frac{2}{5}$, $1\frac{3}{5}$, $1\frac{4}{5}$, 2

c. 0, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, 1, $1\frac{1}{8}$, $1\frac{2}{8}$, $1\frac{3}{8}$, $1\frac{4}{8}$, $1\frac{5}{8}$, $1\frac{6}{8}$, $1\frac{7}{8}$, 2

d. 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2

e. 0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{2}{4}$, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, $2\frac{2}{4}$, $2\frac{3}{4}$

f. Question 2 has only proper fractions, Question 4 has both proper and mixed fractions.

Oral questions

Which faction is the smaller, $\frac{3}{10}$ or $\frac{3}{100}$?

Which faction is the larger, $\frac{15}{10}$ or $\frac{15}{100}$?

Q5 Say whether it is a proper or improper fraction, or a mixed number: Answers:

a. Proper fraction
b. Improper fraction
c. Mixed number
d. Improper fraction
e. Proper fraction
f. Improper fraction

Write down: Answers: These are examples

a. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$ b. $\frac{5}{2}$; $\frac{6}{3}$; $\frac{7}{4}$; $\frac{8}{5}$; $\frac{9}{6}$ c. $1\frac{1}{2}$; $1\frac{1}{3}$; $1\frac{1}{4}$; $1\frac{1}{5}$; $1\frac{1}{6}$

Problem solving

Name five fractions that are between one quarter and two quarters.

Answer: These are examples $\frac{3}{8}$; $\frac{5}{12}$; $\frac{2}{5}$; $\frac{2}{6}$; $\frac{2}{7}$

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31 Equivalent fractions

Topic: Fractions **Content links:** R7, 30, 32-39
Grade 8 links: 5 **Grade 9 links:** 5

Objectives

- Recognise and use equivalent forms of common fractions

Dictionary

Equivalent: having the same value or amount
Equivalent fractions: fractions which have the same value, even though they may look different, e.g. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$

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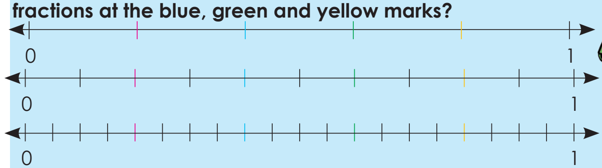
Introduction

Ask the learners what the same colour fraction markings on the number lines have in common?

- $\frac{1}{5}; \frac{2}{10}; \frac{4}{20}$ are all equivalent
- $\frac{2}{5}; \frac{4}{10}; \frac{8}{20}$ are all equivalent
- $\frac{3}{5}; \frac{6}{10}; \frac{12}{20}$ are all equivalent

Fill in the correct fraction at each of the coloured marks on the number lines below. What do the fractions at the red colour marks have in common? What about the fractions at the blue, green and yellow marks?



DEFINITION

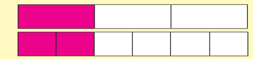
Equivalent fractions have the same value, even though they look different.

Example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent, because they are both 'half'.

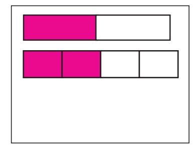
Q1

What fraction equals __: Draw a diagram to show that the two fractions are equivalent. Answers:

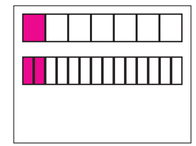
Example: $\frac{1}{3} = \frac{2}{6}$



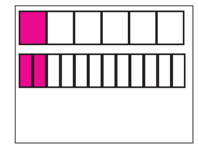
a. $\frac{1}{2} = \frac{2}{4}$



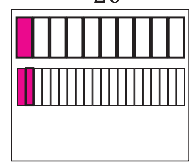
b. $\frac{1}{7} = \frac{2}{14}$



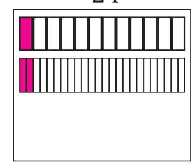
c. $\frac{1}{6} = \frac{2}{12}$



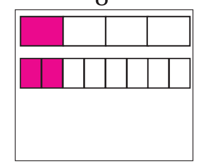
d. $\frac{1}{10} = \frac{2}{20}$



e. $\frac{1}{12} = \frac{2}{24}$



f. $\frac{1}{4} = \frac{2}{8}$



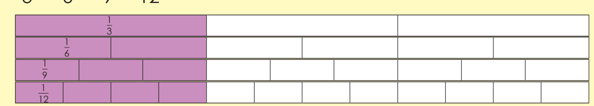
How can you use these measuring spoons to explain equivalent fractions to a friend?

Answer: It will take two spoonfuls of a $\frac{1}{8}$ spoon to fill a $\frac{1}{4}$ spoon (so $\frac{2}{8} = \frac{1}{4}$) and so on.

Q2

Write the next or previous equivalent fraction for:

Example: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$



One third is equivalent to two sixths, this again is equivalent to three ninths, and this again to four twelfths.

31 Equivalent fractions *continued*

Topic: Fractions **Content links:** R7, 30, 32-39
Grade 8 links: 5 **Grade 9 links:** 5

Answers:

a. $\frac{2}{4} = \frac{1}{2}$

b. $\frac{3}{4} = \frac{6}{8}$

c. $\frac{2}{7} = \frac{4}{14}$

d. $\frac{8}{10} = \frac{4}{5}$

e. $\frac{2}{5} = \frac{4}{10}$

f. $\frac{4}{5} = \frac{8}{10}$

Q3

What happened to the numerator and denominator in question 2:

Answers:

a. Halved

b. Doubled

c. Halved

d. Halved

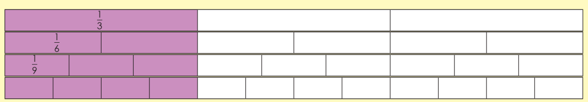
e. Halved

f. Doubled

Q4

Write down three equivalent fraction for: Make a drawing.

Example: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$



One third is equivalent to two sixths, this again is equivalent to three ninths, and this again to four twelfths.

Answers:

a. $1\frac{1}{2} = 1\frac{2}{4} = 1\frac{4}{8} = 1\frac{6}{12}$

b. $3\frac{2}{3} = 3\frac{4}{6} = 3\frac{8}{12} = 3\frac{16}{24}$

c. $4\frac{1}{2} = 4\frac{2}{4} = 4\frac{4}{8} = 4\frac{6}{12}$

d. $6\frac{1}{3} = 6\frac{2}{6} = 6\frac{4}{12} = 6\frac{6}{18}$

e. $2\frac{3}{4} = 2\frac{6}{8} = 2\frac{9}{12} = 2\frac{12}{16}$

f. $2\frac{4}{5} = 2\frac{8}{10} = 2\frac{12}{15} = 2\frac{16}{20}$

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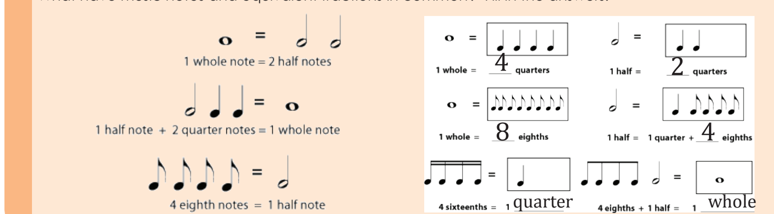
Problem solving

What have music notes and equivalent fractions in common? Fill in the answers.



Reflection questions

Did learners meet the objectives?

32 Simplest form

Topic: Fractions **Content links:** R7, 30-31, 33-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Objectives

- Determine the Highest Common Factor (HCF)
- Write fractions in their simplest form

Dictionary

Highest common factor: E.g. the highest common factor of 2, 3 and 4 is 12.

Common Fraction: A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. Also called a vulgar fraction.

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Introduction

Ask the learners the following questions:

- Are $\frac{8}{16}$ and $\frac{1}{2}$ the same?
- What happened to the numerator from the first to the second fractions? (It is simplified by being divided by 8. It is important that learners should notice that they are equivalent fractions.
- Denominator? (See previous answer)
- Why do you think we need to know how to use the HCF?

Are $\frac{8}{16}$ and $\frac{1}{2}$ the same?

What happened to the numerator from the first to the second fractions?

What happened to the denominator?

Why do you think we need to know how to use the HCF?

DEFINITION

Highest common factor (HCF)
The highest number that divides exactly into two or more numbers.
If you find all the factors of two or more numbers, and you find some factors are the same ("common"), then the largest of those common factors is the Highest Common Factor.

The HCF is sometimes also called the **Greatest Common Factor (GCF)** or the **Greatest Common Divisor (GCD).**

Q1

What is the highest common factor?

Example:
Highest common factor (HCF)
Factors of 4: {1, 2, 4}
Factors of 6: {1, 2, 3, 6}
HCF = 2
So 2 is the biggest number that can divide into 4 and 6.

Answers:

a. Factors of 3 = {1; 3}	Factors of 4={1;2;4}	HCF = 1
b. Factors of 5 ={1;5}	Factors of 6 ={1;2;3;6}	HCF = 1
c. Factors of 6={1;2;3;6}	Factors of 12 ={1;2;3;4;6;12}	HCF = 6
d. Factors of 3 ={1;3}	Factors of 9 ={1;3;9}	HCF = 3
e. Factors of 7 ={1;7}	Factors of 8 = {1;2;4;8}	HCF = 1
f. Factors of 11 ={1;11}	Factors of 10 ={1;2;5;10}	HCF = 1

32 Simplest form *continued*

Topic: Fractions **Content links:** R7, 30-31, 33-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Q2

Write in the simplest form.

Answers:

a. Factors of 6: {1; 2; 3; 6} Factors of 18: {1; 2; 3; 6; 18}

$\frac{6}{18} \div \frac{6}{6} = \frac{1}{3}$

b. Factors of 15: {1; 3; 5; 15} Factors of 25: {1; 5; 25}

$\frac{15}{25} \div \frac{5}{5} = \frac{3}{5}$

c. Factors of 3: {1; 3} Factors of 9: {1; 3; 9}

$\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$

d. Factors of 7: {1; 7} Factors of 21: {1; 3; 7; 21}

$\frac{7}{21} \div \frac{7}{7} = \frac{1}{3}$

e. Factors of 4: {1; 2; 4} Factors of 36: {1; 2; 3; 4; 6; 9; 18; 36}

$\frac{4}{36} \div \frac{4}{4} = \frac{1}{9}$

f. Factors of 18: {1; 2; 3; 6; 9; 18} Factors of 36: {1; 2; 3; 4; 6; 9; 18; 36}

$\frac{18}{36} \div \frac{18}{18} = \frac{1}{2}$

Q3

Fill in the missing words.
(common factor, numerator and denominator)

Answers:

a. Fractions can be simplified when the numerator and denominator have a common factor in them.

b. Give five examples of fractions that could be simplified.

$\frac{3}{6} = \frac{1}{2}$, $\frac{12}{36} = \frac{1}{3}$, $\frac{18}{54} = \frac{1}{3}$, $\frac{9}{27} = \frac{1}{3}$, $\frac{6}{18} = \frac{1}{3}$

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Problem solving

What is $\frac{324}{414}$ in its simplest form?

Answer:

$\frac{324}{414} \div \frac{18}{18} = \frac{18}{23}$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7, 30-32, 34-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

33 Add common fractions with the same and different denominators

Objectives

- Addition and subtraction of common fractions, including mixed numbers, limited to fractions with the same denominator or where one denominator is a multiple of another
- Extend addition and subtraction of fractions where one denominator is not multiple of the other

Dictionary

Proper Fraction: A proper fraction is a fraction in which the numerator (the top number) is smaller than the denominator (the bottom number). It is less than one. E.g.: $\frac{3}{4}$

Improper Fraction: An improper fraction is where the numerator (the top number) is greater than or equal to the denominator (bottom number). E.g.: $\frac{5}{2}$

Mixed Fraction: A mixed fraction is a whole number and proper fraction combined into one "mixed number". It is larger than one. Also called a mixed number. E.g.: $2\frac{1}{4}$.
A mixed fraction can be changed into an improper fraction and vice versa.

Common Fraction: A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. Also called a **vulgar fraction**.

Dictionary

Adding and subtracting fractions: You can add and subtract fractions with the same denominators, e.g.: $\frac{1}{4} + \frac{3}{4} - \frac{1}{4} = \frac{3}{4}$

You need to find the lowest common multiple of the denominators where the fractions have different denominators, e.g.

$$\frac{2}{8} + \frac{2}{4} - \frac{3}{8} = \frac{2}{8} + \frac{4}{8} - \frac{3}{8} = \frac{3}{8}$$

If the answer is an improper fraction it should be written as a mixed number, e.g.

$$\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$$

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Introduction

Ask the learners to:

- Give five fractions where the denominators are the same.
- Give five fractions where the denominators are different.

Ask learners to look at the example of changing a mixed fraction to an improper fraction. How did we do it? Give learners enough time to explore this and come up with a solution. Will changing a mixed fraction always give you an improper fraction or can you get a proper fraction too?

Topic: Fractions **Content links:** R7, 30-32, 34-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

33 Add common fractions with the same and different denominators

Q1

Add the following, write it as a mixed number and simplify if necessary.

Example: $\frac{1}{3} + \frac{4}{3}$
$= \frac{5}{3}$
$= 1\frac{2}{3}$

When we add fractions the denominators should be the same.

Answers:

a. $\frac{2}{5} + \frac{4}{5}$
$= \frac{2+4}{5}$
$= \frac{6}{5}$
$= 1\frac{1}{5}$

b. $\frac{5}{9} + \frac{6}{9}$
$= \frac{5+6}{9}$
$= \frac{11}{9}$
$= 1\frac{2}{9}$

c. $\frac{3}{4} - \frac{2}{4}$
$= \frac{3-2}{4}$
$= \frac{1}{4}$

d. $\frac{7}{10} + \frac{5}{10}$
$= \frac{7+5}{10}$
$= \frac{12}{10}$
$= 1\frac{1}{5}$

e. $\frac{5}{6} + \frac{3}{6}$
$= \frac{5+3}{6}$
$= \frac{8}{6}$
$= 1\frac{1}{3}$

f. $\frac{5}{7} + \frac{6}{7}$
$= \frac{5+6}{7}$
$= \frac{11}{7}$
$= 1\frac{4}{7}$

Q2

Calculate and simplify if necessary.

Example: $\frac{1 \times 2}{2 \times 2} + \frac{1}{4}$
$= \frac{2}{4} + \frac{1}{4}$
$= \frac{3}{4}$

Remember when we add fractions the denominators should be the same. To make the denominators the same we need to find the Lowest Common Multiple (LCM)

Multiples of: {2, 4, 6, 8, ...}
Factors of : {4, 8, 12, 16, ...}

Note that in this case the denominators are multiples of each other (2 is a multiple of 4).

a. $\frac{1}{4} + \frac{1}{2}$
$= \frac{1+2}{4}$
$= \frac{3}{4}$

b. $\frac{1}{5} + \frac{1}{10}$
$= \frac{2+1}{10}$
$= \frac{3}{10}$

c. $\frac{1}{3} + \frac{1}{6}$
$= \frac{2+1}{6}$
$= \frac{1}{2}$

d. $\frac{1}{8} + \frac{1}{4}$
$= \frac{1+2}{8}$
$= \frac{3}{8}$

e. $\frac{1}{5} + \frac{1}{4}$
$= \frac{4+5}{20}$
$= \frac{9}{20}$

f. $\frac{1}{2} + \frac{1}{3}$
$= \frac{3+2}{6}$
$= \frac{5}{6}$

Q3

In your own words write down how you will add.
a. Fractions with the same denominators.
b. Fractions with denominators that are multiples of each other.

Answers: Learner's own answer.

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Problem solving

What is $\frac{5}{10} + \frac{3}{10}$ in its simplest form?

Answer: $\frac{8}{10} = \frac{4}{5}$

Topic: Fractions **Content links:** R7, 30-33, 35-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

34 Multiply unit fractions by unit fractions

Objectives

- Multiply common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another

Dictionary

Unitary fraction: A unit or unitary fraction is a fraction where the numerator is one. E.g. $\frac{1}{4}$

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Introduction

Tell the learners to multiplying fractions you simply multiply the numerators with each other, and the denominators with each other. Give learners one example (see question 1) and ask them to come up with another five. Solve 10 examples with learners on the board.

Compare the two calculations on the right. What do you notice?

A unit (or unitary) fraction is a fraction with a numerator of 1.
E.g.. $\frac{1}{4}$

$\frac{1}{2} + \frac{1}{4}$
LCM = 4
$\frac{2}{4} + \frac{1}{4}$
$= \frac{3}{4}$

$\frac{1}{2} \times \frac{1}{4}$
$= \frac{1}{8}$

When you are multiplying fractions you simply multiply the numerators with each other, and the denominators with each other. In this example the sum means $\frac{1}{2}$ OF $\frac{1}{4}$ which is $\frac{1}{8}$.

Q1

First add and then multiply the two fractions.

Example: $\frac{1}{2}, \frac{1}{3}$

Addition
$\frac{1}{2} + \frac{1}{3}$
LCM = 6
$\frac{3}{6} + \frac{2}{6}$
$= \frac{5}{6}$

Multiplication
$\frac{1}{2} \times \frac{1}{3}$
$= \frac{1}{6}$

I see that when multiplying proper fractions the answer gets smaller. The denominator of the answerg gets bigger. So $\frac{1}{6}$ is less than $\frac{1}{2}$ and less than $\frac{1}{3}$.

That is true. Think about it. If I multiply a six pack of juice by 2 then I get twelve juices. But if I take half ($\frac{1}{2}$) of a six pack of juice I get three.

a. $\frac{1}{2} + \frac{1}{12} = \frac{7}{12}$
$\frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$

b. $\frac{1}{2} + \frac{1}{11} = \frac{13}{22}$
$\frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$

c. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

d. $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$
$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

e. $\frac{1}{4} \times \frac{1}{10} = \frac{7}{20}$
$\frac{1}{4} \times \frac{1}{10} = \frac{1}{40}$

f. $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$
$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$

Q2

Calculate:

a. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$

b. $\frac{1}{4} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{40}$

c. $\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{24}$

d. $\frac{1}{3} \times \frac{1}{6} \times \frac{1}{2} = \frac{1}{36}$

e. $\frac{1}{3} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{30}$

f. $\frac{1}{2} \times \frac{1}{5} \times \frac{1}{9} = \frac{1}{90}$

Topic: Fractions **Content links:** R7, 30-33, 35-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

34 Multiply unit fractions by unit fractions *cont...*

Q3

What two fractions when multiplying, will give you the answer of $\frac{1}{32}$?

Answer:

$\frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$ or $\frac{1}{2} \times \frac{1}{16} = \frac{1}{32}$

What three fractions when multiplying will give you this answer?

Answer:

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{8} = \frac{1}{32}$

Q4

What do you notice when you extend this unitary fraction pattern?

Answer:

The numerator stays the same (1), the denominator is a square number ($2^2, 3^2, 4^2, 5^2, \ldots$), and the resulting fraction gets smaller ($\frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$).

Problem solving

Can two unit (or unitary) fractions give you a single unit fraction with a numerator of 1 if you:

- add them together?
- multiply them?

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Answers:

- add them together? No, e.g. $\frac{1}{3} + \quad = \frac{2}{3}$, except in the case of $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = \frac{1}{1}$ (which is a whole rather than a fraction).
- multiply them? Yes, $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7, 30-34, 36-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

35 Multiply common fractions by common fractions with the same and different denominators

Objectives

Multiply common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another

Dictionary

Denominator: The bottom number in a fraction is the denominator. E.g. $\frac{2}{3}$ 3 is the denominator. It tells you how many parts make up the whole (in this example, 3 parts).

Numerator: The top number in a fraction is the numerator. E.g. $\frac{2}{3}$, 2 is the numerator. It tells you how many parts of the whole there are (in this example, 2 parts).

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Introduction

Ask the learners to look at the fractions in the two blocks. Ask the learners to multiply the numbers with the same colour together in the block. What do you notice?

Look at the fractions and compare the two blocks. What differs between the numbers in the two blocks?

$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{6}$

$\frac{2}{4}$	$\frac{5}{6}$	$\frac{3}{8}$	$\frac{5}{7}$
$\frac{2}{8}$	$\frac{2}{7}$	$\frac{1}{4}$	$\frac{3}{6}$

A unit fraction nominator is always 1 and a non-unit fraction nominator is always more than one.

Multiply the numbers of the same colour in each block together. Compare the two sets of calculations.

$\frac{1}{2} \times \frac{1}{3} = \frac{2}{6}$ $\qquad$ $\frac{2}{8} \times \frac{3}{8} = \frac{6}{64}$

$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$ $\qquad$ $\frac{5}{6} \times \frac{3}{6} = \frac{15}{36}$

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ $\qquad$ $\frac{2}{7} \times \frac{5}{7} = \frac{10}{49}$

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ $\qquad$ $\frac{2}{4} \times \frac{1}{4} = \frac{2}{16}$

What happens with the denominators if you multiply them?
Remember:
- If you multiply unit (unitary) fractions, the product is a unit fraction.
- If you multiply non–unit fractions together, or a non–unit fraction with a unit fraction, the product is a non–unit fraction.

Q1

Calculate the following:

Example 1: $\frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$

Example 2: $\frac{6}{7} \times \frac{5}{6} = \frac{30}{42}$

a. $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ $\qquad$ b. $\frac{2}{4} \times \frac{1}{4} = \frac{2}{16}$ $\qquad$ c. $\frac{1}{6} \times \frac{3}{7} = \frac{3}{42}$

d. $\frac{1}{2} \times \frac{4}{6} = \frac{4}{12}$ $\qquad$ e. $\frac{7}{8} \times \frac{2}{4} = \frac{14}{32}$ $\qquad$ f. $\frac{8}{5} \times \frac{4}{5} = \frac{32}{25}$

Q2

Write down two different multiplication sums that will give the fractions shown as the answer. State what fractions you have multiplied together.

Example: ___ x ___ = $\frac{12}{18}$

$\frac{3}{3} = 1$

$\frac{3}{3} \times \frac{4}{6} = \frac{12}{18}$ A whole number x a proper fraction.

$\frac{2}{9} \times \frac{6}{2} = \frac{12}{18}$ A proper fraction x an improper fraction.

a. _ x _ = $\frac{4}{9}$

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ proper fraction x proper fraction

or $\frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$ proper fraction x improper fraction

b. _ x _ = $\frac{8}{4}$

$\frac{4}{2} \times \frac{2}{2} = \frac{8}{4}$ improper fraction x whole number

or $\frac{2}{1} \times \frac{4}{4} = \frac{8}{4}$ improper fraction x whole number

Topic: Fractions **Content links:** R7, 30-34, 36-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

35 Multiply common fractions by common fractions with the same and different denominators

c. $\_ \times \_ = \frac{6}{8}$

$\frac{2}{2} \times \frac{3}{4} = \frac{6}{8}$ whole number × proper fraction

or $\frac{1}{1} \times \frac{6}{8} = \frac{6}{8}$ whole number × proper fraction

d. $\_ \times \_ = \frac{12}{16}$

$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$ proper fraction × whole number

or $\frac{2}{2} \times \frac{6}{8} = \frac{12}{16}$ whole number × proper fraction

e. $\_ \times \_ = \frac{10}{64}$

$\frac{2}{8} \times \frac{5}{8} = \frac{10}{64}$ proper fraction × proper fraction

or $\frac{1}{4} \times \frac{10}{16} = \frac{10}{64}$ proper fraction × proper fraction

f. $\_ \times \_ = \frac{9}{12}$

$\frac{3}{2} \times \frac{3}{6} = \frac{9}{12}$ improper fraction × proper fraction

or $\frac{1}{3} \times \frac{9}{4} = \frac{9}{12}$ proper fraction × improper fraction

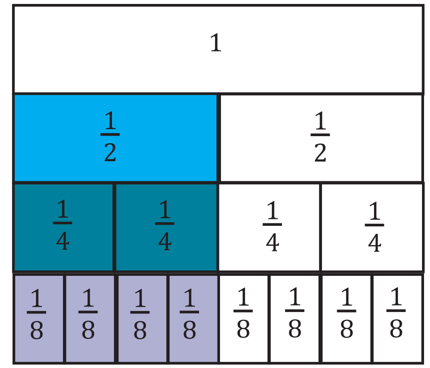
or $\frac{3}{1} \times \frac{3}{12} = \frac{9}{12}$ improper fraction × proper fraction

Q3 What is one quarter of a half? Make use of diagrams to show your calculation.

Answer:

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

In the fraction chart $\frac{1}{8}$ is $\frac{1}{4}$ the size of $\frac{1}{2}$.



Problem solving

What two fractions can you multiply to get the answer $\frac{42}{99}$?

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Answer: $\frac{6}{3} \times \frac{7}{33}$; $\frac{21}{11} \times \frac{2}{9}$; etc.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7, 30-35, 37-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

36 Multiply whole numbers by common fractions

Objectives

- Multiply common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another

Dictionary

Whole number: an integer or natural number

Common fractions: A common fraction is a fraction in which the numerator and denominator are both integers, as opposed to fractions. Also called a vulgar fraction.

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Introduction

Ask the learners to look at the following examples and discuss it with a friend.

$2 = \frac{2}{1}$ $78 = \frac{78}{1}$ $356 = \frac{356}{1}$ $1\ 245 = \frac{1\ 245}{1}$

Ask the learners to write the following as fractions:

Look at the following and discuss it with a friend.

$8 \div 1 = 8$ **$\frac{8}{1} = 8$** So we can write the whole number 8 as the fraction $\frac{8}{1}$

How would I write the following whole numbers as fractions?

2	**78**	**356**	**1 245**	**23 432**	**978 323**

Q1

Calculate the following:

Example:

$8 \times \frac{1}{4}$
$= \frac{8}{1} \times \frac{1}{4}$
$= \frac{8}{4}$
$= 2$

$\frac{8}{4}$
$= 8 \div 4$
$= 2$

Answers:

a. $2 \times \frac{3}{5}$
$= \frac{2}{1} \times \frac{3}{5}$
$= \frac{6}{5}$
$= 1\frac{1}{5}$

b. $4 \times \frac{5}{6}$
$= \frac{4}{1} \times \frac{5}{6}$
$= \frac{20}{6}$
$= 3\frac{2}{6}$

c. $11 \times \frac{3}{10}$
$= \frac{11}{1} \times \frac{3}{10}$
$= \frac{33}{10}$
$= 3\frac{3}{10}$

d. $9 \times \frac{1}{2}$
$= \frac{9}{1} \times \frac{1}{2}$
$= \frac{9}{2}$
$= 4\frac{1}{2}$

e. $\frac{2}{3} \times 2$
$= \frac{2}{3} \times \frac{2}{1}$
$= \frac{4}{3}$
$= 1\frac{1}{3}$

f. $8 \times \frac{6}{7}$
$= \frac{8}{1} \times \frac{6}{7}$
$= \frac{48}{7}$
$= 6\frac{6}{7}$

Topic: Fractions **Content links:** R7, 30-35, 37-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

36 Multiply whole numbers by common fractions *cont...*

Q2

What multiplicatin sums using a whole number and a fraction, will give you the following answers?

Example: $\_\_\_ \times \_\_\_ = \frac{2}{3}$

$= \frac{2}{1} \times \frac{1}{3}$

$= 2 \times \frac{1}{3}$

Answers: these are some of the possible answers.

a. $\frac{2}{1} \times \frac{2}{6} = 2 \times \frac{2}{6}$

b. $\frac{9}{1} \times \frac{1}{10} = 9 \times \frac{1}{10}$

c. $\frac{1}{1} \times \frac{3}{8} = 1 \times \frac{3}{8}$

or $\frac{3}{1} \times \frac{1}{8} = 3 \times \frac{1}{8}$

d. $\frac{15}{1} \times \frac{3}{10} = 15 \times \frac{1}{50}$

or $\frac{5}{1} \times \frac{3}{50} = 5 \times \frac{3}{50}$

e. $\frac{7}{1} \times \frac{1}{21} = 7 \times \frac{1}{21}$

f. $\frac{6}{1} \times \frac{1}{24} = 6 \times \frac{1}{24}$

Q3

One fifth of 15 cell phones on special were sold. How many were not sold? Answers:

$\frac{4}{5}$ were not sold

$\frac{4}{5}$ of 15

$= \frac{4}{5} \times \frac{15}{1}$

= 12 phones were not sold

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Problem solving

If ___ (whole number) x __ fraction = , how many possible solutions are there for this multiplication sum?

Answers: these are the only four possible solutions if the denominator of 12 remains unchanged.

$8 \times \frac{1}{12}$

$4 \times \frac{2}{12}$

$2 \times \frac{4}{12}$

$1 \times \frac{8}{12}$

Reflection questions

Did learners meet the objectives?

Topic: Fractions **Content links:** R7, 30-36, 38-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

37 Multiply common fractions and simplify

Objectives
- Multiply common fractions, including mixed numbers, not limited to fractions where one denominator is a multiple of another
- Simplify fractions by dividing numerators and denominators by common factors
- Use knowledge of multiples and factors to write fractions in the simplest form before or after calculations

Dictionary
Simplify fractions: Simplifying fractions means to make the fraction as simple as possible.

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Introduction
Tell learners simplifying fractions means to make the fraction as simple as possible. Why say four-eighths ($\frac{4}{8}$) when you really mean half ($\frac{1}{2}$) ?

Ask learners to explain the simplification on the right hand side of the introduction.

Simplifying fractions means to make the fraction as simple as possible. Why say four eighths ($\frac{4}{8}$) when you really mean half ($\frac{1}{2}$) ?

Show a friend or family member how this fraction was simplified.

Explain this:

$$\frac{24}{108} \overset{\div 2}{\underset{\div 2}{=}} \frac{12}{54} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{27} \overset{\div 3}{\underset{\div 3}{=}} \frac{2}{9}$$

Q1
Simplify the following:

Example: $\frac{15}{20}$

$= \frac{15}{20} \div \frac{5}{5}$

$= \frac{3}{4}$

Answers:

a. $\frac{4}{12} \div \frac{4}{4} = \frac{1}{3}$

b. $\frac{8}{16} \div \frac{8}{8} = \frac{1}{2}$

c. $\frac{5}{20} \div \frac{5}{5} = \frac{1}{4}$

d. $\frac{16}{24} \div \frac{8}{8} = \frac{2}{3}$

e. $\frac{7}{2} \div \frac{7}{7} = \frac{1}{3}$

f. $\frac{24}{64} \div \frac{8}{8} = \frac{3}{8}$

Q2
Multiply and simplify if possible.

a. $\frac{4}{16} \div \frac{4}{4} = \frac{1}{4}$

b. $\frac{21}{42} \div \frac{21}{21} = \frac{1}{2}$

c. $\frac{80}{120} \div \frac{40}{40} = \frac{2}{3}$

d. $\frac{5}{15} \div \frac{5}{5} = \frac{1}{3}$

e. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

f. $\frac{2}{14} \div \frac{2}{2} = \frac{1}{7}$

Topic: Fractions **Content links:** R7, 30-36, 38-39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

37 Multiply common fractions and simplify *continued*

Q3 Simplify the improper fraction if necessary and then write as a mixed number.

Example: $\frac{14}{4}$

Answers:

a. $6\frac{1}{3}$ b. $4\frac{1}{5}$ c. $3\frac{1}{3}$

d. $3\frac{1}{7}$ e. $1\frac{2}{8} = 1\frac{1}{4}$ f. $2\frac{3}{9} = 2\frac{1}{3}$

Q4 Multiply and simplify.

Example: $\frac{6}{4} \times \frac{5}{2}$

$= \frac{30}{8}$

$= 3\frac{6}{8}$ HCF is 2

$= 3\frac{3}{4}$

Answers:

a. $\frac{21}{12} = 1\frac{9}{12} = 1\frac{3}{4}$

b. $\frac{36}{15} = 2\frac{6}{15} = 2\frac{3}{5}$

c. $\frac{48}{28} = 1\frac{20}{28} = 1\frac{5}{7}$

d. $\frac{45}{32} = 1\frac{13}{32}$

e. $\frac{54}{40} = 1\frac{14}{40} = 1\frac{7}{20}$

f. $\frac{54}{21} = 2\frac{12}{21}$

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Problem solving

a. What is $\frac{16}{20} \times \frac{2}{4}$ in its simplest form?

b. Multiply any two improper fractions and simplify your answer if necessary.

Answers:

a. $\frac{32}{80} \div \frac{16}{16} = \frac{2}{5}$

An example:

b. $\frac{7}{3} \times \frac{5}{4} = \frac{35}{12} = 2\frac{10}{12} = 2\frac{5}{6}$

38 Solve fraction problems

Topic: Fractions **Content links:** R7, 30-37, 39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Objectives

Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers

Dictionary

Problem solving: to work out an answer to a problem

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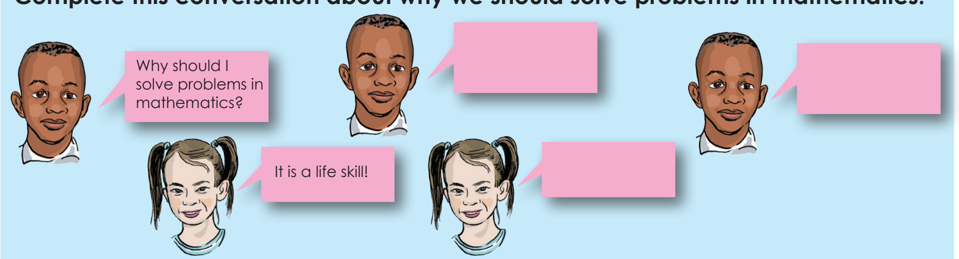
Introduction

Ask the learners what problem solving is? Discuss with learners how they feel when solving problems in maths. Tell them that solving problems is a skill that we can apply in daily life. Ask learners why we say doing maths problems is teaching us a life skill. Ask learners to complete the comic. Give learners the opportunity to read comic strips to the class. Write down key words on the board on problem solving.

Complete this conversation about why we should solve problems in mathematics.



Q1

Calculate the following. You may need extra paper to do your calculations.

Example 1: One half of an hour

$= \frac{1}{2}$ of 60 minutes

$= \frac{1}{2} \times 60$

$= \frac{1}{2} \times \frac{60}{1}$

$= \frac{60}{2}$

$=$ 30 minutes

Which word tells you it is a multiplication sum?

Answers:

a. $\frac{1}{2} \times \frac{7}{1}$
$= \frac{7}{2}$
$= 3,5$ days

b. $\frac{1}{4} \times \frac{24}{1}$
$= \frac{24}{4}$
$= 6$ hours

c. $\frac{1}{5} \times \frac{10}{1}$
$= \frac{10}{5}$
$= 2$ years

d. $\frac{1}{3} \times \frac{60}{1}$
$= \frac{60}{3}$
$= 20$ mins

e. $\frac{1}{2} \times 100$
$= \frac{100}{2}$
$= 50$ years

f. $\frac{1}{2} \times 1\,000$
$= \frac{1\,000}{2}$
$= 500$ years

g. $\frac{2}{(9 \times 7)}$
$= \frac{2}{63}$

h. $\frac{3}{108} = \frac{3}{108} \div \frac{3}{3}$
$= \frac{1}{36}$

i. $\frac{15}{60}$
$= \frac{15}{60} \div \frac{5}{5} = \frac{3}{12} = \frac{1}{4}$

38 Solve fraction problems *continued*

Topic: Fractions **Content links:** R7, 30-37, 39
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Q2

A number of children have R150 each to spend. How much of the R150 did they have left?

Answers:

a. R150 – R75 = R75
b. R150 – R25 = R125
c. R150 – R15 = R135
d. R150 – R18,75 = R131,25
e. R150 – R37,50 = R112,50
f. R150 – R50 = R100

Q3

You have R120 to spend on clothing. You can get discounts at different stores. Work out how much discount you can get at each.

Answers:

a. R60
b. R15
c. R10
d. R30
e. R20
f. R24

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

39 Solve more fraction problems

Topic: Fractions **Content links:** R7, 30-38
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Objectives

Solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing and finding fractions of whole numbers.

Dictionary

Problem solving: To work out an answer to a problem.

92

Introduction

Ask the learners what problem solving is?Ask learners why we say doing maths problems is teaching us a life skill. Ask learners to complete the comic. Give learners the opportunity to read comic strips to the class. Write down key words on the board on problem solving.

Q1

Solve these measurement of distance problems: Answers:

a. $\frac{1}{2}$ of a km
$= \frac{1}{2}$ of 1 000 m
$= \frac{1}{2} \times 1\,000$
$= 500$ m

b. $\frac{1}{4}$ of a km
$= \frac{1}{4}$ of 1 000 m
$= \frac{1}{4} \times 1\,000$
$= 250$ m

c. $\frac{1}{4}$ of a cm
$= \frac{1}{4}$ of 10 mm
$= \frac{1}{4} \times 10$
$= \frac{10}{4}$ mm $= 2{,}5$ mm

d. $\frac{1}{5}$ of a km
$= \frac{1}{5}$ of 1 000 m
$= \frac{1}{5} \times 1\,000$
$= 200$ m

e. $\frac{1}{4}$ of a metre
$= \frac{1}{4}$ of 100 cm
$= \frac{1}{4} \times 100$
$= 25$ cm

f. $\frac{1}{2}$ of a cm
$= \frac{1}{2}$ of 10 mm
$= \frac{1}{2} \times 10$
$= 5$ mm

Q2

Solve these travel distance problems. If I completed __ of the distance, how far do I still have to travel? Answers:

a. $\frac{1}{4}$ of 500 km = 50 km ∴ I still need to travel 450 km
b. $\frac{1}{12}$ of 500 km = 41, 67 km ∴ I still need to travel 458,33 km
c. $\frac{1}{2}$ of 500 km = 250 km ∴ I still need to travel 250 km
d. $\frac{1}{3}$ of 500 km = 166, 67 km ∴ I still need to travel 333,33 km
e. $\frac{1}{4}$ of 500 km = 125 km ∴ I still need to travel 375 km
f. $\frac{1}{6}$ of 500 km = 83,33 km ∴ I still need to travel 416,67 km

39 Solve more fraction problems *cont...*

Topic: Fractions **Content links:** R7, 30-38
Grade 8 links: R5, 65-67, 73 **Grade 9 links:** R5, 11-14

Q3

My friends and I competed in a cycling race of 120 km. We had to finish the race in eight hours. After five hours, we still needed to travel the remaining quarter of the distance. How far did we still need to go to the finishing line? Did we finish the race in time?

Answer:
Distance to end of race: 120 km ÷ 4 = 30 km
Time left to finish: 3 hours
Speed required to finish: $\frac{30}{3}$ = 10 km per hour
Speed of the first 90 km: $\frac{90}{5}$ = 18 km per hour

Therefore it is very likely that they finished in time.

Q4

Solve: What is ____ of a kg?

Answers:
a. 500 g
b. 250 g
c. 200 g
d. 100 g
e. 125 g
f. 10 g

Q5

Solve: How many grams of the 150 g of food did I eat?

Answers:
a. 18,75 g
b. 75 g
c. 25 g
d. 30 g
e. 25 g
f. 7,5 g

Q6

Solve: How many millilitres did I drink?

Answers:
a. 500 ml
b. 250 ml
c. 500 ml
d. 800 ml
e. 375 ml
f. 300 ml

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Problem solving

Write your own word problem on a separate piece of paper, using capacity and fractions. Use the previous questions to guide you.

Reflection questions

Did learners meet the objectives?

Topic: Fractions **Content links:** R7, R8, 30-39, 41-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6,15-16

40 Fractions, decimals and percentages

Objectives

Revise the following done in Grade 6:

- Recognize equivalence between common fraction and decimal fraction forms of the same number
- Find percentages of whole numbers
- Calculate the percentage of part of a whole
- Solve problems in contexts involving percentages

Dictionary

Percent/percentage: A value expressed as a fraction of 100. Symbol for percentage: % Per-cent means 'per hundred'.

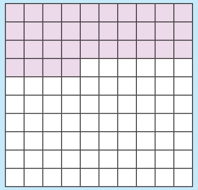
Equivalence between common fractions, decimal fractions and percentage: Common fractions, decimal fractions and percentages with the same value may look different but are the same,
E.g. = 25% = 0,25 = $\frac{25}{100}$

94

Introduction

Ask the learners to explain the diagram in the introduction section.

Explain the following:



$\frac{34}{100}$ = 0,34 = 34%

Quick quiz: What does the following mean:
- **Cent**?
- **Cent**ury?
- **Cent**ipede?
- Per**cent**age?

100

Quick quiz:

What does the following mean:

- **Cent?** There are 100 cents in a Rand. Cent means 100.
- **Century?** There are 100 years in a century.
- **Centipede?** A creature with 100 legs.
- **Percentage?** Per cent means per hundred.

Q1

Write the following as a fraction and a decimal fraction:

Example: 18% or $\frac{18}{100}$ or 0,18 = $\frac{9}{50}$

If possible write the fraction in the simplest form.

$\frac{18}{100}$ Simplified is $\frac{9}{50}$

Answers:

a. $\frac{37}{100}$ = 0,37   b. $\frac{25}{100}$ = 0,25   c. $\frac{83}{100}$ = 0,83   d. $\frac{90}{100}$ = 0,9   e. $\frac{55}{100}$ = 0,55   f. $\frac{3}{100}$ = 0,03

Topic: Fractions **Content links:** R7, R8, 30-39, 41-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6,15-16

40 Fractions, decimals and percentages *continued*

Q2

Write the following as a fraction in its simplest form:

Answers: The pattern is 10% increases from 10% to 100%

Percentage	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Fraction	$\frac{10}{100}$	$\frac{20}{100}$	$\frac{30}{100}$	$\frac{40}{100}$	$\frac{50}{100}$	$\frac{60}{100}$	$\frac{70}{100}$	$\frac{80}{100}$	$\frac{90}{100}$	$\frac{100}{100}$
Simplest form	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	$\frac{1}{1}$

Q3

Calculate

a. $\frac{20}{100} \times \frac{R24}{1}$
$= \frac{480}{100} \div \frac{20}{20}$
$= \frac{24}{5} = 4\frac{4}{5}$
$=$ R4,80

b. $\frac{70}{100} \times \frac{R15}{1} = 1$
$\frac{1\,050}{100} \div \frac{50}{50}$
$= \frac{21}{2} = 10\frac{1}{2}$
$=$ R10,50

c. $\frac{60}{100} \times \frac{R95}{1}$
$\frac{5\,700}{100} \div \frac{100}{100}$
$=$ R57

d. $\frac{80}{100} \times \frac{R74}{1}$
$= \frac{5\,920}{100}$
$=$ R59,20

e. $\frac{30}{100} \times \frac{R90}{1}$
$= \frac{2\,700}{100}$
$=$ R27

f. $\frac{50}{100} \times \frac{R65}{1}$
$= \frac{3\,250}{100}$
$=$ R32,50

Q4

Calculate:

a. $\frac{30}{100} \times \frac{R1,80}{1}$
$= \frac{3}{10} \times \frac{R1,80}{1}$
$=$ R0,54

b. $\frac{80}{100} \times \frac{R1,60}{1}$
$= \frac{8}{10} \times \frac{R1,60}{1}$
$=$ R1,28

c. $\frac{90}{100} \times \frac{R8,10}{1}$
$= \frac{9}{10} \times \frac{R8,10}{1}$
$=$ R7,29

d. $\frac{20}{100} \times \frac{R4,60}{1}$
$= \frac{2}{10} \times \frac{R4,60}{1}$
$=$ R0,92

e. $\frac{60}{100} \times \frac{R5,40}{1}$
$= \frac{6}{10} \times \frac{R5,40}{1}$
$=$ R3,24

f. $\frac{20}{100} \times \frac{R6,40}{1}$
$= \frac{2}{10} \times \frac{R6,40}{1}$
$=$ R1,28

95

Problem solving

I bought a pair of shoes for R150. I got 25% discount. How much did I pay for it?

Answer: R150,00 – R37,50 (has discounted) = R112,50

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R8, 40
Grade 8 links: R6, 69-70 **Grade 9 links:** R6,15-16

41 Percentage increase and decrease

Objectives

- Recognize equivalence between common fraction and decimal fraction forms of the same number.
- Calculate percentage increase or decrease of a whole.
- Solve problems in contexts involving percentages.

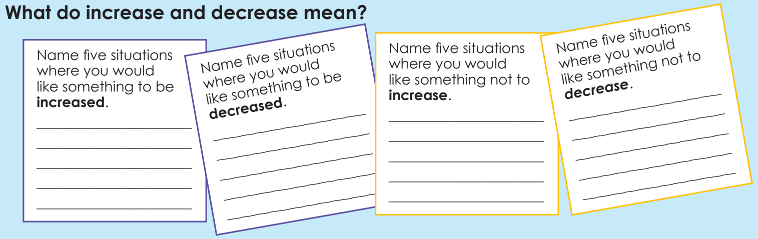
Dictionary

Decrease: make something smaller (in size or quantity)
Increase: make something bigger (in size or quantity)

96

Introduction

What do increase and decrease mean?



Q1

Calculate the percentage increase.

Example: Calculate the **percentage** increase if the price of a bus ticket of R60 is **increased** to R84.

$\frac{24}{60} \times \frac{100}{1}$

$= \frac{240}{60}$

$= 40\%$

We first need to ask by how much the bus ticket price was increased.

It was increased by R24 because R84 minus R60 is R24.

24/60 is the price increase.

To work out the percentage increase we multiply by 100.

Answers:

a. R20
$= \frac{20}{50} \times \frac{100}{1}$
$= 40\%$

b. R40
$= \frac{40}{80} \times \frac{100}{1}$
$= 50\%$

c. R3
$= \frac{3}{15} \times \frac{100}{1}$
$= 20\%$

d. R5
$= \frac{5}{25} \times \frac{100}{1}$
$= 20\%$

e. R20
$= \frac{20}{100} \times \frac{100}{1}$
$= 20\%$

f. R18
$= \frac{18}{36} \times \frac{100}{1}$
$= 50\%$

Q2

Name an item which you really like, the price of which was increased recently. What was the percentage increase?
Anwer: Learner's own answer

Topic: Fractions **Content links:** R8, 40
Grade 8 links: R6, 69-70 **Grade 9 links:** R6,15-16

41 Percentage increase and decrease *continued*

Q3

Calculate the percentage decrease.

Example: 18% of R20

$= \frac{18}{100} \times \frac{R20}{1}$

$= \frac{R360}{100}$

$= R3,60$

Answers:

a. R5

$= \frac{5}{20} \times \frac{100}{1}$

$= \frac{500}{20}$

$= 25\%$

b. R5

$= \frac{5}{50} \times \frac{100}{1}$

$= \frac{500}{50}$

$= 10\%$

c. R3

$= \frac{3}{18} \times \frac{100}{1}$

$= \frac{300}{18}$

$= 17\%\ (16,67\%)$

d. R6

$= \frac{6}{24} \times \frac{100}{1}$

$= \frac{600}{24}$

$= 25\%$

e. R10

$= \frac{10}{90} \times \frac{100}{1}$

$= \frac{1\,000}{90}$

$= 11\%\ (11,11\%)$

f. R7

$= \frac{7}{28} \times \frac{100}{1}$

$= \frac{700}{28}$

$= 25\%$

Q4

What item you want do you like to be decrease in price? If it is decreased by 20% what would the price be?
Answer: learner's own answer

97

Problem solving

Calculate the percentage decrease if the price of petrol goes down from 960 cents to 840 cents per litre.

Answer:

$\frac{120}{960} \times \frac{100}{1}$

$= \frac{12\,000}{960} = 12,5\%$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7-R8, 40, 43-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

42 Place value, ordering and comparing decimals

Objectives

Revise the following done in Grade 6:

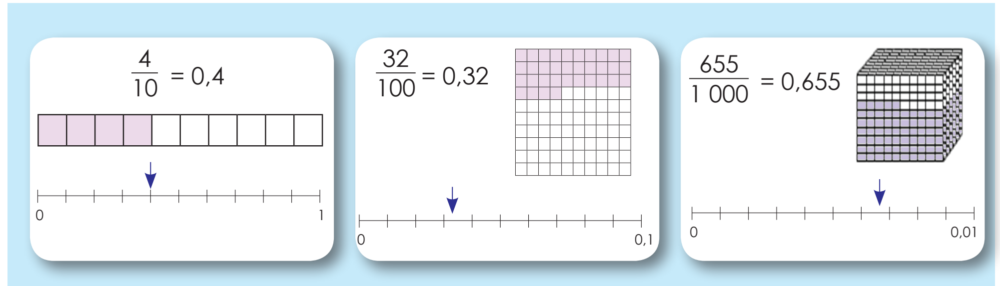
- Compare and order decimal fractions to at least two decimal place
- Place value of digits to at least two decimal places
- Count forwards and backwards in decimal fractions to at least two decimal places
- Use knowledge of place value to estimate the number of decimal places in the result before performing calculations

Dictionary

Decimal fraction: A decimal fraction is a fraction where the denominator (the bottom number in a common fraction) is a power of ten (such as 10, 100, 1 000, etc). Decimal fractions are written with a decimal comma (or point) and no denominator.
This makes it a lot easier to do calculations like addition and multiplication with fractions. e.g 2,45 = 2 + 0,4 + 0,05

98

Introduction
Ask the learners to look at the introduction and explain each block using words such as tenths, hundredths, and thousandths.



Q1

Write the following in expanded notation:
Answers:

a. 4 + 0,3 + 0,07 + 0,008
b. 5 + 0,2 + 0,01 + 0,003
c. 14 + 0,6 + 0,07 + 0,008
d. 5 + 0,03 + 0,006
e. 8 + 0,3 + 0,005
f. 9 + 0,006

Q2

Write the following in words:
Answers:

a. 5 units + 3 tenths + 7 hundredths + 6 thousandths
b. 8 units + 2 tenths + 9 hundredths + 1 thousandth
c. 3 units + 5 tenths + 8 hundredths + 9 thousandths
d. 7 units + 3 hundredths + 6 thousandths
e. 8 units + 5 thousandths

Topic: Fractions **Content links:** R7-R8, 40, 43-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

42 Place value, ordering and comparing decimals *continued*

Q3

Write the following in the correct column:

Answers:

Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths
			4	,	7	6	5
		1	8	,	3	4	6
		1	9	,	0	0	5
	2	3	1	,	0	4	
7	6	8	5	,	2		

Q4

Write down the value of the underlined digit:

Answers:

a. 0,05 or 5 hundredths
b. 0,02 or 2 hundredths
c. 5 or 5 units
d. 0,09 or 9 hundredths
e. 8 or 8 units
f. 0,002 or 2 thousandths

Q5

Write the following in ascending order:

Answers:

a. 0,004; 0,04; 0,4
b. 0,011; 0,1; 0,11
c. 0,9; 0,99; 0,999
d. 0,753; 0,8; 0,82
e. 0,007; 0,06; 0,67

Q6

Fill in $<$, $>$, $=$

Answers:

a. $0,4 > 0,04$
b. $0,05 > 0,005$
c. $0,1 = 0,10$
d. $0,62 > 0,26$
e. $0,58 < 0,85$
f. $0,37 < 0,73$
g. $0,123 < 0,321$
h. $0,2 = 0,20$
i. $0,4 = 0,40$
j. $0,05 = 0,050$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7-R8, 40, 42, 44-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

43 Writing common fractions as decimals

Objectives

- Recognize equivalence between common fraction and decimal fraction forms of the same number
- Count forwards and backwards in decimal fractions to at least two decimal places
- Use knowledge of place value to estimate the number of decimal places in the result before performing calculations
- Solve problems in contexts involving percentages

Dictionary

Decimal fraction: A decimal fraction is a fraction where the denominator (the bottom number in a common fraction) is a power of ten (such as 10, 100, 1 000, etc). Decimal fractions are written with a decimal comma (or point) and no denominator.
This makes it a lot easier to do calculations like addition and multiplication with fractions. e.g 2,45 = 2 + 0,4 + 0,05

98

Introduction
Ask the learners to look at the introduction and explain it.

Q1 Write as a decimal fraction: Answers:

a. 0,6 b. 0,7 c. 0,008
d. 0,4 e. 0,005 f. 0,003

Q2 Write as a decimal fraction: Answers:

a. 0,45 b. 0,76 c. 0,98 d. 0,36 e. 0,476 f. 0,075

Q3 Write as a decimal fraction: Answers:

a. 3,6 b. 67,05 c. 8,8 d. 32 e. 76,5 f. 93,47

Q4 Write as a common fraction: Answers:

a. $\frac{95}{10}$ b. $\frac{1\,515}{100}$ c. $\frac{8\,934}{1\,000}$ d. $\frac{376}{100}$ e. $\frac{32\,004}{1\,000}$ f. $\frac{76}{10}$

Q5 Write as a decimal fraction: Answers:

a. $\frac{1}{5} = \frac{2}{10} = 0{,}2$ b. $\frac{1}{4} = \frac{25}{100} = 0{,}25$ c. $\frac{1}{2} = \frac{5}{10} = 0{,}5$

d. $\frac{3}{5} = \frac{6}{10} = 0{,}6$ e. $\frac{2}{4} = \frac{5}{10} = 0{,}5$ f. $\frac{1}{25} = \frac{4}{100} = 0{,}04$

101

Problem solving

[You can use a calculator if you want to.]
a. What would you do to change the decimal fraction 7,345 to 7,305?
b. Then to change it to 7,005 and then to 7?
c. If the tenths digit is nine and the units digit is five, what should I do to get an answer of 5,932?

Answer: a. Subtract 0,04 b. Subtract 0,3 and then 0,005 c. Add 0,032

Topic: Fractions **Content links:** R7-R8, 40, 42-43, 45-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

44 Writing common fractions as decimals

Objectives

Revise

- Count in decimal fractions
- Rounding off decimal fractions to at least 1 decimal place
- Recognize equivalence between common fraction and decimal fraction forms of the same number
- Count forwards and backwards in decimal fractions to at least two decimal places
- Use knowledge of place value to estimate the number of decimal places in the result before performing calculations
- Solve problems in contexts involving percentages

Dictionary

Rounding (decimals): Rounding means shortening a number. The value of the last digit of the shortened number is increased by 1 if the first of the discarded digits is 5 or more. Rounded numbers are less accurate but easier to use.

- 3,6 rounded off to the nearest unit is 4
- 2,32 rounded off to the nearest tenth is 2,3
- 1,738 rounded off to the nearest hundredth is 1,74

102

Introduction

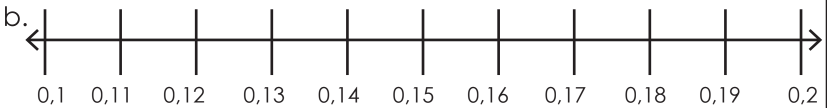
Ask the learners how fast can they count.

Q1

Complete the number lines:

a.

b.



c.

Q2

Complete the following:

Example: 0,34; 0,35; 0,36; …; 0,39
= 0,34; 0,35; 0,36; **0,37**; **0,38**; 0,39

Answers:

a. 0,1; 0,2; 0,3; **0,4**; 0,5; 0,6; 0,7; 0,8; 0,9
b. 0,21; 0,22; 0,23; **024**; 0,25; 0,26; 0,27; 0,28; 0,29
c. 0,31; 0,32; 0,33; **034**; **0,35**; 0,36; 0,37; 0,38; 0,39

Topic: Fractions **Content links:** R7-R8, 40, 42-43, 45-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

44 Writing common fractions as decimals *continued*

Q3

Extend the pattern with five decimal fractions:

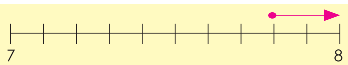
Example: 5,36; 5,37; 5,38; ...
= 5,36; 5,37; 5,38; **5,39; 5,4; 5,41; 5,42; 5,43**

Answers:
a. 7,7; 7,8; 7,9; **8,0; 8,1; 8,2; 8,3; 8,4**
b. 3,64; 3,65; 3,66; **3,67; 3,68; 3,69; 3,70; 3,71**
c. 2,173; 2,174; 2,175; **2,176; 2,177; 2,178; 2,179; 2,180**
d. 5,4; 5,5; 5,6; **5,7; 5,8; 5,9; 6,0; 6,1**
e. 9,6; 9,5; 9,4; **9,3; 9,2; 9,1; 9,0; 8,9**
f. 3,874; 3,873; 3,872; **3,871; 3,870; 3,869; 3,868; 3,867**

Q4

Round off to the nearest unit:

Example: 7,8
Rounded off to 8

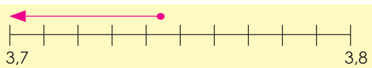


Answers:
a. 3 b. 3 c. 5 d. 5 e. 4 f. 7

Q5

Round off to the nearest tenth:

Example: 3,745
Rounded off to 3,7



Answers:
a. 6,1 b. 3,6 c. 5,6 d. 68,5 e. 7,2 f. 4,3

Q6

Round off to the nearest tenth:

		Unit	Tenth
a.	3,84	4	3,8
b.	3,89	4	3,9
c.	14,27	14	14,3
d.	999,31	999	999,3
e.	4,09	4	4,1
f	51,781	52	51,8

103

Problem solving

a. Give five examples of decimal fractions that will be between 0,08 and 0,09.
b. Give five examples of numbers you could have rounded off to 5.

Answer: Examples of answers.
a. 0,081; 0,082; 0,083; 0,084; 0,085
b. Round off to the nearest 5
i. 5,2 ii. 4,98 iii. 4,52 iv. 5,48 v. 5,09

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7-R8, 40, 42-44, 46-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

45 Addition and subtraction with decimal fractions

Objectives

- Addition and subtraction of decimal fractions with at least two decimal places
- Solve problems in context involving decimal fractions

Dictionary

Decimal fraction: A decimal fraction is a fraction where the denominator (the bottom number in a common fraction) is a power of ten (such as 10, 100, 1 000, etc). Decimal fractions are written with a decimal comma (or point) and no denominator.
This makes it a lot easier to do calculations like addition and multiplication with fractions. e.g. 2,45 = 2 + 0,4 + 0,05

104

Introduction
Learners look at the pictures and make up addition and/or subtraction sums. Possible examples:
$1{,}25\ l + 0{,}5\ l + 1\ l = 2{,}75\ l$
$3{,}5 + 3{,}5 + 4{,}7 + 4{,}7 = 16{,}4$ m
$2{,}5$ kg $+ 0{,}5$ kg $+ 1$ kg $= 4$ kg

Q1

Calculate using both methods. Check your answer. Answers:

a. $\begin{array}{r} 3,12 \\ +\ 4,57 \\ \hline 7,69 \\ \hline \end{array}$ $\begin{aligned} &3{,}12 + 4{,}57 \\ &= (3 + 4) + (0{,}1 + 0{,}5) + (0{,}02 + 0{,}07) \\ &= 7 + 0{,}6 + 0{,}09 \\ &= 7{,}69 \end{aligned}$

b. $\begin{array}{r} 5,34 \\ +\ 2,26 \\ \hline 7,60 \\ \hline \end{array}$ or $\begin{aligned} &5{,}34 + 2{,}26 \\ &= (5 + 2) + (0{,}3 + 0{,}2) + (0{,}04 + 0{,}06) \\ &= 7 + 0{,}5 + 0{,}1 \\ &= 7{,}6 \end{aligned}$

c. $\begin{array}{r} 1,46 \\ +\ 2,28 \\ \hline 3,74 \\ \hline \end{array}$ or $\begin{aligned} &1{,}46 + 2{,}28 \\ &= (1 + 2) + (0{,}4 + 0{,}2) + (0{,}06 + 0{,}08) \\ &= 3 + 0{,}6 + 0{,}14 \\ &= 3 + 0{,}6 + 0{,}1 + 0{,}04 \\ &= 3{,}74 \end{aligned}$

d. $\begin{array}{r} 3,45 \\ +\ 4,67 \\ \hline 8,12 \\ \hline \end{array}$ or $\begin{aligned} &3{,}45 + 4{,}67 \\ &= (3 + 4) + (0{,}4 + 0{,}6) + (0{,}05 + 0{,}07) \\ &= 7 + 1 + 0{,}12 \\ &= 8 + 0{,}12 \\ &= 8{,}12 \end{aligned}$

e. $\begin{array}{r} 6,58 \\ +\ 5,78 \\ \hline 12,36 \\ \hline \end{array}$ or $\begin{aligned} &6{,}58 + 5{,}78 \\ &= (6 + 5) + (0{,}5 + 0{,}7) + (0{,}08 + 0{,}08) \\ &= 11 + 1{,}2 + 0{,}16 \\ &= 12 + 0{,}3 + 0{,}06 \\ &= 12{,}36 \end{aligned}$

f. $\begin{array}{r} 9,99 \\ +\ 9,97 \\ \hline 19,96 \\ \hline \end{array}$ or $\begin{aligned} &9{,}99 + 9{,}97 \\ &= (9 + 9) + (0{,}9 + 0{,}9) + (0{,}09 + 0{,}07) \\ &= 18 + 1{,}8 + 0{,}16 \\ &= 18 + 1 + 0{,}8 + 0{,}1 + 0{,}06 \\ &= 19 + 0{,}9 + 0{,}06 \\ &= 19{,}96 \end{aligned}$

Topic: Fractions **Content links:** R7-R8, 40, 42-44, 46-47
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

45 Addition and subtraction with decimal fractions *continued*

Calculate using both methods.

a.
1, 15
+ 2, 21
3, 36
– 1, 21
2, 15

Method 2:
1,15 + 2,21 – 1,21
= (1 + 2 – 1) + (0,1 + 0,2 – 0,2) + (0,05 + 0,01 – 0,01)
= 2 + 0,1 + 0,05
= 2,15

b.
2, 34
+ 3, 42
5, 76
– 2, 34
3, 42

Method 2:
2,34 + 3,42 – 2,34
= (2 + 3 – 2) + (0,3 + 0,4 – 0,3) + (0,04 + 0,02 – 0,04)
= 3 + 0,4 + 0,02
= 3,42

c.
3, 24
+ 3, 35
6, 59
– 5, 36
1, 23

Method 2:
3,24 + 3,35 – 5,36
= (3 + 3 – 5) + (0,2 + 0,3 – 0,3) + (0,04 + 0,05 – 0,06)
= 1 + 0,2 + 0,03
= 1,23

d.
4, 760
+ 6, 112
10, 87
– 3, 52
7, 35

Method 2:
4,76 + 6,11 – 3,52
= (4 + 6 – 3) + (0,7 + 0,1 – 0,5) + (0,06 + 0,01 – 0,02)
= 7 + 0,3 + 0,05
= 7,35

e.
2, 36
+ 5, 42
7, 78
– 3, 47
4, 31

Method 2:
2,36 + 5,42 – 3,47
= (2 + 5 – 3) + (0,3 + 0,4 – 0,4) + (0,06 + 0,02 – 0,017
= 4 + 0,3 + 0,01
= 4,31

f.
6, 89
+ 9, 10
15, 99
– 5, 19
10, 80

Method 2:
6,89 + 9,10 – 5,19
= (6 + 9 – 5) + (0,8 + 0,1 – 0,1) + (0,059+ 0 – 0,9)
= 10 + 0,8 + 0
= 10,80

Make five different number sentences using the following decimals. Solve it. 2,56; 1,99 and 3,47. Answers:

a. 2,56 – 1,99 = 0,57
b. 1,99 + 3,47 = 5,46
c. 3,47 – 2,56 = 0,91
d. 3,47 – 1,99 = 1,48
e. 256 + 1,99 = 4,55

104

Problem solving

My friend went on a diet and lost 2,5 kg the first week, and 1,25kg the second week. He gained 0,75kg the third week and lost 0,5 kg the fourth week. How much did he lose in the four weeks? (Remember it is not healthy to lose too much weight in a short period of time.)

Answer:
2,5 + 1,25 – 0,75 + 0,5 = 3,5 kg. He lost a total of 3,5 kg for four weeks

Topic: Fractions **Content links:** R7-R8, 40, 42-45, 47
Grade 8 links: 74 **Grade 9 links:** 18

46 Multiplication of decimal fractions

Objectives

- Multiplication of decimal fractions by 10 and 100
- Multiply decimal factions to include decimal fractions to at least 2 decimal places by decimal fractions to at least 1 decimal place
- Multiply decimal factions to include decimal fractions to at least 3 decimal places by whole numbers
- Solve problems in contexts involving decimal fractions

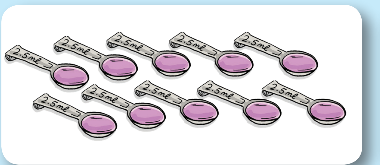
Dictionary

Decimal fraction: A decimal fraction is a fraction where the denominator (the bottom number in a common fraction) is a power of ten (such as 10, 100, 1 000, etc). Decimal fractions are written with a decimal comma (or point) and no denominator.
This makes it a lot easier to do calculations like addition and multiplication with fractions. e.g 2,45 = 2 + 0,4 + 0,05

106

Introduction

Look at the following pictures. Make up your own addition, subtraction and multiplication sum for each.



- 10 x 2,5 ml = 25 ml
- 4 x 25,5 cm = 102 cm
- 6 x 1,5 kg = 9 kg

Q1

Calculate. Check your answers using a calculator.

Example:
- 0,2 x 0,3 = 0,06
- 0,02 x 0,3 = 0,006
- 0,02 x 0,03 = 0,0006

Do you notice the pattern? Describe it.

Answers:
a. 0,08 b. 0,03 c. 0,20 d. 0,42 e. 0,0008 f. 0,005

Q2

Calculate. Check your answers using a calculator.

Example 1: 0,2 x 4 = 0,8 **Example 2:** 0,02 x 4 = 0,08 **Example 3:** 0,4 x 3 = 1,2

Answers
a. 0,5 x 3 = 1,5
b. 0,8 x 3 = 2,4
c. 0,6 x 4 = 2,4
d. 0,02 x 9 = 0,18
e. 0,07 x 6 = 0,42
f. 0,003 x 8 = 0,024

Q3

Calculate. Check your answers using a calculator.

Example 1: 0,3 x 0,2 x 100
= 0,06 x 100
= 6

Example 2: 0,3 x 0,2 x 10
= 0,06 x 10
= 0,6

Answers:
a. 0,4 x 0,2 x 10
= 0,08 x 10
= 0,8

b. 0,5 x 0,02 x 10
= 0,01 x 10
= 0,1

c. 0,3 x 0,3 x 100
= 0,09 x 100
= 9

d. 0,6 x 0,03 x 100
= 0,018 x 100
= 1,8

e. 0,5 x 0,2 x 100
= 0,1 x 100
= 10

f. 0,7 x 0,01 x 100
= 0,007 x 100
= 0,7

Topic: Fractions **Content links:** R7-R8, 40, 42-45, 47
Grade 8 links: 74 **Grade 9 links:** 18

46 Multiplication of decimal fractions *continued*

Q4 Calculate. Check your answers using a calculator.

Answers:

a. 1,123 x 10
= (1 x 10) + (0,1 x 10) + (0,02 x 10) + (0,003 x 10)
= 10 + 1 + 0,2 + 0,03 = 11,23

b. 4,886 x 30
= (4 x 30) + (0,8 x 30) + (0,08 x 30) + (0,006 x 30)
= 120 + 24 2,4 + 0,18 = 146,58

c. 2,932 x 40
= (2 x 40) + (0,9 x 40) + (0,03 x 40) + (0,002 x 40)
= 80 + 36 1,2 + 0,08 = 117,28

d. 7,457 x 60
= (7 x 60) + (0,4 x 60) + (0,05 x 60) + (0,007 x 60)
= 420 + 24 + 3 + 0,42 = 447,42

e. 8,234 x 20
= (8 x 20) + (0,2 x 20) + (0,03 x 20) + (0,004 x 20)
= 160 + 4 + 0,6 + 0,08 = 164,68

f. 6,568 x 80
= (6 x 80) + (0,5 x 80) + (0,06 x 80) + (0,008 x 80)
= 480 + 40 + 4,8 + 0,64 = 525,44

g. 11,23; 117,28; 146,58; 164,68; 447,42; 525,44

Q5 Now redo the problems in question 4 using the column method to solve it. Answers:

a.
$$\begin{array}{r} 1,123 \\ \times \quad 10 \\ \hline 0,000 \\ 11,230 \\ \hline 11,230 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 4,886 \\ \times \quad 30 \\ \hline 0,000 \\ 146,580 \\ \hline 146,580 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 2,932 \\ \times \quad 40 \\ \hline 0,000 \\ 117,280 \\ \hline 117,280 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 7,457 \\ \times \quad 60 \\ \hline 0,000 \\ 447,420 \\ \hline 447,420 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 8,234 \\ \times \quad 20 \\ \hline 0,000 \\ 164,680 \\ \hline 164,680 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 6,568 \\ \times \quad 80 \\ \hline 0,000 \\ 525,440 \\ \hline 525,440 \\ \hline \end{array}$$

107

Problem solving

Multiply the number that is exactly between 1,15 and 1,16 by the number that is equal to ten times three.

Answer:
1,15 + 1,16 = 2,31 ÷ 2 = 1,155 x 30 = 34,65

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Fractions **Content links:** R7-R8, 40, 42-46
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

47 Division, rounding off and flow diagrams

Objectives

- Solve problems in contexts involving decimal fractions.
- Divide decimal fractions including decimal fractions to at least 2 decimal places by whole numbers.
- Round off decimal fractions.

Dictionary

Rounding(decimals): Rounding means reducing or increasing the digit in a number while trying to keep it's value similar. The result is less accurate, but easier to use. E.g.:
3,6 rounded off to the nearest unit is 3
2,32 rounded off to the nearest tenth is 2,3
1,738 rounded off to the nearest hundredth is 1,74

108

Introduction

Ask the learners to look at the patterns and describe them. Ask the learners to explain to a friend what rounding off to the nearest whole number and tenth mean if you work with decimals.

Look at the following two patterns and describe them.

800 ÷ 4 = 200	80 ÷ 4 = 20	8 ÷ 4 = 2	0,8 ÷ 4 = 0,2	0,08 ÷ 4 = 0,02

150 ÷ 3 = 50	15 ÷ 3= 5	1,5 ÷ 3 = 0,5	0,15 ÷ 3 = 0,05	0,015 ÷ 3 = 0,005

Explain to a friend what rounding off to the nearest whole number or to a tenth means if you work with decimals.

Q1

Calculate the following: Answers:
a. 0,2
b. 0,2
c. 0,3
d. 0,4
e. 0,6
f. 0,3

Q2

Round off your answers to the nearest whole number. Answers:
a. 0
b. 0
c. 0
d. 0
e. 1
f. 0

Q3

Calculate the following. Answers:
a. 0,09
b. 0,05
c. 0,09
d. 0,09
e. 0,03
f. 0,17

Topic: Fractions **Content links:** R7-R8, 40, 42-46
Grade 8 links: R6, 69-75 **Grade 9 links:** R6, 16-18

47 Division, rounding off and flow diagrams *continued*

Q4

Round off your answers to the nearest tenth.

Answers:

a. 0,1
b. 0,1
c. 0,1
d. 0,1
e. 0
f. 0,2

Q5

Complete the flow diagram. Round off to the nearest whole number.

Answers:

a. R0,50 → Divide by 2 → Round off to the nearest rand → 0,25 R0

b. 2 m → Divide by 8 → Round off to the nearest metre → 0,3 0 m

c. 5,4 kg → Divide by 9 → Round off to the nearest kilogram → 0,6 1 kg

d. R3,75 → Divide by 25 → Round off to the nearest rand → 0,15 R0

e. 2,5 ℓ → Divide by 5 → Round off to the nearest litre → 0,5 0 ℓ

f. 1,44 kg → Divide by 12 → Round off to the nearest kilogram → 0,12 0 kg

109

Problem solving

- You need seven equal pieces from 28,7 m of rope. How long will each piece be?
- I have R45,75. I have to divide it by five. What will my answer be?
- My mother bought 12,8 m of string. She has to divide it into four pieces. How long will each piece be?

Answer:
28,7m ÷ 7 = 4,1m (each piece); R45,75 ÷ 5 = R9,15 (each person will have); 12,8m ÷ 4 = 3,2m (each piece)

Reflection questions
Did learners meet the objectives?

48 Flow diagrams

Topic: Input and output values **Content links:** R9, 49-51, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

Objectives

- Determine input values, output values or rules for patterns and relationships using:
  – Flow diagrams
  – Formulae
- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
  – In flow diagrams
  – By formulae
  – By number sentences
  – Verbally

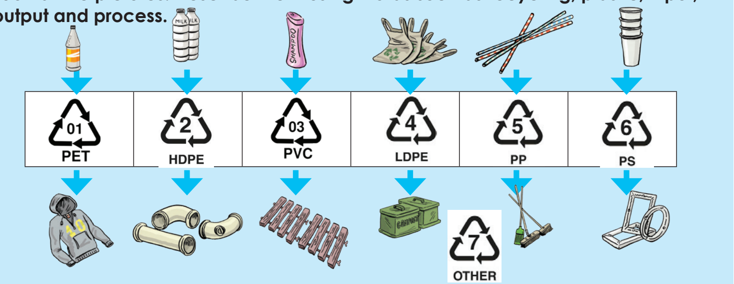
Dictionary

Functions: A function is the means of matching or relating one set of values with another set of values. For example, if we have a function + 3 then the value 1 in the one set will match the value 4 in the other set. Input (1) -> Function (+ 3) -> Output (4)
Flow diagram: A diagram of the sequence of operations.
Formula: An equation using numbers and symbols that shows you how to convert a measurement or measurements into another.

110

Introduction

Look at the pictures. Describe them using words such as recycling, plastic, input, output and process.



Q1

How fast can you complete the flow diagrams.

Answers:

a.

input	rule	output
1	x 6	6
5		30
7		42
9		54
12		72

The rule is x 6

b.

input	rule	output
3	x 4	12
6		24
8		32
4		16
5		20

The rule is x 4

c.

input	rule	output
12	x 8	96
11		88
9		72
6		48
5		40

The rule is x 8

d.

input	rule	output
3	x 9	27
10		90
12		108
9		81
8		72

The rule is x 9

48 Flow diagrams *continued*

Topic: Input and output values **Content links:** R9, 49-51, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

e.

7	x 7	49
10		70
6		42
4		28
3		21

The rule is x 7

f.

2	x 5	10
4		20
8		40
9		45
3		15

The rule is x 5

Q2

Use the given rule to calculate the value of b. Answers:

a.

a		b
4	$b = a \times 6$	24
5		30
6		36
2		12
3		18

The rule is $b = a \times 6$

b.

a		b
2	$b = a \times 10$	20
12		120
10		100
11		110
15		150

The rule is $b = a \times 10$

c.

x		y
11	$y = x - 9$	2
10		1
12		3
20		11
100		91

The rule is $y = x - 9$

d.

r		s
4	$s = r + 11$	15
7		18
9		20
20		31
5		16

The rule is s $= r + 11$

e.

m		n
5	$n = m - 4$	1
7		3
9		5
10		6
15		11

The rule is $n = m - 4$

f.

a		b
3	$b = 10 + a$	13
7		17
100		110
5		15
8		18
4		14

The rule is $b = 10 + a$

Q3

Prepare to present any flow diagram done in this lesson in a future lesson period.

Answer: Learner's own preparation

111

Problem solving

Draw a flow diagram where a = b + 7.

2	$a = b + 7$	9
3		10
4		11
5		12
6		13

Reflection questions

Did learners meet the objectives?

49 More flow diagrams

Topic: Input and output values **Content links:** R9, 49-51, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

Objectives

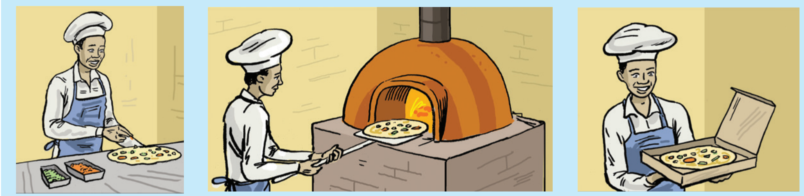
- Determine input values, output values or rules for patterns and relationships using:
  – Flow diagrams
  – Formulae
- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
  – In flow diagrams
  – By formulae
  – By number sentences
  – Verbally

Dictionary

Functions: A function is the means of matching or relating one set of values with another set of values. For example, if we have a function + 3 then the value 1 in the one set will match the value 4 in the other set.
Input (1) -> Function (+ 3) -> Output (4)
Flow diagram: A diagram of the sequence of operations.
Formula: An equation using numbers and symbols that shows you how to convert a measurement or measurements into another.

112 **Introduction**

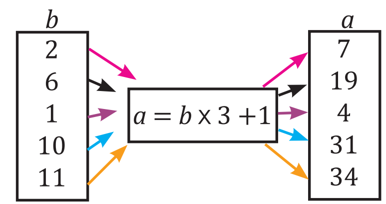
Let us look at Input and Output again. What do you think this is?



Answer: A Pizza showing the "input", "process" and "output".

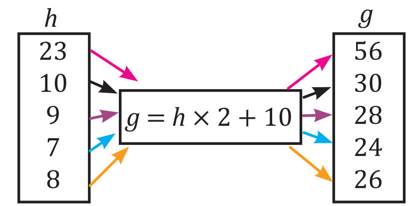
Q1 Complete the flow diagrams. Show all your calculations.
Answers:

a.



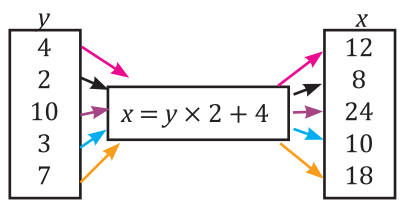
The rule is $a = b \times 3 + 1$
i. $2 \times 3 = 6 + 1 = 7$
ii. $6 \times 3 = 18 + 1 = 19$
iii. $1 \times 3 = 3 + 1 = 4$
iv. $10 \times 3 = 30 + 1 = 31$
v. $11 \times 3 = 33 + 1 = 34$

b.



The rule is $g = h \times 2 + 10$
i. $23 \times 2 = 46 + 10 = 56$
ii. $10 \times 2 = 20 + 10 = 30$
iii. $9 \times 2 = 18 + 10 = 28$
iv. $7 \times 2 = 14 + 10 = 24$
v. $8 \times 2 = 16 + 10 = 26$

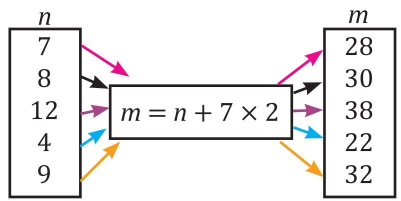
c.



The rule is $x = y \times 2 + 4$

i. $4 \times 2 = 8 + 4 = 12$
ii. $2 \times 2 = 4 + 4 = 8$
iii. $10 \times 2 = 20 + 4 = 24$
iv. $3 \times 2 = 6 + 4 = 10$
v. $7 \times 2 = 14 + 4 = 18$

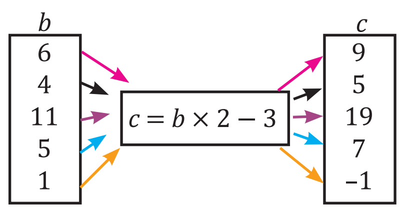
d.



The rule is $m = n + 7 \times 2$

i. $7 + 7 = 14 \times 2 = 28$
ii. $8 + 7 = 15 \times 2 = 30$
iii. $12 + 7 = 19 \times 2 = 38$
iv. $4 + 7 = 11 \times 2 = 22$
v. $9 + 7 = 16 \times 2 = 32$

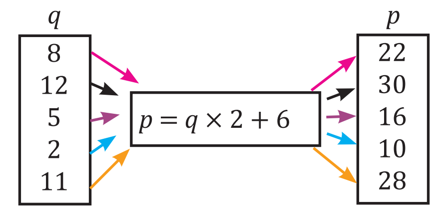
e.



The rule is $c = b \times 3 + 1$

i. $2 \times 3 = 6 + 1 = 7$
ii. $6 \times 3 = 18 + 1 = 19$
iii. $1 \times 3 = 3 + 1 = 4$
iv. $10 \times 3 = 30 + 1 = 31$
v. $11 \times 3 = 33 + 1 = 34$

f.



i. $8 \times 2 = 16 + 6 = 22$
ii. $12 \times 2 = 24 + 6 = 30$
iii. $5 \times 2 = 10 + 6 = 16$
iv. $2 \times 2 = 4 + 6 = 10$
v. $11 \times 2 = 22 + 6 = 28$

Q2

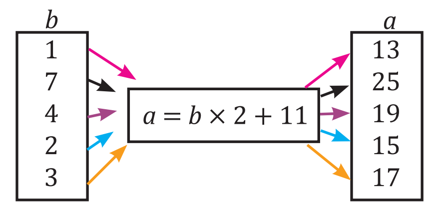
Prepare a flow diagram to present to the class. Change the flow diagram to an "input" and "output" device.
Answer: learners' own diagrams

113

Problem solving

Draw your own flow diagram where a = b x 2 + 11. Learner's own answer.

For example:



Reflection questions

Did learners meet the objectives?

50 Tables

Topic: Input and output values **Content links:** 48-49, 51, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

Objectives

Revise:

- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
  – in flow diagrams
  – verbally
  – in tables
- Determine input values, output values or rules for patterns and relationships using:
  – formulae
  – tables

Dictionary

Functions: A function is the means of matching or relating one set of values with another set of values. For example, if we have a function + 3 then the value 1 in the one set will match the value 4 in the other set. Input (1) -> Function (+ 3) -> Output (4)
Flow diagram: A diagram of the sequence of operations.
Formula: An equation using numbers and symbols that shows you how to convert a measurement or measurements into another.
Table: A way of presenting data in rows and columns.

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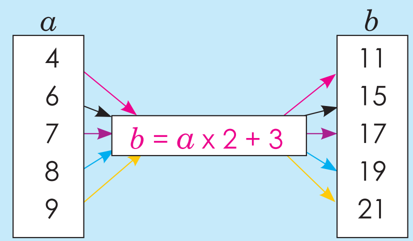
Introduction
Ask the learners to look at the example, and discuss the following:

- Flow diagram,
- Table, and
- Rule

What are the examples under the table showing us?

Complete the following:

The rule: $b = a \times 2 + 3$



$4 \times 2 + 3 = 11$
$6 \times 2 + 3 = 15$
$7 \times 2 + 3 = 17$
$8 \times 2 + 3 = 19$
$9 \times 2 + 3 = 21$

a	4	6	7	8	9
b	11	15	17	19	21

Q1

Complete the tables and show your calculations.
Answers:
a. $y = x + 2$

x	2	4	6	8	10	20
y	4	6	8	10	12	22

50 Tables *continued*

Topic: Input and output values **Content links:** 48-49, 51, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

b. $b = a + 7$

a	1	2	3	4	5	10
b	8	9	10	11	12	17

c. $n = m + 4$

m	4	5	6	7	10	100
n	8	9	10	11	14	104

d. $z = x \times 2$

x	2	3	4	5	6	7
z	4	6	8	10	12	14

e. $y = 2x - 1$

x	1	2	3	4	5	6	7
y	1	3	5	7	9	11	13

f. $n = 3m + 2$

m	1	5	10	20	25	100
n	5	17	32	62	77	302

Q3

Prepare any table to present to the class.
Answer: learner's own answer

115

Problem solving

If $x = 2y + 4$ and $y = 2, 3, 4, 5, 6$, draw a table to show it.

Answers:

y	2	3	4	5	6
x	8	10	12	14	16

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

51 Input and output values

Topic: Input and output values **Content links:** 48-50, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

Objectives

Revise:

- Determine input values, output values or rules for patterns and relationships using:
  – formulae
  – tables
- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
  – in flow diagrams
  – verbally
  – in tables

Dictionary

Number sentence: an equation expressed using numbers and common mathematical symbols
Verbally: communicate in the form of spoken words

116

Introduction
Tell the learners that you got these notes from two of your friends. Compare them.

Q1

Solve m and n.

x	1	2	3	4		18	n	51
y	8	9	10	11		25	39	m

Determine the rule:
$y = x + 7$

m?
$y = x + 7$
$y = 51 + 7$
$y = 58$
m is 58

n?
$x = n$ and $y = 39$
$y = x + 7$
$39 = n + 7$
$39 - 7 = n + 7 - 7$
$32 = n$
$n = 32$

a.

Rule:
$y = x + 9$

m?
$y = x + 9$
$= 25 + 9$
$= 34$
m is 34

n?
$x = n$ and $y = 39$
$y = x + 9$
$39 = n + 9$
$39 - 9 = n + 9 - 9$
$30 = n$
$n = 30$

51 Input and output values *continued*

Topic: Input and output values **Content links:** 48-50, 72, 118-119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R7

b.

Rule:	m?	n?
$y = x \times 2$	$x = m$ and $y = 22$ $y = x \times 2$ $22 = m \times 2$ $22 \div 2 = m \times 2 \div 2$ $11 = m$ $m = 11$	$y = x \times 2$ $= 30 \times 2$ $= 60$ $n = 60$

c.

Rule:	m?	n?
$y = x \times 5$	$x = m$ and $y = 90$ $y = x \times 5$ $90 = m \times 5$ $90 \div 5 = \mathrm{m} \times 5 \div 5$ $18 = m$ $m = 18$	$y = x \times 5$ $= 15 \times 5$ $= 75$ $n = 75$

d.

Rule:	m?	n?
$y = x + 12$	$x = m$ and $y = 24$ $y = x + 12$ $24 = m + 12$ $24 - 12 = m + 12 - 12$ $12 = m$ $m = 12$	$y = x + 12$ $= 46 + 12$ $= 58$ $n = 58$

e.

Rule:	m?	n?
$y = x \times 3$	$x = m$ and $y = 60$ $y = x \times 3$ $60 = m \times 3$ $60 \div 3 = \mathrm{m} \times 3 \div 3$ $20 = m$ $m = 20$	$y = x \times 3$ $= 10 \times 3$ $= 30$ $n = 30$

f.

Rule:	m?	n?
$y = x + 10$	$x = m$ and $y = 28$ $28 = m + 10$ $28 - 10 = m + 10 - 10$ $18 = m$ $m = 18$	$y = x + 10$ $= 41 + 10$ $= 51$ $n = 51$

117

Problem solving

What is the 10th pattern for 3 x 4: 4 x 4: 5 x 4: . . .

Answer:
$3 \times 4, 4 \times 4, 5 \times 4, 6 \times 4, 7 \times 4, 8 \times 4, 9 \times 4, 10 \times 4, 11 \times 4, 12 \times 4$

Reflection questions
Did learners meet the objectives?

52 Perimeter and area

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, R14, 53-55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

Objectives

- Calculate to at least 1 decimal place.
- Use appropriate formulae to calculate perimeter and area of:
  - Rectangles
  - Squares
  - Solve problems involving perimeter and area of polygons

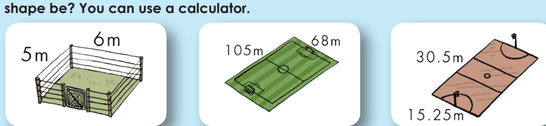
Dictionary

Perimeter: The perimeter of a shape is the distance around it.
Formula for the perimeter of square: $4l$
Formula for the perimeter of a rectangle: $2l+2b$
Area: The amount of surface scovering a 2-dimensional space.
Formula for the area of a square: l^2
Formula for the area of a rectangle: $l \times b$

118

Introduction
Ask the learners to look a the picture and say what the perimeter is. What will the area of each shape be.? Learners can use calculators.

Look at the pictures and say what the perimeters are. What will the area of each shape be? You can use a calculator.



Draw these on grid paper where: 1 m, 1 m, 1 cm represents 1 m

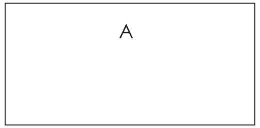
Answers

A fence	A soccer field	Netball court
Perimeter: $2\,l + 2\,b$	Perimeter: $2\,l + 2\,b$	Perimeter: $2\,l + 2\,b$
$= 2(6\text{ m}) + 2(5\text{ m})$	$= 2(105\text{ m}) + 2(68\text{ m})$	$= 2(30{,}5\text{ m}) + 2(15{,}25\text{ m})$
$= 12\text{ m} + 10\text{ m}$	$= 210\text{ m} + 136\text{ m}$	$= 61\text{ m} + 30{,}5\text{ m}$
$= 22\text{ m}$	$= 346\text{ m}$	$= 91{,}5\text{ m}$
Area: $l \times b$	Area: $l \times b$	Area: $l \times b$
$= 6\text{ m} \times 5\text{ m}$	$= 105\text{ m} \times 68\text{ m}$	$= 30{,}5\text{ m} \times 15{,}25\text{ m}$
$= 30\text{ m}^2$	$= 7\,140\text{ m}^2$	$= 465{,}125\text{ m}^2$

Q1

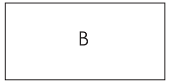
Calculate the perimeter and the area of the following polygons:

a. 4,5 cm



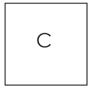
2,2 cm

b. 2,9 cm



1,4 cm

c. 1,5 cm



a. Perimeter: $2\,l + 2\,b$
$= 2(4{,}5) + 2(2{,}2)$
$= 9 + 4{,}4 = 13{,}4\text{ cm}$
Area: $l \times b$
$= 4{,}5\ cm \times 2{,}2\ cm$
$= 9{,}9\ cm^2$

b. Perimeter: $2\,l + 2\,b$
$= 2(2{,}9) + 2(1{,}4)$
$= 5{,}8 + 2{,}8 = 8{,}6\text{ cm}$
Area: $l \times b$
$= 2{,}9\ cm \times 1{,}4\ cm$
$= 4{,}06\ cm^2$

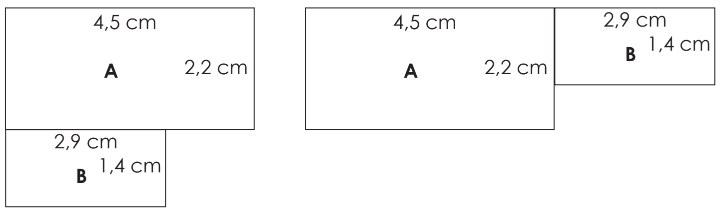
c. Perimeter: $2\,l + 2\,b$
$= 2(1{,}5) + 2(1{,}5)$
$= 3 + 3 = 6\text{ cm}$
Area: $l \times b$
$= 1{,}5\ cm \times 1{,}5\ cm$
$= 2{,}25\ cm^2$

Q2 Using the polygons A, B, and C above, draw each set of the polygons in two different ways so when joined together, they have

- the shortest possible perimeter
- the longest possible perimeter

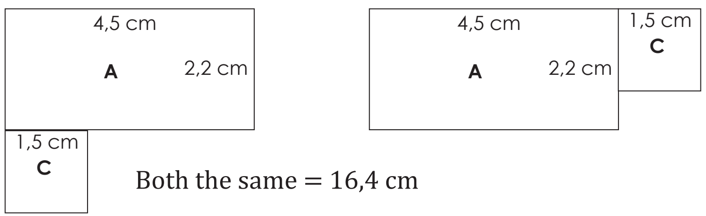
Answers:

a.



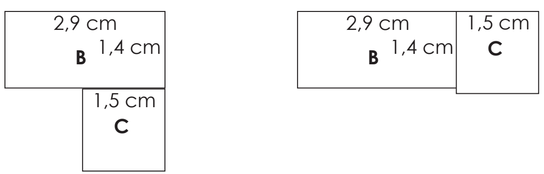
Shortest = 16,2 cm   Longest = 19,2 cm

b.



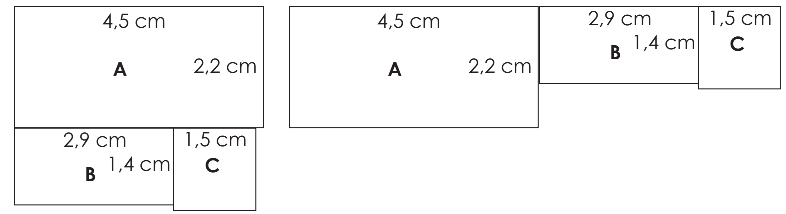
Both the same = 16,4 cm

c.



Shortest = 11,6 cm   Longest = 11,8 cm

d.



Shortest = 16,4 cm   Longest = 22,4 cm

Q3 If the area is ____, what could the perimeter be?

Answers: these are some possible answers

a. (i) $P = 2(6) + 2(6)$
$= 12 + 12$
$= 24$ cm

(ii) $P = 2(9) + 2(4)$
$= 18 + 8$
$= 26$ cm

b. (i) $P = 2(4) + 2(3)$
$= 8 + 6$
$= 14$ cm

(ii) $P = 2(6) + 2(2)$
$= 12 + 4$
$= 16$ cm

c. (i) $P = 2(10) + 2(10)$
$= 20 + 20$
$= 40$ cm

(ii) $P = 2(50) + 2(2)$
$= 100 + 4$
$= 104$ cm

d. (i) $P = 2(25) + 2(5)$
$= 50 + 10$
$= 60$ cm

(ii) $P = 2(125) + 2(1)$
$= 250 + 2$
$= 252$ cm

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, R14, 53-55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

e. (i) $P = 2(6) + 2(5)$
$= 12 + 10$
$= 22$ cm

(ii) $P = 2(10) + 2(3)$
$= 20 + 6$
$= 26$ cm

f. (i) $P = 2(6) + 2(3)$
$= 12 + 6$
$= 18$ cm

(ii) $P = 2(9) + 2(2)$
$= 18 + 4$
$= 22$ cm

Q4

Measure the perimeter and calculate the area of each shape. Give your answer in cm and mm. Answers: [note that these dimensions are approximate due to variations in size due to the printing process].

a. Perimeter:
3 cm + 3 cm + 1,5 cm + 1 cm + 1,5 cm + 2 cm = 12 cm = 120 mm
Area:
(2 cm × 1,5 cm) + (3 cm × 1,5 cm) = 7,5 cm^2 = 750 mm^2

b. Perimeter:
0,75 cm + 1 cm + 1 cm + 1cm + 0,75 cm + 3 cm + 0,75 + 1 cm + 1 cm + 1 cm + 0,75 + 3 cm = 15 cm = 150 mm
Area:
(1 cm × 1 cm) + (3 × 2,5 cm) + (1 cm × 1 cm) = 9,5 cm^2 = 950 mm^2

c. Perimeter:
2 cm + 2 cm + 2 cm + 1 cm + 4 cm + 3 cm = 14 cm = 140 mm
Area:
(2 cm × 2 cm) + (1 cm × 4 cm) = 4 cm^2 + 4 cm^2
= 8 cm^2 = 800 mm^2

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Problem solving

a. Draw a square and a rectangle each of which has a perimeter of 9 cm.
b. If the perimeter of a square is 22 cm, what is the length of each side?
c. What is the perimeter of a regular octagon if the length of each side is 17 cm?
d. What is the perimeter of a square if its area is 225 cm^2?

Answer:
a. Drawing of a square 2,25 cm × 2,25 cm
Drawing of a rectangle (many possible dimensions), e.g. 3 cm × 1, 5 cm
b. 5,5 cm
c. 136 cm
d. 60 cm

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

53 Area of triangles

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, R14, 53-55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

Objectives

- Use appropriate formulae to calculate perimeter and area of triangles
- Solve problems, with or without a calculator involving perimeter and area of triangles

Dictionary

Area of a triangle: the size of the surface inside the boundary of a triangle

Area of a triangle formula = $\frac{1}{2}$ base x vertical height

Perimeter of a triangle: The perimeter, P, of a triangle is given by the formula P = a + b + c where a, b and c are the side lengths of the triangle.

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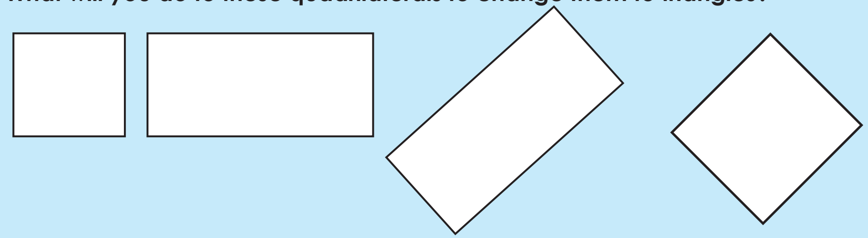
Introduction
Ask the learners what will they do with these quadrilaterals to change them into triangles.

What will you do to these quadrilaterals to change them to triangles?

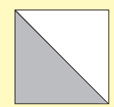


Q1

What is the area of the triangles? Use both methods.

Example:

2 cm



2 cm

length^2
$= (2 \text{ cm})^2$
$= 2 \text{ cm} \times 2 \text{ cm}$
$= 4 \text{ cm}^2$

The triangle is <u>half</u> of the square.

Method 1
$\frac{1}{2}$ of 4 cm^2
$= \frac{1}{2} \times 4 \text{ cm}^2$
$= \frac{1}{2} \times \frac{4}{1} \text{ cm}^2$
$= \frac{4}{2} \text{ cm}^2$
$= 2 \text{ cm}^2$

Method 2
$4 \text{ cm}^2 \div 2$
$= 2 \text{ cm}^2$

Answers:
Area:

a. $l^2 = (3 \text{ cm})^2 = 9 \text{ cm}^2$
$9 \text{ cm}^2 \div 2 = 4{,}5 \text{ cm}^2$
$\frac{1}{2}$ of $9 \text{ cm}^2 = \frac{1}{2} \times \frac{9}{1}$
$\frac{9}{2} = 4{,}5 \text{ cm}^2$

b. $l^2 = (4 \text{ cm})^2 = 16 \text{ cm}^2$
$\frac{1}{2} \times \frac{16}{1} \text{cm}^2$
$= 8 \text{ cm}^2$
$= 16 \text{ cm}^2 \div 2$
$= 8 \text{ cm}^2$

53 Area of triangles *continued*

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, R14, 53-55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

What is the area of the triangles?

Answers:

a. $5 \text{ cm} \times 3 \text{ cm} = 15 \text{ cm}^2$

$= \frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (5 \text{ cm}) \times 3 \text{ cm}$

$= 2{,}5 \text{ cm} \times 3 \text{ cm} = 7{,}5 \text{ cm}^2$

b. $4 \text{ cm} \times 2{,}5 \text{ cm} = 10 \text{ cm}^2$

$= \frac{1}{2}\, b \times h$

$= \frac{1}{2}\, (4 \text{ cm}) \times 2{,}5 \text{ cm}$

$= 2 \text{ cm} \times 2{,}5 \text{ cm} = 5 \text{ cm}^2$

c. $= 6 \text{ cm} \times\ 5 \text{ cm} = 30 \text{ cm}^2$

$= \frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (6 \text{ cm}) \times 5 \text{ cm}$

$= 3 \text{ cm} \times 5 \text{ cm} = 15 \text{ cm}^2$

d. $8 \text{ cm} \times 4 \text{ cm} = 32 \text{ cm}^2$

$= \frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (8 \text{ cm}) \times 4 \text{ cm}$

$= 4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$

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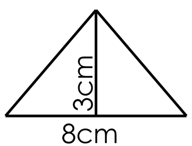
Problem solving

What is the area of a triangle if the base is 8 cm and the height is 3 cm?

Answers:



Area:

$\frac{1}{2}\ b \times h$

$\frac{1}{2}\ (8 \text{ cm}) \times 3 \text{ cm}$

$4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

54 More area of triangles

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, 52-53, 55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

Objectives

- Use appropriate formulae to calculate perimeter and area of triangles
- Solve problems, with or without a calculator involving perimeter and area of triangles

Dictionary

Area of a triangle: the size of the surface inside the boundary of a triangle

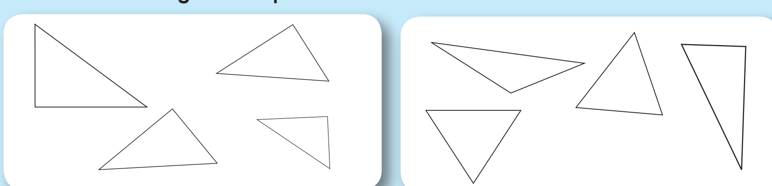
Area of a triangle formula = $\frac{1}{2}$ base x vertical height

Perimeter of a triangle: The perimeter, P, of a triangle is given by the formula P = a + b + c where a, b and c are the side lengths of the triangle.

Introduction

122

Look at these triangles. Compare them.

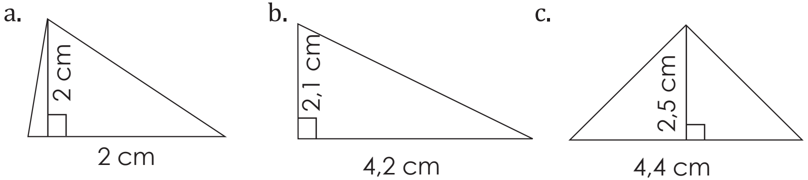


Right angles triangles

Not right angled triangles

Q1 Draw a perpendicular line showing the height of the triangle.
Answers:

a. b. c.



Q2 Calculate the area of the triangles. Answers:

a. $\frac{1}{2}\ b \times h$

$\frac{1}{2}(2 \text{ cm}) \times 2 \text{ cm}$

$1 \text{ cm} \times 2 \text{ cm}$

$= 2 \text{ cm}^2$

b. $\frac{1}{2}\ b \times h$

$\frac{1}{2}\ (4{,}2 \text{ cm}) \times 2{,}\ 1 \text{ cm}$

$2{,}1 \times 2{,}1 \text{ cm}$

$= 4{,}41 \text{ cm}^2$

c. $\frac{1}{2}\ b \times h$

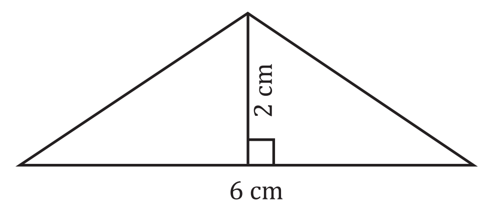
$\frac{1}{2}\ (4{,}4 \text{ cm}) \times 2{,}5 \text{ cm}$

$2{,}2 \text{ cm} \times 2{,}5 \text{ cm}$

$= 5{,}5 \text{ cm}^2$

Q3 Draw a triangle with the given measurements and then calculate the area. Answers: [Note that these dimensions are approximate due to variations in size due to the printing process.]

a.



$\frac{1}{2}\ b \times h$

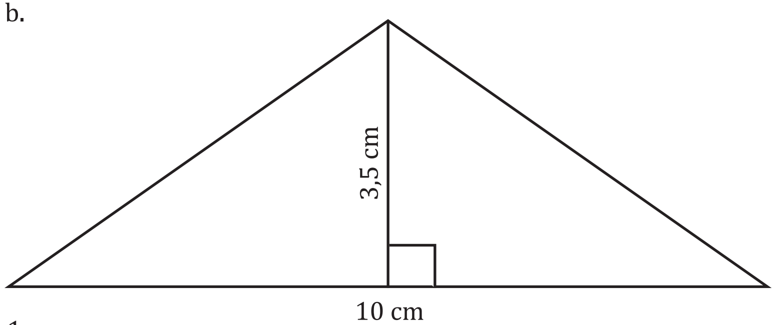
$\frac{1}{2}\ (6 \text{ cm}) \times 2 \text{ cm}$

$3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$

54 More area of triangles *cont...*

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, 52-53, 55
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

b.

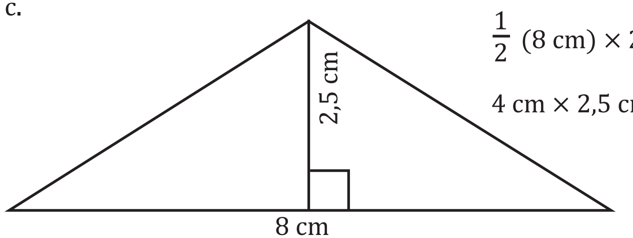


$\frac{1}{2}\ b \times h$

$\frac{1}{2}\ (10\text{ cm}) \times 3{,}5\text{ cm}$

$5\text{ cm} \times 3{,}5\text{ cm} = 17{,}5\text{ cm}^2$

c.



$\frac{1}{2}\ b \times h$

$\frac{1}{2}\ (8\text{ cm}) \times 2{,}5\text{ cm}$

$4\text{ cm} \times 2{,}5\text{ cm} = 10\text{ cm}^2$

Q4 Measure and calculate the area. Give your answer in cm^2 and mm^2. Answers: [note that these dimensions are approximate due to variations in size due to the printing process].

a. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}(22\text{ cm}) \times 16\text{ cm}$

$= 176\text{ cm}^2$

b. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}(32\text{ cm}) \times 16\text{ cm}$

$=256\text{ cm}^2$

c. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}(24\text{ cm}) \times 22\text{ cm}$

$= 276\text{ cm}^2$

d. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (36\text{ cm}) \times 16\text{ cm}$

$= 288\text{ cm}^2$

e. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (32\text{ cm}) \times 16\text{ cm}$

$= 346\text{ cm}^2$

f. $\frac{1}{2}\ b \times h$

$= \frac{1}{2}\ (37\text{ cm}) \times 22\text{ cm}$

$= 407\text{ cm}^2$

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Problem solving

What is the area of a triangle if the base equals 3,5 cm and the height equals 1,5cm?

Answer: $\frac{1}{2}\ b \times h = \frac{1}{2}\ (3{,}5\text{ cm}) \times 1{,}5\text{ cm} = 2{,}625\text{ cm}^2$

55 Area conversion

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, 52-54
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

Objectives

- Calculate to at least 1 decimal place
- Use and convert between appropriate International System of Units (SI units) including:
  - $mm^2 \longleftrightarrow cm^2$
  - $cm^2 \longleftrightarrow m^2$
- Solve problems, with or without a calculator involving perimeter and area of polygons

Dictionary

Convert: To change a value or measurement from one system of units to another.

Convert between SI units: $cm^2 \longleftrightarrow m^2$:

100 cm = 1 m
100 cm x 100 cm = 1 m x 1m
10 000 cm^2 = 1 m^2

Convert between SI units: $mm^2 \longleftrightarrow cm^2$:

10 mm = 1 cm
10 mm x 10 mm = 1 cm x 1 cm
100 mm^2 = 1 cm^2

Convert between SI units: $m^2 \longleftrightarrow km^2$:

1 000 m = 1 km
1 000 m x 1 000 m = 1 km x 1 km
1 000 000 m^2 = 1 km^2

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Introduction

Revise the following with your learners:
1 000 mm = 1 m
100 cm = 1 m
1 000 m = 1 km

Convert the following:

Revision

1 000 mm = ___m
___cm = 1 m
___m = 1 km

How did we get these answers?

cm^2 = 100 mm^2

1 cm = 10 mm
1 cm^2 (1 cm x 1 cm)
= 100 mm^2 (10 mm x 10 mm)

m^2 = 1 000 000 mm^2

1 m = 1 000 mm
1 m^2 (1 m x 1 m)
= 1 000 000 mm^2 (1 000 mm x 1 000 mm)

km^2 = 1 000 000 m^2

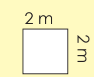
1 km = 1 000 m
1 km^2 (1 km x 1 km)
= 1 000 000 m^2 (1 000 m x 1 000 m

Q1

Work out the area and give your answers in m^2, cm^2 and mm^2.

Example: Length = 2 m, breadth = 1m

l x b	l x b	l x b
= 2 m x 1 m	= 200 cm x 100 cm	= 2 000 mm x 1 000 mm
=2 m^2	=20 000 cm^2	=2 000 000 mm^2



Answers:

a. $l \times b$
$= 5 \text{ m} \times 3 \text{ m}$
$= 15 \text{ m}^2$

$l \times b$
$= 500 \text{ cm} \times 300 \text{ cm}$
$= 150\ 000 \text{ m}^2$

$l \times b$
$= 5000 \text{ mm} \times 3000 \text{ mm}$
$= 15\ 000\ 000 \text{ mm}^2$

55 Area conversion *continued*

Topic: Size, area and perimeter of 2-D shapes **Content links:** R12, 52-54
Grade 8 links: R14, 82-86 **Grade 9 links:** R14, 60-64

b. $l \times b$
$= 3\text{m} \times 1{,}5 \text{ m}$
$= 4{,}5 \text{ m}^2$

$l \times b$
$= 300 \text{ cm} \times 150 \text{ cm}$
$= 45\ 000 \text{ cm}^2$

$l \times b$
$= 3000 \text{ mm} \times 1\ 500 \text{ mm}$
$= 4\ 500\ 000 \text{ mm}^2$

c. $l \times b$
$= 6 \text{ m} \times 3{,}2 \text{ m}$
$= 19{,}2 \text{ m}^2$

$l \times b$
$= 600 \text{ cm} \times 320 \text{ cm}$
$= 192\ 000 \text{ cm}^2$

$l \times b$
$= 6000 \text{ mm} \times 3\ 200 \text{ mm}$
$= 19\ 200\ 000 \text{ mm}^2$

d. $l \times b$
$= 4{,}5 \text{ m} \times 2{,}1 \text{ m}$
$= 9{,}45 \text{ m}^2$

$l \times b$
$= 450 \text{ cm} \times 210 \text{ cm}$
$= 94\ 500 \text{ cm}^2$

$l \times b$
$= 4\ 500 \text{ mm} \times 2\ 100 \text{ mm}$
$= 9\ 450\ 000 \text{ mm}^2$

e. $l \times b$
$= 7{,}2 \text{ m} \times 5 \text{ m}$
$= 36 \text{ m}^2$

$l \times b$
$= 720 \text{ cm} \times 500 \text{ cm}$
$= 360\ 000 \text{ cm}^2$

$l \times b$
$= 7\ 200 \text{ mm} \times 5000 \text{ mm}$
$= 36\ 000\ 000 \text{ mm}^2$

Q2 Given the area of a rectangle, find a possible length and breadth in cm and m.

Answers:

a. Calculation:
$5\ 000 \times 3\ 000$ mm
$= 500 \text{ cm} \times 300 \text{ cm}$
$= 5 \text{ m} \times 3 \text{ m} = 15 \text{ m}^2$
length = 500 cm = 5 m
breadth = 300 cm = 3 m

b. Calculation:
$9\ 000 \times 7\ 000$
$= 900 \text{ cm} \times 700 \text{ cm}$
$= 9 \text{ m} \times 7 \text{ m}$
length = 900 cm = 9 m
breadth = 700 cm = 7 m

c. Calculation:
$9\ 000 \times 3\ 000$
$= 900 \text{ cm} \times 300 \text{ cm}$
$= 9 \text{ m} \times 3 \text{ m}$
$= 27$ m
length = 900 cm = 9 m
breadth = 300 cm = 3 m

d. Calculation:
$7\ 000 \times 4\ 000$
$= 700 \text{ cm} \times 400 \text{ cm}$
$= 7 \text{ m} \times 4 \text{ m}$
$= 28$ m
length = 700 cm = 7 m
breadth = 400 cm = 4 m

e. Calculate:
$6\ 000 \times 6\ 000$
$= 600 \text{ cm} \times 600 \text{ cm}$
$= 6 \text{ m} \times 6 \text{ m}$
$= 36$ m
length = 600 cm = 6 m
breadth = 600 cm = 6 m

f. Calculate:
$4\ 000 \times 4\ 000$
$= 400 \text{ cm} \times 400 \text{ cm}$
$= 4 \text{ m} \times 4 \text{ m}$
$= 16$ m
length = 400 cm = 4 m
breadth = 400 cm = 4 m

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Problem solving

If the base of a triangle is 4 m and the height 3 m, calculate the area and give your answer in m², cm² and mm².

Answer: $\frac{1}{2} b \times h = \frac{1}{2} (4 \text{ m}) \times 3 \text{ m} = 6 \text{ m}^2 = 600 \text{ cm}^2 = 6\ 000 \text{ mm}^2$

Reflection questions

Did learners meet the objectives?

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 57-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

56 Understanding the volume of cubes

Objectives
- Use appropriate formulae to calculate the surface area, volume and capacity of cubes
- Solve problems including volume

Dictionary
Volume: Volume is the measure of the amount of space within or occupied by a solid figure. It is the space actually occupied by an object or some substance.

Capacity: Capacity is a containing space. It is amount of room available to hold something. So if a bottle has the capacity of 1 litre you will need a volume of 1 litre of water to fill it.

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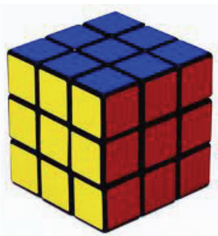
Introduction
Ask the learners to look at the pictures and answer the following questions:
- How many containers are on the truck? (216)
- How did you work it out? (Length × width × height)
- Is their a quicker way of working it out? Explain it.

Slower way
$36 + 36 + 36 + 36 + 36 + 36 = 216$
Faster way
$6 \times 6 \times 6$
$= 36 \times 6$
$= 216$

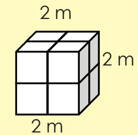
How many containers are on the truck?

How many cubes do you count in this block?
$3 \times 3 \times 3$
$= 27$



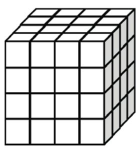
Q1 Label the diagram. Count the cubes. Write the number of cubes in exponential form.

Example

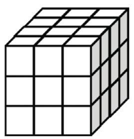


$2 \times 2 \times 2 = 2^3$
$= 2\ m \times 2\ m \times 2\ m$
$= 8\ m^3$

Answers:
a. $l \times b \times h$
$= 4 \times 4 \times 4$
$= 4\ m \times 4\ m \times 4\ m$
$= 64\ m^3$



b. $l \times b \times h$
$= 3 \times 3 \times 4$
$= 3\ m \times 3\ m \times 4\ m$
$= 36\ m^2$

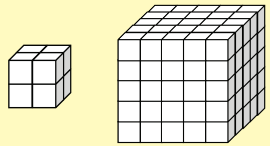


56 Understanding the volume of cubes *cont...*

Q2

Write down a sum in exponential form for each diagram and then calculate the total number of cubes used.

Example:
2 cubes^3 + 5 cubes^3
= 8 cubes + 125 cubes
= 133 cubes



Answers:

a. $4 \text{ cm}^3 + 2 \text{ cm}^3$
$= 64$ cubes $+ 8$ cubes
$= 72$ cubes

b. $3 \text{ cm}^3 + 3 \text{ cm}^3 + 3 \text{ cm}^3$
$= 27$ cubes $+ 27$ cubes $+ 27$ cubes
$= 81$ cubes

Q3

Calculate the volume of the buildings. Show your calculations.

Answers:

a. $5^3 + 4^3$
$= 125 + 64$
$= 189$ units

b. $4^3 + 4^3 + 2^3 + 2^3$
$= 64 + 64 + 8 + 8$
$= 144$ units

c. $4^3 + 3^3 + 2^3$
$= 64 + 27 + 8$
$= 99$ units

d. $2^3 + 2^3 + 2^3 + 2^3$
$= 8 + 8 + 8 + 8$
$= 32$ units

e. $5^3 + 5^3$
$= 125 + 125$
$= 250$ units

Q4

Make a drawing and calculate the following:

Answers:

a. $l \times b \times h$
$= 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm}$
$= 8 \text{ cm}^3$

b. $l \times b \times h$
$= 4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$
$= 64 \text{ cm}^3$

c. $l \times b \times h$
$= 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$
$= 125 \text{ cm}^3$

d. $l \times b \times h$
$= 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$
$= 27 \text{ cm}^3$

e. $l \times b \times h$
$= 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$
$= 1 \text{ cm}^3$

f. $l \times b \times h$
$= 7 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm}$
$= 343 \text{ cm}^3$

127

Problem solving

If a block has 1 728 cubic units, what will its dimensions be?

Answer: One possible answer is $12 \times 12 \times 12 = 1\,728$ cubic units

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

57 Volume of cubes

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56, 58-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100

Objectives

- Use and convert between appropriate SI Units including: $mm^3 \leftrightarrow cm^3$, $mm^2 \leftrightarrow cm^2$, $cm^2 \leftrightarrow m^2$
- Use appropriate formulae to calculate the area, volume and capacity of cubes

Dictionary

Volume: Volume is the measure of the amount of space inside of a solid figure.
Volume of a cube: V = length x length x height or area x height
Capacity: Capacity is the amount of space within a container.

128

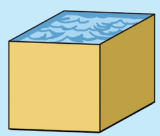
Introduction

Ask the leaners what the difference between volume and capacity is. They should use the picture to support their answer.

What is the difference between volume and capacity?

The **volume** of a solid is the amount of space it occupies.

Capacity is the amount of liquid a container can hold.



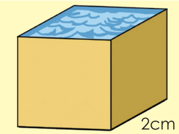
10cm

10 cm x 10 cm x 10 cm
= 1 000 cm^3
= 1000 ml
= 1 ℓ

Q1

Use a formula to calculate the volume of water that will fill each cube.

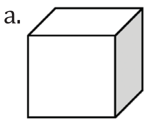
Example:
The formula for the volume of a cube is l^3



2cm

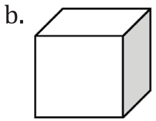
2 cm x 2 cm x 2 cm
= 8 cm^3
= 8 ml
= 0,008 ℓ

Answers:

a. 

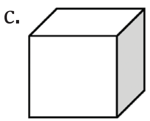
3 cm

$3\ cm \times 3\ cm \times 3\ cm$
$= 27\ cm^3$
$= 27\ ml$
$= 0{,}027\,\ell$

b. 

5 cm

$5\ cm \times 5\ cm \times 5\ cm$
$= 125\ cm^3$
$= 125\ ml$
$= 0{,}125\,\ell$

c. 

4 cm

$4\ cm \times 4\ cm \times 4\ cm$
$= 64\ cm^3$
$= 64\ ml$
$= 0{,}064\,\ell$

Q2

What will the dimensions of a cube be if the volume is:

Example:
8 cm^3 = 2 cm x 2 cm x 2 cm

Answers:

a. $3\ cm \times 3\ cm \times 3\ cm$
b. $4\ cm \times 4\ cm \times 4\ cm$
c. $5\ cm \times 5\ cm \times 5\ cm$
d. $1\ cm \times 1\ cm \times 1\ cm$
e. $6\ cm \times 6\ cm \times 6\ cm$

57 Volume of cubes *cont...*

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56, 58-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100

Q3

Use the example to guide you in completing these volume calculations for these cubes:

Answers:

a. 4 m
$4\text{ m} \times 4\text{ m} \times 4\text{ m}$
64 m^3

400 cm
$400\text{cm} \times 400\text{ cm} \times 400\text{ cm}$
$64\ 000\ 000\text{ cm}^3$

4 000 mm
$4\ 000\text{ mm} \times 4\ 000\text{ mm} \times 4\ 000\text{ mm}$
$64\ 000\ 000\ 000\text{ mm}^3$

b. 3 m
$3\text{ m} \times 3\text{ m} \times 3\text{ m}$
27 m^3

300 cm
$300\text{ cm} \times 300\text{ cm} \times 300\text{ cm}$
$27\ 000\ 000\text{ cm}^3$

3 000 mm
$3\ 000\text{ mm} \times 3\ 000\text{ mm} \times 3\ 000\text{ mm}$
$27\ 000\ 000\ 000\text{ mm}^3$

c. 5 m
$5\text{ m} \times 5\text{ m} \times 5\text{ m}$
125 m^3

500 cm
$500\text{ cm} \times 500\text{ cm} \times 500\text{ cm}$
$125\ 000\ 000\text{ cm}^3$

5 000 mm
$5\ 000\text{ mm} \times 5\ 000\text{ mm} \times 5\ 000\text{ mm}$
$125\ 000\ 000\ 000\text{ mm}^3$

d. 1 m
$1\text{ m} \times 1\text{ m} \times 1\text{ m}$
1 m^3

100 cm
$100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}$
$1\ 000\ 000\text{ cm}^3$

1 000 mm
$1\ 000\text{ mm} \times 1\ 000\text{ mm} \times 5\ 000\text{ mm}$
$1\ 000\ 000\ 000\text{ mm}^3$

Q4

Look at the example showing how to calculate the dimensions of a cube with a particular volume. Rewrite all the volumes below showing the dimensions of the cubes in mm, cm and m.

Answers:

a. $216\text{ m}^3 = 6\text{ m} \times 6\text{ m} \times 6\text{ m}$
$216\ 000\ 000\text{ cm}^3 = 600\text{ cm} \times 600\text{ cm} \times 600\text{ cm}$
$216\ 000\ 000\ 000\text{ mm}^3 = 6\ 000\text{ mm} \times 6\ 000\text{ mm} \times 6\ 000\text{ mm}$

b. $343\ 000\ 000\ 000\text{ mm}^3 = 7\ 000\text{ mm} \times 7\ 000\text{ mm} \times 7\ 000\text{ mm}$
$343\ 000\ 000\text{ cm}^3 = 700\text{ cm} \times 700\text{ cm} \times 700\text{ cm}$
$343\text{ m}^3 = 7\text{ m} \times 7\text{ m} \times 6\text{ m}$

c. $512\ 000\ 000\text{ cm}^3 = 800\text{ cm} \times 800\text{ cm} \times 800\text{ cm}$
$512\ 000\ 000\ 000\text{ mm}^3 = 8\ 000\text{ mm} \times 8\ 000\text{ mm} \times 8\ 000\text{ mm}$
$512\text{ m}^3 = 8\text{ m} \times 8\text{ m} \times 8\text{ m}$

d. $125\ 000\ 000\text{ mm}^3 = 5\ 000\text{ mm} \times 5\ 000\text{ mm} \times 5\ 000\text{ mm}$
$125\text{ m}^3 = 5\text{ m} \times 5\text{ m} \times 5\text{ m}$

131

Problem solving

a. If the volume of a cube is 125 cm³, what are its dimensions in mm and m?
b. With a family member think of five everyday objects that are cubes.

Answer:

a. 125 cm^3
$= 5\text{ cm} \times 5\text{ cm} \times 5\text{ cm}$
$= 50\text{ mm} \times 50\text{ mm} \times 50\text{ mm}$
$= 0{,}05\text{ m} \times 0{,}05\text{ m} \times 0{,}05\text{ m}$

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-57, 59-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

58 Volume of rectangular prisms

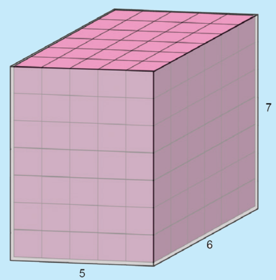
Objectives

- Use appropriate formulae to calculate the surface area, volume of rectangular prisms
- Solve problems involving volume

Dictionary

Volume: Volume is the measure of the amount of space inside of a solid figure.

How many cubes are in the large container?



132

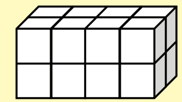
Introduction

Ask the learners to look at the pictures and answer the following questions:

- How many cubes are in the container? (210)
- How did you work it out? (Multiplying length by width by height: $6 \times 5 \times 7 = 210$)
- Is there a quicker way of working it out? Explain it.

Q1 Write a multiplication sum to calculate the number of cubes making up each rectangular object.

Example:



4 x 2 x 2 = 16 cubes

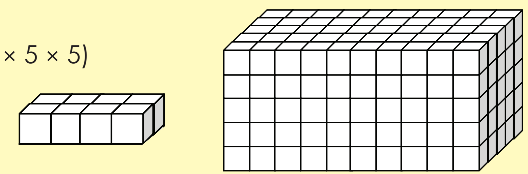
Answers:

a. $8 \times 4 \times 4 = 128$ cubes
b. $6 \times 4 \times 3 = 72$ cubes

Q2 Write multiplication sums to calculate the number of cubes in each pair of rectangular objects.

Example:

(4 × 1 × 2) + (10 × 5 × 5)
= 8 + 250
= 258 cubes



Answers:

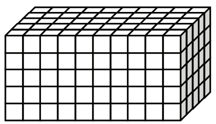
a. $(8 \times 4 \times 4) + (4 \times 2 \times 2)$
$= 128 + 16$
$= 144$ cubes

b. $(6 \times 3 \times 3) + (6 \times 6 \times 3)$
$= 54 + 108$
$= 162$

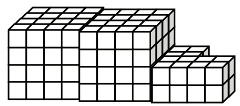
58 Volume of rectangular prisms *continued*

Q3

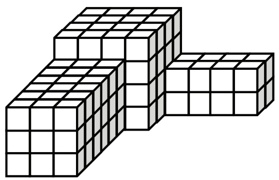
Calculate the volume of each of these buildings. Show your calculations. Answers:



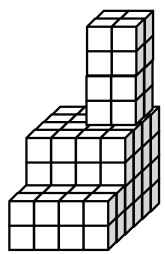
a. $10 \times 5 \times 5$
$= 250$ cubes



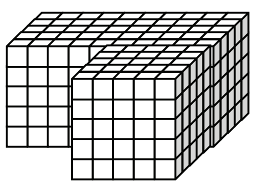
b. $4 \times 4 \times 4 = 64$
$4 \times 4 \times 4 = 64$
$2 \times 4 \times 2 = 16$
$2 \times 2 \times 2 = 8$
$64 + 64 + 16 + 8 = 152$ cubes



c. $3 \times 6 \times 3 = 64$
$4 \times 4 \times 4 = 64$
$4 \times 2 \times 2 = 16$
$64 + 64 + 16 = 134$ cubes



d. $4 \times 6 \times 2 = 48$
$4 \times 4 \times 2 = 32$
$2 \times 2 \times 4 = 16$
$48 + 32 + 16 = 96$

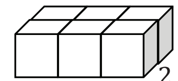


e. $5 \times 5 \times 5 = 125$
$10 \times 5 \times 5 = 250$
$125 + 250 = 375$

Q4

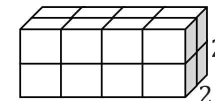
Calculate the volume of rectangular prisms with the following dimensions and make a drawing of each rectangular prism showing the dimensions. Answers:

a. 6 cm^3



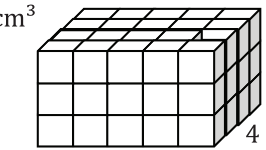
1 cm
2 cm
3 cm

b. 16 cm^3



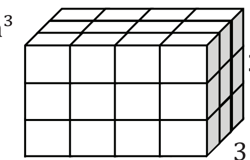
2 cm
2 cm
4 cm

c. 60 cm^3



3 cm
4 cm
5 cm

d. 36 cm^3



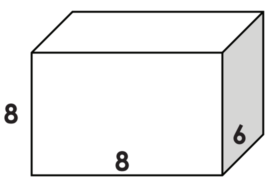
3 cm
3 cm
4 cm

133

Problem solving

If a rectangular prism has 384 cubic units, what will its dimensions be?

Answer: Various answers are possible, e.g.
$3 \times 2 \times 64$
$3 \times 4 \times 32$
$3 \times 8 \times 16$
$6 \times 4 \times 16$
$6 \times 8 \times 8$
$12 \times 4 \times 8$
$12 \times 2 \times 16$ etc.



Reflection questions
Did learners meet the objectives?

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-58, 60-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

59 Volume of rectangular prisms again

Objectives

- Use and convert between appropriate SI Units including:
  $mm^3 \leftrightarrow cm^3$, $mm^2 \leftrightarrow cm^2$, $cm^2 \leftrightarrow m^2$
- Solve problems involving volume

Dictionary

Volume: Volume is the measure of the amount of space inside of a solid figure.
Volume of a cube: V = length x length x height or area x height
Capacity: Capacity is the amount of space or a substance a container can hold.

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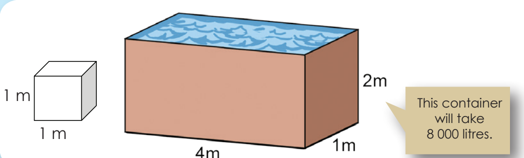
Introduction

Ask the learners how many small cubes (1 m x 1 m) will fit into the rectangular prism (1 m x 2 m x 4 m) 8 m^3.

How many small containers will fit in the large container? How did you work it out? Why do we know the large container can hold 8 000 litres?

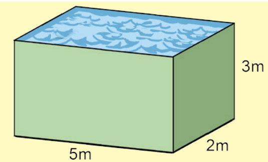


We know that
10 cm x 10 cm x 10 cm
= 1000 cm^3
= 1000 ml
= 1 l

Q1

Calculate the volume of the following and give your answer in m^3, cm^3 and mm^3. Also say what the capacity of each container is when filled with water.

Example:



This contaimer will hold 30 000 000 ml or 30 000 ℓ of water.

m^3	cm^3	mm^3
$l \times b \times h$	$l \times b \times h$	$l \times b \times h$
= 5 m × 2 m × 3 m	= 500 cm × 200 cm × 300 cm	= 5 000 mm × 2 000 mm × 3 000 mm
= 30 m^3	= 30 000 000 cm^3	= 30 000 000 000 mm^3

Answers:

a. m^3: $l \times b \times h$
$= 6 \text{ m} \times 1 \text{ m} \times 3 \text{ m} = 18 \text{ m}^3$
cm^3: $l \times b \times h$
$= 600 \text{ cm} \times 100 \text{ cm} \times 300 \text{ cm} = 18\ 000\ 000 \text{ cm}^3$
mm^3: $l \times b \times h$
$= 6\ 000 \text{ mm} \times 1\ 000 \text{ mm} \times 3\ 000 \text{ mm} = 18\ 000\ 000\ 000 \text{ mm}^3$

b. m^3: $l \times b \times h$
$= 2 \text{ m} \times 1 \text{ m} \times 9 \text{ m} = 18 \text{ m}^3$
cm^3: $l \times b \times h$
$= 200 \text{ cm} \times 100 \text{ cm} \times 900 \text{ cm} = 18\ 000\ 000 \text{ cm}^3$
mm^3: $l \times b \times h$
$= 2\ 000 \text{ mm} \times 1\ 000 \text{ mm} \times 9\ 000 \text{ mm} = 18\ 000\ 000\ 000 \text{ mm}^3$

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-58, 60-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

59 Volume of rectangular prisms again *cont...*

c. m^3: $l \times b \times h$
$= 2\text{ m} \times 2\text{ m} \times 5\text{ m} = 20\text{ m}^3$
cm^3: $l \times b \times h$
$= 200\text{ cm} \times 200\text{ cm} \times 500\text{ cm} = 20\ 000\ 000\text{ cm}^3$
mm^3: $l \times b \times h$
$= 2\ 000\text{ mm} \times 2\ 000\text{ mm} \times 5\ 000\text{ mm} = 20\ 000\ 000\ 000\text{ mm}^3$

d. m^3: $l \times b \times h$
$= 9\text{ m} \times 3\text{ m} \times 5\text{ m} = 135\text{ m}^3$
cm^3: $l \times b \times h$
$= 900\text{ cm} \times 300\text{ cm} \times 500\text{ cm} = 135\ 000\ 000\text{ cm}^3$
mm^3: $l \times b \times h$
$= 9\ 000\text{ mm} \times 3\ 000\text{ mm} \times 5\ 000\text{ mm} = 135\ 000\ 000\ 000\text{ mm}^3$

e. m^3: $l \times b \times h$
$= 2\text{ m} \times 2\text{ m} \times 7\text{ m} = 28\text{ m}^3$
cm^3: $l \times b \times h$
$= 200\text{ cm} \times 200\text{ cm} \times 700\text{ cm} = 28\ 000\ 000\text{ cm}^3$
mm^3: $l \times b \times h$
$= 2\ 000\text{ mm} \times 2\ 000\text{ mm} \times 7\ 000\text{ mm} = 28000\ 000\ 000\text{ mm}^3$

f. m^3: $l \times b \times h$
$= 4\text{ m} \times 2\text{ m} \times 6\text{ m} = 48\text{ m}^3$
cm^3: $l \times b \times h$
$= 400\ ccm \times 200\text{ cm} \times 600\text{ cm} = 48\ 000\ 000\text{ cm}^3$
mm^3: $l \times b \times h$
$= 4\ 000\text{ mm} \times 2\ 000\text{ mm} \times 6\ 000\text{ mm} = 48\ 000\ 000\ 000\text{ mm}^3$

135

Problem solving

- What is the volume if the dimensions of a rectangular prism are the following: length = 2,4 cm, breadth = 3 m and height = 10cm? What type of geometric object is it?
- With a family member think of five everyday objects that are rectangular prisms.

Answer:
$l \times b \times h$
$= 2{,}4\text{ cm} \times 300\text{ cm} \times 10\text{ cm}$
$= 7\ 200\text{ cm}^3$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

60 Volume problems

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-60, 62-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

Objectives

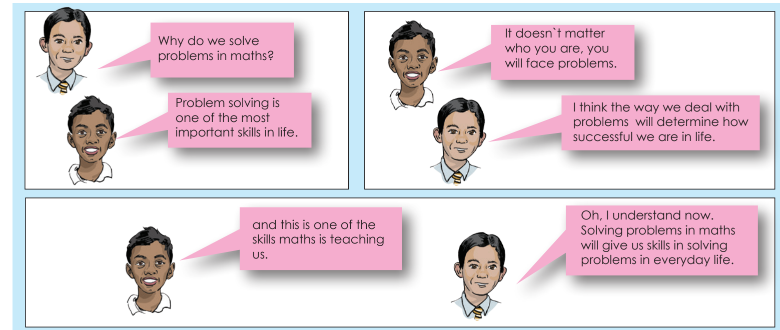
- Solve appropriate problems involving surface area, volume and capacity.

Dictionary

Word problem: A mathematical activity where background or context of the problem is presented as text or narrative story rather than as a mathematical notation.
Volume: Volume is the measure of the amount of space inside of a solid figure
Volume of a cube: V = length x length x height or area x height
Volume of a rectangular prism: V = length x breadth x height
Capacity: Capacity is a containing space. It is amount of room available to hold something in a container.

136

Introduction
Ask the learners to read the comic strip. Ask them why problem solving is such important skill in day-to-day life. Write learners answers on the board and summarise it for them.



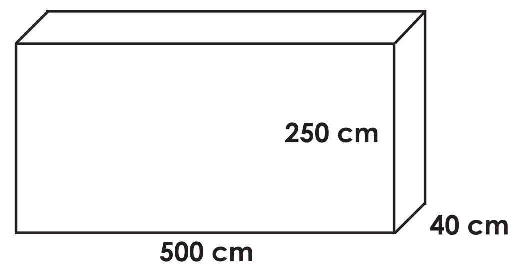
Q1

Calculate the volume (in cubic centimetres) of a rectangular prism that is 5 m long, 40 cm wide and 2 500 mm high. Make a drawing.

Answers:

a. $(l \times b \times h)$
$= 500\ cm \times 40\ cm \times 250\ cm$
$= 5\ 000\ 000\ cm$

$5\ m = 500\ cm$
$40\ cm = 40\ cm$
$2\ 500\ mm = 250\ cm$



60 Volume problems *cont...*

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-60, 62-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

Q2

A swimming pool is 8 m long, 6 m wide and 1,5 m deep. The water resistant paint needed for the pool costs R50 per square meter.

Answers:

a. Inside surface area:
$2 \times (8\text{ m} \times 1{,}5\text{ m}) + 2 \times (6\text{ m} \times 1{,}5\text{ m}) + 1 \times (8\text{ m} \times 6\text{ m})$
$= (2 \times 12\text{ m}^2) + (2 \times 9\text{ m}^2) + 48\text{ m}^2 = 24\text{ m}^2 + 18\text{ m}^2 + 48\text{ m}^2$
$= 90\text{ m}^2$
Cost: $90 \times \text{R}50 = \text{R}450$

b. Volume water:
$8\text{ m} \times 6\text{ m} \times 1{,}5\text{ m} = 72\text{ m}^3 = 72\ 000$ litres

Q3

At a factory they are trying to store boxes in a storage room with a length of 5 m, width of 3 m and height of 2 m. How many boxes can fit in this space if each box is 10 cm long, 6 cm wide and 4 cm high? Answers:

Storage
$l \times b \times h$
$= 5\,m \times 3\,m \times 2\,m$
$= 30\,m^3$
$\therefore 500\,cm \times 300\,cm \times 200\,cm$
$= 30\ 000\ 000\,cm^3$
$= 30\ 000\ 000\,cm^3 \div 240\,cm^2$
$= 125\ 000\ boxes$

Boxes
$l \times b \times h$
$= (10\,cm \times 6\,cm) \times 4\,cm$
$= 240\,cm^3$

The boxes fit exactly as the length, width and height of the box divides exactly into the length, width and height of the storage room (50 times respectively). Therefore $50 \times 50 \times 50 = 125\ 000$ boxes.

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Problem solving

Solve this with a family member or members.
- Assume we each create a cube of 30 cm x 30 cm x 30 cm of waste per day.
- We have a classroom with dimensions of 5,1m x 4,5m. x 3 m.
- We are 30 children in the class.

How long will we take to fill the class with waste?

Answer:
Waste
$l \times b \times h$
$= 30\,cm \times 30\,cm \times 30\,cm$
$= \ 27\ 000\,cm^3\ \ per\ child\ per\ day$
$\therefore 27\ 000\,cm\ per\ child \times 30\ children$
$= 810\ 000\,cm^3\ \ per\ class\ per\ day$

Classroom:
$l \times b \times h$
$= \ 5{,}1\,m \times 4{,}5\,m \times 3\,m$
$= 68{,}85\,m^3$
$\therefore 68{,}85\,m^3 \times 1\ 000\ 000\text{ cm} = 68\ 850\ 000\text{ cm}^3$
$\therefore 68\ 850\ 000\,cm^3 \div \ 810\ 000\text{ cm}^3$
85 days to fill the class

Reflection questions
Did learners meet the objectives?

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-60, 62-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

61 Volume and capacity

Objectives
- Solve problems involving surface area, volume and capacity
- Use appropriate formulae to calculate the surface area of a cube

Dictionary
Volume: Volume is the measure of the amount of space inside of a solid figure. It is the space actually occupied by an object or some substance.

Capacity: Capacity is a containing space. It is amount of room available to hold something.

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Introduction
Ask the learners to look at the picture: This person needs to collect information, what do you notice? A person thinking, searching on a computer, searching in a library, reading a book, having a lot of resources around him or her, talking to his or her teacher and holding his or her maths workbook.

This person needs to collect information. What do you notice?

Q1

Show that the following statements are true:
1 cm^3 = 1 millilitre
1 000 cm^3 = 1 litre
1 m^3 = 1 000 litre

Answers:
- Measurement for liquids
- A volume measured in cubic centimetres, e.g. a box of A4 paper.
- A volume measured in cubic metres, e.g. a pallet of bricks
- Various answers
- Show that a millilitre has the same volume as a cm^3
- Show that a litre has the same volume as a 1 000 cm^3. Show that a 1 000 litres has the same volume as a m^3.

$1 \text{ cm}^3 = 1 \text{ ml}$
$1\,000 \text{ cm}^3 = 1\,\ell$
$1 \text{ m}^3 = 1\,000\,\ell$
If: $1 \text{ cm}^3 = 1 \text{ ml}$ then $1\,000 \text{ ml} = 1\,\ell$
$\therefore 1\,000 \text{ cm}^3 = 1\,\ell$
If: $1\,000 \text{ cm}^3 = 1\,\ell$
$\therefore 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$
$= 1\,000\,000 \text{ cm}^3 = 1\,000\,000 \text{ ml} = 1\,000\,\ell$

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Problem solving
Share this process step by step with a friend or a family member.

62 Surface area of a cube

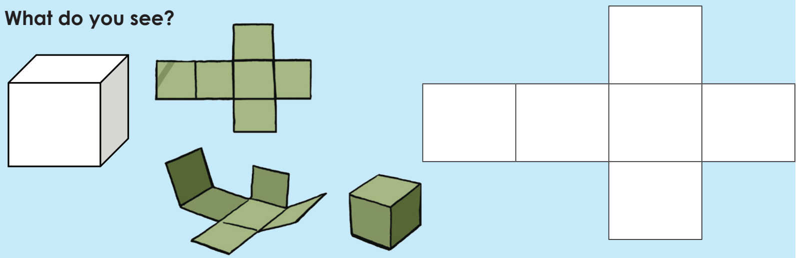
Objectives
- Solve appropriate problems involving surface area, volume and capacity
- Use appropriate formulae to calculate the surface area of a cube

Dictionary
Surface area: The total area of the surface of a geometric object.
Formula: The surface area of a prism = the sum of the area of all its faces.
Formula for the surface area of a cube: = length^2 × total faces

140

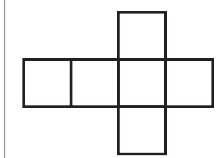
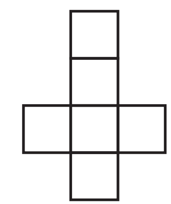
Introduction
Ask the learners to look at the three pictures, and ask them what do they see?
- a cube
- a cube unfolded
- the net of a cube

What do you see?



Q1

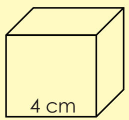
Revision: Calculate the volume of these cubes.
Answers:

	cm	cm^3	mm^3	Make a drawing of the net. Describe in words the geometric figures (2–D shapes) in the net.
a.	4 cm x 4 cm x 4 cm $l \times b \times h$ 4 cm × 4 cm × 4 cm = 64 cm^3	64	64 000	
b.	4 cm x 4 cm x 4 cm $l \times b \times h$ = 2,5 cm × 2,5 cm × 2,5 cm = 15,625 cm^3	15,625	15 625	

62 Surface area of a cube *continued*

Q2 Calculate the surface area of the following cubes.

Example: The surface area of a cube is length x length x total number of faces.



= l^2 x total faces
= $(4cm)^2$ x total faces
= 16 cm^2 x 6
= 96 cm^2

Answers:

a. $l^2 \times total\ faces$
$= (3\text{ cm})^2 \times total\ faces$
$= 9\text{ cm}^2 \times 6$
$= 54\text{ cm}^2$

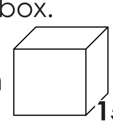
b. $l^2 \times total\ faces$
$= (2\text{ cm})^2 \times total\ faces$
$= 4\text{ cm}^2 \times 6$
$= 24\text{ cm}^2$

c. $l^2 \times total\ faces$
$= (4{,}5\text{ cm})^2 \times total\ faces$
$= 20{,}25\text{ cm}^2 \times 6$
$= 121{,}50\text{ cm}^2$

d. $l^2 \times total\ faces$
$= (1{,}8\text{ cm})^2 \times total\ faces$
$= 3{,}24\text{ cm}^2 \times 6$
$= 19{,}44\text{ cm}^2$

Q3 You want to make a gift box in the shape of a cube. The gift is 15 cm high and 9 wide. How much card board do you need to make a cube gift box.

Answers:
$15\text{ cm}^2 \times 6$
$= 225\text{ cm}^2 \times 6$
$= 1\,350\text{ cm}^2$



141

Problem solving

If a cube's surface area is 150 cm^2, what will the dimensions of the cube be?

Answer:
$\sqrt{(\mathbf{150}\text{ cm}^2 \div \mathbf{6})}$
$\sqrt{(\mathbf{25}\text{ cm}^2)}$
$= \mathbf{5}\text{ cm}$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

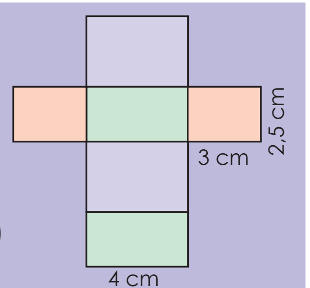
Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-61, 63-64
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

63 Surface area of rectangular prisms

Objectives
- Solve appropriate problems involving surface area, volume and capacity.
- Use appropriate formulae to calculate the surface area of a rectangular prism

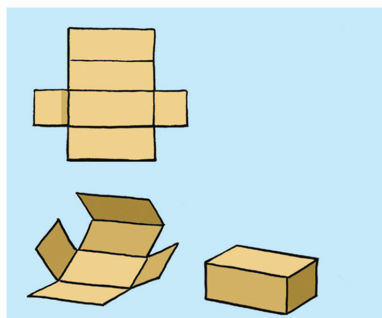
Dictionary
Surface area: The total area of the surface of a geometric object. Formula: The surface area of a prism = the sum of the area of all its faces.

Formula for the surface area of a rectangular prism = (2 x Length x Width) + (2 x Length x Height) + 2 x Width x Height)



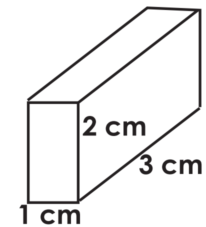
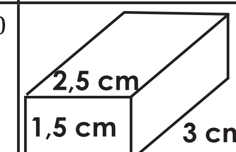
142

Introduction
Ask the learners to look at the three pictures, and ask them what do they see?
- a rectangular prism,
- a rectangular prism slightly unfolded and,
- net of a rectangular prism.



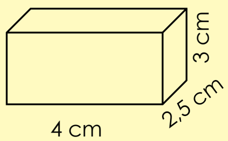
Q1

Revision: Calculate the volume of these cubes.
Answers:

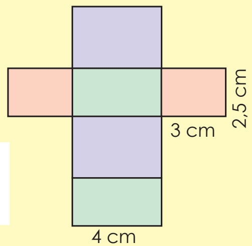
	cm	cm³	mm³	Make a drawing of the net. Describe in words the geometric figures (2-d shapes in the net.
	$l \times w \times h$ 1 cm × 3 cm × 2 cm = 6 cm^3	6	6 000	
	$l \times w \times h$ 2,5 cm × 3 cm × 1,5 cm = 11,25 cm^3	11,25	11 250	

63 Surface area of rectangular prisms *cont...*

Q2 Calculate the surface area of the following rectangular prisms:



Surface area of a rectangular prism = (2 x Length x Width) + (2 x Length x Height) + 2 x Width x Height)



Area of a rectangle:
4 cm x 2,5 cm =10 cm²
2 x 10 cm² = 20 cm²

Area of a rectangle:
4 cm x 3 cm = 12 cm²
2 x 12 cm² = 24 cm²

Area of a rectangle:
2,5 cm x 3 cm = 7,5 cm²
2 x 7.5 cm² = 15 cm²

20 cm² + 24 cm² + 15 cm²
= 59 cm²

Answers:

a. $5 \text{ cm} \times 3{,}1 \text{ cm} = 15{,}5 \text{ cm}^2$
$2 \times 15{,}5 \text{ cm}^2 = 31 \text{ cm}^2$

$3{,}1 \text{ cm} \times 2{,}4\ cm = 7{,}44 \text{ cm}^2$
$2 \times 7{,}44 \text{ cm}^2 = 14{,}88 \text{ cm}^2$

$5 \text{ cm} \times 2{,}4 \text{ cm} = 12 \text{ cm}^2$
$2 \times 12 \text{ cm}^2 = 24 \text{ cm}^2$

$31 \text{ cm}^2 + 14{,}88 \text{ cm}^2 + 24 \text{ cm}^2$
$= 69{,}88 \text{ cm}^2$

b. $7\text{cm} \times 2{,}2 \text{ cm} = 15{,}4 \text{ cm}^2$
$2 \times 15{,}4 \text{ cm}^2 = 30{,}8 \text{ cm}^2$

$2{,}2 \text{ cm} \times 1{,}8 \text{ cm} = 3{,}96 \text{ cm}^2$
$2 \times 3{,}96 \text{ cm}^2 = 7{,}92 \text{ cm}^2$

$7 \text{ cm} \times 1{,}8 \text{ cm} = 12{,}6 \text{ cm}^2$
$2 \times 12{,}6 \text{ cm}^2 = 25{,}2 \text{ cm}^2$

$30{,}8 \text{ cm}^2 + 7{,}92 \text{ cm}^2 + 25{,}2 \text{ cm}^2$
$= 63{,}92 \text{ cm}^2$

c. $6 \text{ cm} \times 4{,}5 \text{ cm} = 27 \text{ cm}^2$
$2 \times 27 \text{ cm}^2 = 54 \text{ cm}^2$

$4{,}5 \text{ cm} \times 3{,}7 \text{ cm} = 16{,}65 \text{ cm}^2$
$2 \times 16{,}65 \text{ cm}^2 = 33{,}30 \text{ cm}^2$

$6 \text{ cm} \times 3{,}7 \text{ cm} = 22{,}2 \text{ cm}^2$
$2 \times 22{,}2 \text{ cm}^2 = 44{,}4 \text{ cm}^2$

$54 \text{ cm}^2 + 33{,}30 \text{ cm}^2 + 44{,}4 \text{ cm}^2$
$= 131{,}70 \text{ cm}^2$

d. $4 \text{ cm} \times 1{,}8 \text{ cm} = 7{,}2 \text{ cm}^2$
$2 \times 7{,}2 \text{ cm}^2 = 14{,}4 \text{ cm}^2$

$1{,}8 \text{ cm} \times 2{,}3 \text{ cm} = 4{,}14 \text{ cm}^2$
$2 \times 4{,}14 \text{ cm}^2 = 8{,}28 \text{ cm}^2$

$4 \text{ cm} \times 2{,}3 \text{ cm} = 9{,}2 \text{ cm}^2$
$2 \times 9{,}2 \text{ cm}^2 = 18{,}4 \text{ cm}^2$

$41{,}4 \text{ cm}^2 + 8{,}28 \text{ cm}^2 + 18{,}4 \text{ cm}^2$
$= 41{,}08 \text{ cm}^2$

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Problem solving

If the surface area of a rectangular prism is 52 cm², what could its dimensions be?

Answer:
$2lw + 2lh + 2wh = 52 \text{ cm}^2$
$2(lw + lh + wh) = 52$
$lw + lh + wh = \dfrac{52}{2} = 26$
Let $l = 3$ and $w = 2$.
Then $(3 \times 2) + (3 \times h) + (2 \times h) = 26$
$6 + 3h + 2h = 26$
$6 + 5h = 26$
$h = 4$
So possible dimensions are length = 3 cm, width = 2 cm and height = 4 cm.
Test: $2(3 \times 2) + 2(3 \times 4) + 2(2 \times 4) = 12 + 24 + 16 = 52$

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-63
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

64 Surface area problem solving

Objectives

- Solve appropriate involving surface area, volume and capacity.

Dictionary

Surface area: The total area of the surface of a geometric object.
Formula: The surface area of a prism = the sum of the area of all its faces.

144

Introduction

Discuss with your learners how you will solve a problem. Write the keywords on the board. Go through this with your learners.

Before solving the problems, make notes on how you will solve a problem.

Revise the formulas for surface area.
Write them down.

Cube: ____________________

Rectangular prism: ____________________

Q1

How many square tiles (20 cm x 20 cm) are needed to cover the sides and base of a pool that is 10 m long, 6 m wide and 3 m deep?

Answers:

a. They want to tile a swimming pool (inside surface)
b. That the amount of tiles depends on the area of the pool.
c. What the area of the tile is, and the surface area of the pool sides and base.
d. 54 tiles are needed to tile the swimming pool.

Swimming pool:

Bottom
$10 \text{ m} \times 6 \text{ m}$
$= 60 \text{ m}^2$

Sides
$2(10 \text{ m} \times 3\text{m}) + 2(6 \text{ m} \times 3 \text{ m})$
$= 60 \text{ m}^2 + 36 \text{ m}^2$
$= 96 \text{ m}^2$

Total area $= 60 \text{ m}^2 + 96 \text{ m}^2$
$= 156 \text{ m}^2$

Tiles: $20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm}^2$
$\therefore$ The total area of the pool is

$$\frac{(156 \text{ m}^2)}{(400 \text{ cm}^2)} = \frac{1\,560\,000 \text{ cm}^3}{400 \text{ cm}^3}$$

$= 3\,900$ tiles

Topic: Size, surface area and volume of 3-D objects **Content links:** R12, R14, 56-63
Grade 8 links: R15, 87-91 **Grade 9 links:** R15, 100-104

64 Surface area problem solving *cont...*

Q2

Four cubes of ice with side lengths of 4 cm each are left to melt in a square box with sides 8 cm long. How high will the water rise when all of them have melted?

What is this problem all about?
Calculating the total volume of some solid cubes and working out how much of a solid square base container that volume will fill.

What do I know?
How to calculate the volume of a cube and how to calculate the area of a square.

What do I need to know more about?
The formulae are given.

Tackle the problem:
The volume of the four cubes of ice:
$4 \times (4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}) = 256 \text{ cm}^3$

The area of the square base of the box:
$8 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$

Height water will rise to: $\frac{256}{64} \text{ cm} = 4 \text{ cm}$

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Problem solving

You are a great problem solver. Share with a family member why you are a great problem solver. Why is maths helping you to become such a problem solver?

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

Teacher's notes

Teacher's notes

Teacher's notes

Grade 7 Book 2

Mathematics
Teacher Guide

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

ENGLISH

Book 2

Contents Grade 7 Book 2

Topic: Numeric and geometric patterns **Content links:** R9, 66-71, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

65 Numeric patterns: constant difference

Objectives

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns in physical or diagrammatic form with or without a constant difference

Dictionary

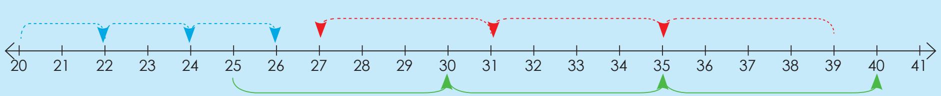
Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Difference: an equal difference between terms in a sequence, e.g.: 2, 4, 6, 8, ... (the constant difference added each time is 2)

2

Introduction

Discuss the patterns with the learners. Tell the learners that we describe patterns by using words like “adding” and “subtracting” or "multiplying by" a certain value.

Describe the patterns involving adding and subtraction shown in the number line below.

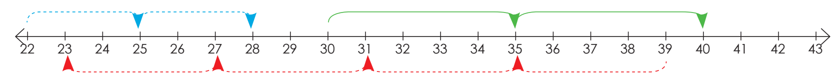


Adding 2: 20, 22, 24, 26 Subtracting 4: 39, 35, 31, 27 Adding 5: 25, 30, 35, 40

Q1

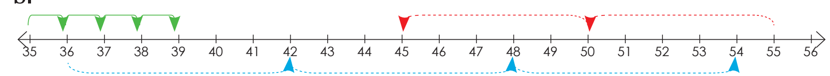
Describe each pattern.

Answers: a.



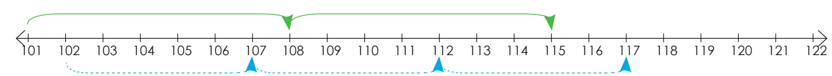
1. Adding 3: 22, 25, 28
2. Adding 5: 30, 35, 40
3. Subtracting 4: 39, 35, 31, 27, 23

b.



1. Adding 1: 35, 36, 37, 38, 39
2. Adding 5: 55, 50, 45
3. Adding 6: 36, 42, 48, 54

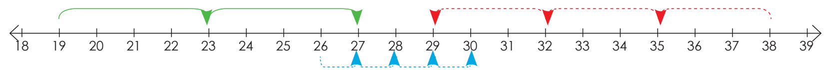
c.



1. Adding 7: 101, 108, 115
2. Adding 5: 102, 107, 112, 117

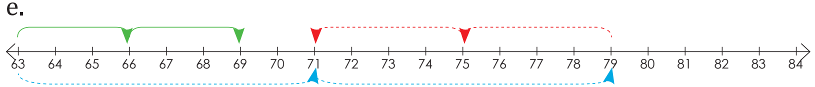
65 Numeric patterns: constant difference *continued*

d.



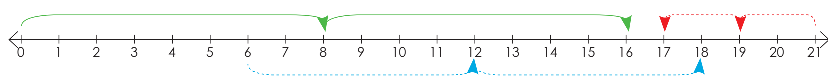
1. Adding 4: 19, 23, 27
2. Subtracting 3: 38, 35, 32, 29
3. Adding 1: 26, 27, 28, 29, 30

e.



1. Adding 3: 63, 66, 69
2. Subtracting 4: 79, 75, 71
3. Adding 8: 63, 71, 79

f.



1. Adding 8: 0, 8, 16
2. Subtracting 2: 21, 19, 17
3. Adding 6: 6, 12, 18

Q2

Describe the rule for each pattern.

Answers:

a. 6, 14, 22, 30
Adding 8
Counting in 8s

b. 2, 6, 10, 14, 18
Adding 4
Counting in 4s

c. 13, 10, 7, 4, 1
Subtracting 3
Counting in −3s

d. 8, 13, 18, 23, 28
Adding 5
Counting in 5s

e. 5, 9, 13, 17, 21
Adding 4
Counting in 4s

f. −20, −15, −10, −5, 0
Adding 5
Counting in 5s

g. 7, 18, 29, 40, 51
Adding 11
Counting in 11s

h. 1, 9, 17, 25, 33
Adding 8
Counting in 8s

i. 4, 5, 6, 7, 8
Adding 1
Counting in 1s

j. −6, −4, −2, 0, 2
Adding 2
Counting in 2s

3

Sharing

The rule is 'adding 11'. Start your pattern with 35.

Answer: 35, 46, 57, 68, 79, 90, 101, 112, 123

66 Numeric patterns: constant ratio

Objectives

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns represented in diagrammatic form not limited to constant difference or ratio

Dictionary

Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Ratio: the value of the ratio between each pair of numbers in a sequence remains the same - constant, e.g. as in the geometrical sequence: 2, 4, 8, 16, the ratio 2:4 = 4:8 = 8:16 is constant

4

Introduction

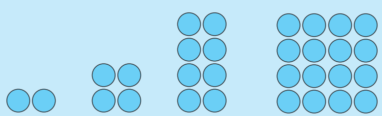
Describe the pattern.

2, 4, 8, 16, ...

Take your time and think carefully when you identify the pattern.

2 x2 4 x2 8 x2 16

Identify the **constant ratio** between consecutive terms. This pattern can be described in your own words as "multiplying the previous number by 2".



Can you still remember what constant ratio means?

Q1

Describe the pattern.

Example: 8, 32, 128, 512

Term 1: 8
Term 2: 32 = 8 x 4
Term 3: 128 = 32 x 4
Term 4: 512 = 128 x 4

Multiply the previous term by 4

Answers:

a. Term 1: 2
Term 2: 2 x 4 = 8
Term 3: 8 x 4 = 32
Term 4: 32 x 4 = 128
Term 5: 128 x 4 = 512

Multiply the previous term by 4.

b. Term 1: 4
Term 2: 4 x 3 = 12
Term 3: 12 x 3 = 36
Term 4: 36 x 3 = 108
Term 5: 108 x 3 = 324

Multiply the previous term by 3.

c. Term 1: 6
Term 2: 6 x 2 = 12
Term 3: 12 x 2 = 24
Term 4: 24 x 2 = 48
Term 5: 48 x 2 = 96

Multiply the previous term by 2.

Topic: Numeric and geometric patterns **Content links:** R9, 65, 67-71, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

66 Numeric patterns: constant ratio *continued*

d. Term 1: 8 Multiply the previous term by 5.
Term 2: 8 x 5 = 40
Term 3: 40 x 5 = 200
Term 4: 200 x 5 = 1 000
Term 5: 1 000 x 5 = 5 000

e. Term 1: 1 Multiply the previous term by 6.
Term 2: 1 x 6 = 6
Term 3: 6 x 6 = 36
Term 4: 36 x 6 = 216
Term 5: 216 x 6 = 1 296

f. Term 1: 3 Multiply the previous term by 3.
Term 2: 3 x 3 = 9
Term 3: 9 x 3 = 27
Term 4: 27 x 3 = 81
Term 5: 81 x 3 = 243

g. Term 1: 5 Multiply the previous term by 4.
Term 2: 5 x 4 = 20
Term 3: 20 x 4 = 80
Term 4: 80 x 4 = 320
Term 5: 320 x 4 = 1 280

h. Term 1: 7 Multiply the previous term by 6.
Term 2: 7 x 6 = 42
Term 3: 42 x 6 = 252
Term 4: 252 x 6 = 1 512

i. Term 1: 9 Multiply the previous term by 5.
Term 2: 9 x 5 = 45
Term 3: 45 x 5 = 225
Term 4: 225 x 5 = 1 125

j. Term 1: 10 Multiply the previous term by 2.
Term 2: 10 x 2 = 20
Term 3: 20 x 2 = 40
Term 4: 40 x 2 = 80
Term 5: 80 x 2 = 160

5

Problem solving

If the rule is "subtracting 9", give the first five terms of the sequence starting with 104.

Answer: ...; 104; 95; 86; 77; 68; 59; 50; 41; ...

Reflection questions

Did learners meet the objectives?

67 Numeric patterns: neither a constant difference nor a constant ratio

Objectives

- Investigate and extend numeric and geometric patterns that are neither a constant difference nor a constant ratio

Dictionary

Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Difference: an equal difference between terms in a sequence, e.g.: 2, 4, 6, 8, ... (the constant difference added each time is 2)
Constant Ratio: the value of the ratio between each pair of numbers in a sequence remains the same - constant, e.g. as in the geometrical sequence: 2, 4, 8, 16, the ratio 2:4 = 4:8 = 8:16 is constant

6

Introduction

What is the difference between constant difference and ratio?
- constant difference, e.g. 21, 23, 25, 27, ...
- constant ratio, e.g. 2, 4, 8, 16, ...

Take your time to figure out the pattern.

Describe the pattern.

1, 2, 4, 7, 11, 16, ...

What will the next three terms be, applying the identified rule?

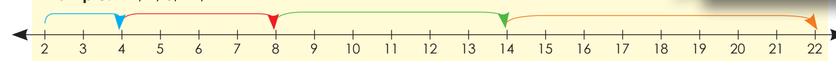
This pattern has neither a constant difference nor a constant ratio. It can be described as "increasing the difference between consecutive terms by one each time" or "adding one more than was added to get the previous term".

Q1

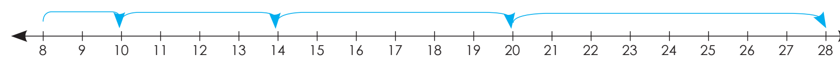
Describe the pattern and draw a number line to show each.
Answers; (Note that in some of the answers (b., g. h., and i.) the number lines have had sections of numbers shortened (...))

Add in multiples of 2, starting at 2 (so 2, 2 + 2 = 4, 4 + 4 = 8, 8 + 6 = 14, 14 + 8 = 22)

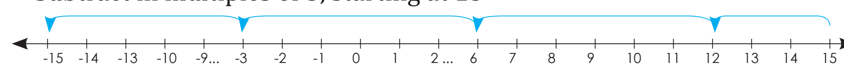
Example: 2, 4, 8, 14, 22



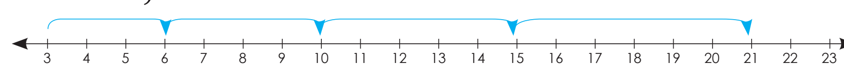
a. 8, 10, 14, 20, 28
Add in multiples of 2



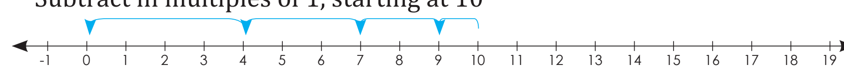
b. 15, 12, 6, −3, −15
Subtract in multiples of 3, starting at 15



c. 3, 6, 10, 15, 21
Add in multiples of 1, starting at 3 with 3 (so 3, 3 + 3 = 6, 6 + 4 = 10 and so on)



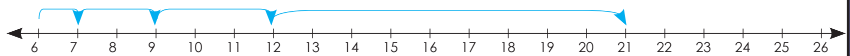
d. 10, 9, 7, 4, 0
Subtract in multiples of 1, starting at 10



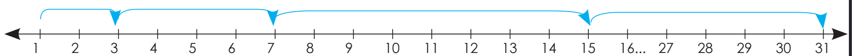
Topic: Numeric and geometric patterns **Content links:** R9, 65-66, 68-71, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

67 Numeric patterns: neither a constant difference nor a constant ratio

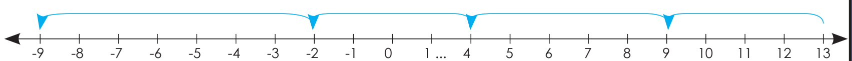
e. 6, 7, 9, 12, 16
Add in multiples of 1, starting at 6 (so 6, $6 + 1 = 7$, $7 + 2 = 9$, $9 + 3 = 12$ and so on)



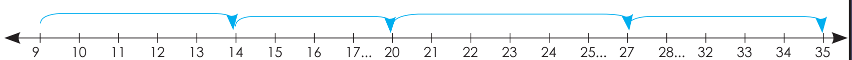
f. 1, 3, 7, 15, 31
Double the previous term and add 1



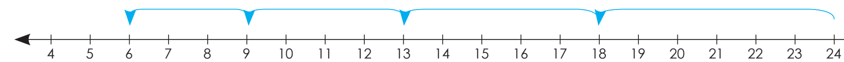
g. 13, 9, 4, –2, –9
Subtract multiples of 1, starting at 13 with 4 (so 13, $13 - 4 = 9$, $9 - 5 = 4$ and so on)



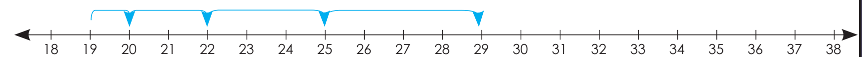
h. 9, 14, 20, 27, 35
Add 4 + multiples of 1, starting at 9 with 5 (so 9, $9 + 4 + 1 = 14$, $14 + 4 + 2 = 20$, and so on)



i. 24, 18, 13, 9, 6
Subtract 6 – multiples of 1, starting at 24 with –6 (so 24, $24 - 6 = 18$, $18 - 6 + 1 = 13$, $13 - 6 + 2 = 9$, $9 - 6 + 3 = 6$)



j. 19, 20, 22, 25, 29
Add multiples of 1, starting at 19 with 1



7

Problem solving

Create your own sequence without a constant ratio.

Learner's own answer. One possible answer is 1, 2, 4, 7, 11, 16
Add multiples of 1 starting at 1

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

Topic: Numeric and geometric patterns **Content links:** R9, 65-67, 69-71, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

68 Numeric patterns: tables

Objectives

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns represented in tables and of learners' own creation.

Dictionary

Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 4, 8, 16, 32, 64 ... (starts at 4 and each following term is 2 times the term before)

8

Introduction

Give a rule to describe the relationship between the numbers in this sequence: 2, 4, 6, 8, ... Use the rule to find the value of the tenth term.

Position in the sequence	1	2	3	4		10
Value of term	2	4	6	8		?

We can represent a sequence in a table.

The "tenth term" refers to position 10 in the number sequence. You have to find a rule in order to determine the value of the tenth term (rather than continuing the sequence up to the value of the tenth term). You should recognise that each term in the bottom row is obtained by doubling the number in the top row. So double 10 is 20. The tenth term is 20.

Q1

Find the value of the tenth term in each table and fill in the blank answer spaces showing how the value of each term is obtained. Answers:

a.

Position in the sequence	1	2	3	4		10
Term	4	8	12	16		40
	1 x 4	2 x 4	3 x 4	4 x 4		10 x 4

b.

Position in the sequence	1	2	3	4		10
Term	8	16	24	32		80
	1 x 8	2 x 8	3 x 8	4 x 8		10 x 8

c.

Position in the sequence	1	2	3	4		10
Term	12	24	36	48		120
	1 x 12	2 x 12	3 x 12	4 x 12		10 x 12

d.

Position in the sequence	1	2	3	4		10
Term	7	14	21	28		70
	1 x 7	2 x 7	3 x 7	4 x 7		10 x 7

e.

Position in the sequence	1	2	3	4		10
Term	5	10	15	20		50
	1 x 5	2 x 5	3 x 5	4 x 5		10 x 5

68 Numeric patterns: tables *continued*

Q2

Write down the rule and find the value of the final term in the table. Answers

Example: 5, 10, 15, 20. Position of the term × 5.

Position in the sequence	1	2	3	4		15
Term	5	10	15	20		75

a.

Position in the sequence	1	2	3	4		20
Term	10	20	30	40		**200**

Position of the term x 10

b.

Position in the sequence	1	2	3	4		28
Term	3	6	9	12		**84**

Position of the term x 3

c.

Position in the sequence	1	2	3	4		35
Term	8	16	24	32		**280**

Position of the term x 8

d.

Position in the sequence	1	2	3	4		100
Term	12	24	36	48		**1200**

e.

Position in the sequence	1	2	3	4		10
Term	15	30	45	60		**150**

Position of the term x 15

f.

Position in the sequence	1	2	3	4		50
Term	1	8	27	64		**125 000**

Position of term cubed (^{3})

9

Problem solving

Thabelo is building a model house from matches. If he uses 400 matches in the first section, 550 in the second and 700 in the third section, how many matches will he need to complete the fourth section, if the pattern continues?

Answer:

a. 4 + 150 = 550
550 + 150 = 700
750 + 150 = 900
900 matches

Topic: Numeric and geometric patterns **Content links:** R9, 65-68, 70-71, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

69 Number sequences and words

Objectives

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including learner's own patterns

Dictionary

Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 5, 10, 15, 20, ... (starts at 5 and adds 5 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 1, 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 1 and each following term is 2 times the term before)

10

Introduction

Look at this pattern:

4, 7, 10, 13, ...

If you consider only the relationship between consecutive terms, then you can continue the pattern ("adding 3 to previous number") up to the 20th term to find the answer. However, if you look for a relationship or rule between the term and the position of the term, you can predict the answer without continuing the pattern. Using number sequences can be useful for finding the rule.

First term:	4	= 3(1) + 1
Second term:	7	= 3(2) + 1
Third term:	10	= 3(3) + 1
Fourth term:	13	= 3(4) + 1

The number in brackets corresponds with the position of the term in the sequence.

What will the 20^{th} term be?

Q1

Look at the following sequences:
Describe the rule in your own words. Calculate the 20th term using a number sequence.

Answers:

a. Number sequence: 2, 5, 10, 17, ...
Rule: Square the position of the term, then add one
20^{th} term: (20)2 + 1
= 400 + 1
= 401

b. Number sequence: –8, –6, –4, –2, ...
Rule: Two multiplied by the position of the term, then subtract ten
15^{th} term: (2 x 15) – 10
= 30 – 10
= 20

c. Number sequence: –1, 2, 5, 8, ...
Rule: Three multiplied by the position of the term, then subtract four
12^{th} term: (3 x 12) – 4
= 36 – 4
= 32

d. Number sequence: 6, 9, 12, 15, ...
Rule: Three multiplied by the position of the term, then add three
19^{th} term: (3 x 19) + 3
= 57 + 3
= 60

69 Number sequences and words *continued*

e. Number sequence: –6, –2, 2, 6, ...
Rule: Four multiplied by the position of the term, then subtract ten
18^{th} term: (4 x 18) – 10
= 72 – 10 = 62

f. Number sequence: 7, 12, 17, 22, ...
Rule: Five multiplied by the position of the term, then add two
12^{th} term: (5 x 12) + 2
= 60 + 2 = 62

g. Number sequence: 2,5, 3,0, 3,5, 4,0, ...
Rule: 0,5 multiplied by the position of the term, then add two
21^{th} term: (0,5 x 21) + 2
= 10,5 + 2 = 12,5

h. Number sequence: –3, –1, 1, 3, ...
Rule: Two multiplied by the position of the term, then subtract five
15^{th} term: (2 x 15) – 5
= 30 – 5 = 25

i. Number sequence: 3, 7, 11, 15, ...
Rule: Four multiplied by the position of the term, then subtract one
14^{th} term: (4 x 14) – 1
= 56 – 1 = 55

j. Number sequence: 14, 24, 34, 44, ...
Rule: Ten multiplied by the position of the term, then add four
25^{th} term: (10 x 25) + 4
= 250 + 4 = 254

11

Problem solving

Miriam collects stickers for her sticker album. If she collects 4 stickers on day 1, 8 on day 2, 16 on day 3 and 32 on day 4, how many will she collect on day 5 if the pattern continues?

Helen spends 2 hours playing computer games on the first day of the school holidays. On the second day she plays for 5 hours and on the third day she plays for 8 hours. For how many hours will she play on the fourth day if she kept on playing in this pattern?

Answers:
Number sequence 4, 8, 16, 32, ...
Rule: Double the previous number
∴ Miriam will collect 64 stickers on day 5.

Number sequence: 2, 5, 8, ...
Rule: Add three to the previous number
∴ Helen will play for 11 hours on day 4.

Note that these problems are simple because we know the previous number, unlike in the problems in Question 1 where a more complex rule is needed to find the value of the nth position term.

Reflection questions
Did learners meet the objectives?

70 Geometric number patterns

Objectives

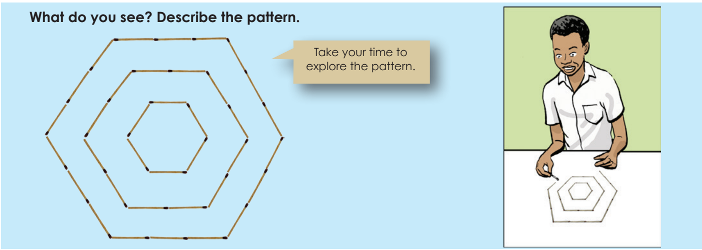
- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns in physical and diagrammatical form, tables and in different sequences.

Dictionary

Numeric pattern: a list of numbers that follow a certain sequence or pattern, e.g.: 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Difference: an equal difference between terms in a sequence, e.g.: 2, 4, 6, 8, ... (the constant difference added each time is 2)
Constant Ratio: the value of the ratio between each pair of numbers in a sequence remains the same - constant, e.g. as in the geometrical sequence: 2, 4, 8, 16, the ratio 2:4 = 4:8 = 8:16 is constant

Introduction

12

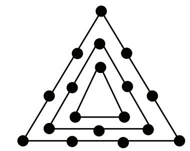


Q1

Create the first three terms of the following patterns with matchsticks and then draw it in your books. Complete the tables.

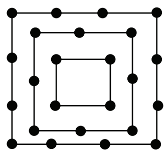
Answers:

a. Triangular pattern



Position of a triangle in pattern	1	2	3	4	5	6	7
Number of matches	3	6	12	24	48	96	192

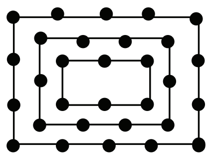
b. Square pattern.



Position of a square in pattern	1	2	3	4	5	6	7
Number of matches	4	8	16	32	64	108	216

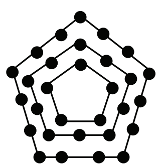
70 Geometric number patterns *continued*

c. Rectangular pattern.



Position of a rectangle in pattern	1	2	3	4	5	6	7
Number of matches	6	10	14	18	22	26	30

d. Pentagonal pattern.



Position of a pentagon in pattern	1	2	3	4	5	6	7
Number of matches	5	10	15	20	25	30	35

Q2

13

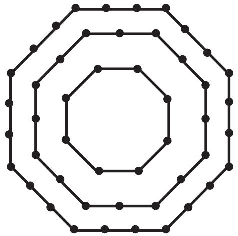
Look at worksheet 81-86 again. Explain and give examples of the following:

Answer: Learners' own answer. Possible answer.

Arithmetic number pattern. A number sequence that has a rule, where there is a constant difference. E.g. 2, 4, 6, 8, 10
+2 +2 +2 +2

Geometric number pattern. A number sequence that has a rule, where there is a constant ratio. E.g. 2, 4, 8, 16, 32
x2 x2 x2 x2

Problem solving:



1 x 8, 2 x 8, 3 x 8, ...
8, 16, 24, ...

Problem solving

Represent an octagonal number pattern. Answer: 8, 16, 24, 32

Reflection questions

Did learners meet the objectives?

Topic: Numeric and geometric patterns **Content links:** R9, 65-70, 114-117
Grade 8 links: 27, 105, 107-108 **Grade 9 links:** 27-28, 65-68

71 Numeric patterns: describe a pattern

Objectives

- Determine input values, output values or rules for patterns and describe and justify the general rules using formulae or in own words

Dictionary

Input value: a number that is inputted into a diagram that determines the output value, for example: □ + 5 = 16 where 11 is the input value

Output value: a number value that is the result of a diagram's input and process

14

Introduction

Look at the example and describe it.

Adding 4 to the previous term

3 7 11 15

4 times the position of the term – 1

Position in the sequence	1	2	3	4
Term	3	7	11	15

1 × 4 – 1 2 × 4 – 1 3 × 4 – 1 4 × 4 – 1

4(*n*) – 1, where n is the position of the term.

First term: 3 = 4(1) – 1
Second term: 7 = 4(2) – 1
Third term: 11 = 4(3) – 1
Fourth term: 15 = 4(4) – 1

Describe the sequence in different ways using the template.

a. 5, 11, 17, 23, ...

i) adding 6 to the previous term

5 11 17 23 29

ii)

Position in the sequence	1	2	3	4
Term	5	11	17	23

6 x 1 –1 6 x 2 –1 6 x 3 –1 6 x 4 –1

iii) $6(n) - 1$, where n is the position of the term.

First term: $6(1) - 1 = 5$ Second term: $6(2) - 1 = 11$

Third term: $6(3) - 1 = 17$ Fourth term: $6(4) - 1 = 23$

b. 5, 7, 9, 11, ...

i) adding 2 to the previous term

5 7 9 11 13

ii)

Position in the sequence	1	2	3	4
Term	3	5	7	9

2(1)+1 2(2)+1 2(3)+1 2(4)+1

iii) $2(n) + 1 =$, where n is the position of the term.

First term: $2(1) + 1 = 3$ Second term: $2(2) + 1 = 5$

Third term: $2(3) + 1 = 7$ Fourth term: $2(4) + 1 = 9$

71 Numeric patterns: describe a pattern *continued*

c. 10, 19, 28, 37, …

i) Adding 9 to the previous term

10 19 28 37 46

ii)

Position in the sequence	1	2	3	4
Term	10	19	28	37

$9(1)+1$ $9(2)+1$ $9(3)+1$ $9(4)+1$

iii) $9(n) + 1$, where n is the position of the term.

First term: $9(1) + 1 = 10$ Second term: $9(2) + 1 = 19$

Third term: $9(3) + 1 = 28$ Fourth term: $9(4) + 1 = 37$

d. 0, 4, 8, 12, …

i) Adding 4 to the previous term

0 4 9 12 16

ii)

Position in the sequence	1	2	3	4
Term	0	4	8	12

$1(4) -4$ $4(4) -4$ $3(4) -4$ $4(4) -4$

iii) $4(n) - 4$, where n is the position of the term.

First term: $4(1) - 4 = 0$ Second term: $4(2) - 4 = 4$

Third term: $4(3) - 4 = 8$ Fourth term: $4(4) - 4 = 12$

e. 14, 25, 36, 47, …

i) Adding 11 to the previous term

14 25 36 47 58

ii)

Position in the sequence	1	2	3	4
Term	14	25	36	47

$11(1)+3$ $11(2)+3$ $11(3)+3$ $11(4)+3$

iii) $11(n) + 3$, where n is the position of the term.

First term: $11(1) + 3 = 4$ Second term: $11(2) + 3 = 25$

Third term: $11(3) + 3 = 36$ Fourth term: $11(4) + 3 = 47$

Q2 Answer: The pattern follows the rule $9(n) + 1$

Q3 Answer: The pattern follows the rule $-9(n)+ 46$

Problem solving

What is the 30th term if the nth position is 8(n) – 7?

17

Answer: $8(30) - 7 = 240 - 7 = 233$

Reflection questions

Did learners meet the objectives?

72 Input values and output values

Objectives

- Determine and interpret input, output values and rules for patterns and relationships using flow diagrams and formula

Dictionary

Input value: a number that is inputted into a diagram that determines the output value, for example: $\square + 5 = 16$ where 11 is the input value

Output value: a number value that is the result of a diagram's input and process

18

Introduction

What do input and output mean? Make a drawing to show a real-life example.

Input → Process → Output

Q1

Complete the flow diagrams. Answers:

a.

1	× 6	6
5		30
7		42
9		54
12		72

b.

3	× 4	12
4		16
5		20
6		24
8		32

c.

12	× 8	96
11		88
9		72
6		48
5		40

d.

3	× 9	27
8		72
9		81
10		90
12		108

Q2

Use the given rule to calculate the value of b. Answers:

a.

a		b
2	$b = a \times 6$	12
3		18
4		24
5		30
6		36

b.

a		b
2	$b = a \times 10$	20
10		100
11		110
12		120
15		150

c.

x		y
10	$y = x - 9$	1
11		2
12		3
20		11
100		91

d.

r		s
4	$s = r + 11$	15
5		16
7		18
9		20
20		31

72 Input values and output values *continued*

Q3 Use the given rule to calculate the unknown variable. Answers:

a. $a = b \times 3 + 1$

b	a
1	4
2	7
6	19
10	31
11	34

b. g = h × 2 + 10

h	g
7	24
8	26
9	28
10	30
23	56

c. $x = y \times 2 + 4$

y	x
2	8
3	10
4	12
7	18
10	24

d. $m = n + 7 \times 2$

n	m
4	18
7	21
8	22
9	23
12	26

e. $y = x + 0{,}5$

x	y
0,2	0,7
0,3	0,8
0,4	0,9
0,7	1,2
0,9	1,4

f. $b = a \times 0{,}2$

a	b
0,01	0,002
0,06	0,012
0,1	0,02
0,3	0,06
0,5	0,1

g. $r = t \times 1 + 5$

t	r
5	10
6	11
7	13
8	12
10	15

h. $v = w \times 3 + 8$

w	v
11	41
9	35
8	32
6	26
3	17

19

Problem solving

Draw your own flow diagram where a = b + 7.
Draw your own flow diagram where a = b × 2 + 11.

Answers: Learners' own answers. Here are two examples.

$a = b + 7$

b	a
2	9
4	11
6	13
8	15
10	17

$h = g \times 2 + 11$

h	g
3	17
6	23
9	29
12	35
15	41

Reflection questions

Did learners meet the objectives?

73 Functions and relationships

Topic: Functions and relationships **Content links:** 48-51, 72, 118-119
Grade 8 links: R7, 29, 110 **Grade 9 links:** None

Objectives

- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented by formulae and number sentences.
- Determine input values, output values or rules for patterns and relationships using tables.

Dictionary

Input value: a number that is inputted into a diagram that determines the output value, for example: $\square$ + 5 = 16 where 11 is the input value

Output value: a number value that is the result of a diagram's input and process

Introduction

Discuss this:
The rule is $y = x + 5$

x	1	2	3	10	100
y	6	7	8	15	105

$y = 1 + 5 = 6$ | $y = 2 + 5 = 7$ | $y = 3 + 5 = 8$ | $y = 10 + 5 = 15$ | $y = 100 + 5 = 105$

Q1

20

Complete the tables.

Answers:

a. $y = x + 2$

x	2	4	6	8	10	20
y	**4**	**6**	**8**	**10**	**12**	**22**

b. $a = b + 7$

b	1	2	3	4	5	10
a	**8**	**9**	**10**	**11**	**12**	**17**

c. $m = n + 4$

n	3	4	5	6	7	10	100
m	**7**	**8**	**9**	**10**	**11**	**14**	**104**

d. $x = z \times 2$

z	2	3	4	5	6	7
x	**4**	**6**	**8**	**10**	**12**	**14**

e. $y = 2x - 2$

x	1	2	3	4	5	6	7
y	**0**	**2**	**4**	**6**	**8**	**10**	**12**

f. $m = 3n + 2$

n	1	5	10	20	25	100
m	**5**	**17**	**32**	**62**	**77**	**302**

Q2

What is the value of m and n?

a.

x	1	2	3	4		25	m	51
y	10	11	12	13		n	39	60

m: $y = x + 9$
$m = 25 + 9$
$\therefore m = 34$

n: $y = x + 9$
$39 = n + 9$
$\therefore n = 30$

b.

x	1	2	3	4		m	30	60
y	2	4	6	8		22	n	120

m: $y = 2x$
$m = 2(30)$
$\therefore m = 60$

n: $y = 2x$
$22 = 2n$
$\therefore n = 11$

c.

x	1	2	3	4		10	15	m
y	5	10	15	20		50	n	90

m: $y = 5x$
$m = 5(15)$
$\therefore m = 75$

n: $y = 5x$
$90 = 5n$
$\therefore n = 18$

d.

x	1	2	3	4		7	m	46
y	13	14	15	16		19	24	n

m: $y = 12 + x$
$m = 12 + 46$
$\therefore m = 58$

n: $y = 12 + x$
$24 = 12 + n$
$\therefore n = 12$

e.

x	1	2	3	4		6	10	m
y	3	6	9	12		18	n	60

m: $y = 3x$
$m = 3(10)$
$\therefore m = 30$

n: $y = 3x$
$60 = 3 + n$
$\therefore n = 20$

21

Problem solving

- What is the tenth term in the pattern? (3 × 7, 4 × 7, 5 × 7, ...)
- If $x = 2y + 9$ and $y = 2, 3, 4, 5, 6$ draw a table to show the values of x and y.

Answer:

a. $12 \times 7 = 84$

b.

x	2	3	4	5	6
y	**13**	**15**	**17**	**19**	**21**

Topic: Algebraic expressions **Content links:** 75-76, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

74 Algebraic expressions and equations

Objectives

- Identify variables and constant in given formulae or equations. Recognize and interpret rules or relationships represented in number form.

Dictionary

Expressions: Input numbers that include the process but have no output. Example: 5 + 4
Equations: Input numbers that have the process and include the result or output. Example: 5 + 4 = 9
Variables: Letters that are in the place of an unknown number
Example: $a + 9 = 17$ … therefore $a = 8$

22

Introduction

Compare the two examples.

5 + 4

5 + 4 = 9

What do you notice?

What is on the left–hand side of the equal sign?

The left–hand side is an **expression**, 5 + 4. It is equal to the value of the expression 9.

What is on the right–hand side?

5 + 4 = 9 is called an **equation**. The left–hand side of an equation is equal to the right–hand side.

An equation is a mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

Q1

Say whether it is an expression or an equation.

Example: 8 + 3 (It is an expression)
8 + 3 = 11 (It is an equation)

Answers:

a. 4 + 8 — Expression

b. 9 + 7 = 16 — Equation

c. 7 + 6 — Expression

d. 3 + 5 = 8 — Equation

e. 11 + 2 — Expression

f. 9 + 7 — Expression

Q2

Describe the following:

Example: 6 + 2 = 8
The **expression**, 6 + 2 is equal to the value of the expression on the right–hand side, 8.
6 + 2 = 8 is called an **equation**. The left–hand side of an equation equals the right–hand side.

Answers:

a. This is an expression, 9 + 1. It is equal to the value on the right-hand side, 10. 9 + 1 = 10 is called an equation. The left-hand side of an equation equals the right-hand side.
b. This is an expression, 3 + 5. It is equal to the value on the right-hand side, 8. 3 + 5 = 8 is called an equation. The left-hand side of an equation equals the right-hand side.

Topic: Algebraic expressions **Content links:** 75-76, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

74 Algebraic expressions and equations *continued*

d. This is an expression, $1 + 6$. It is equal to the value on the left-hand side, 7. $7 = 1 + 6$ is called an equation. The left-hand side of an equation equals the right-hand side.
e. This is an expression, $5 + 6$. It is equal to the value on the left-hand side, 11. $11 = 6 + 5$ is called an equation. The left-hand side of an equation equals the right-hand side.
f. This is an expression, $8 + 9$. It is equal to the value on the right-hand side, 17. $8 + 9 = 17$ is called an equation. The left-hand side of an equation equals the right-hand side.

Q3 Use the variable "a" to create 3 expressions of your own.

Example: $5 + a$

Answers: Learners' own answers. Here are three possible answers.
a. $12 + a$ b. $3 - a$ c. $7 + a$

Q4 Say whether it is an expression or an equation.

Example: $8 + a$ (It is an expression)
$8 + a = 11$ (It is an equation)

Answers:

a. $5 + a$ Expression
b. $6 + a = 12$ Equation
c. $7 + b = 8$ Equation
d. $8 + b$ Expression
e. $9 + a = 18$ Equation
f. $6 + a$ Expression

Q5 What would the value of "a" be in question 4b, and 4e?
Answer:
4b: $a = 6$
4e: $a = 9$

Q6 What would the value of "b" be in question 4c?
Answer: 4c: $b = 1$

23

Problem solving

Write an equation for the following. I have 12 sweets. In total Phelo and I have 18 sweets. How many sweets does Phelo have?

Answer:
$12 + a = 18$
$a = 18 - 12$
$= 6$
Phelo has 6 sweets

Reflection questions
Did learners meet the objectives?

75 Algebraic expressions

Topic: Algebraic expressions **Content links:** 74, 76, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

Objectives

- Identify variables and constant in given formulae or equations
- Recognize and interpret rules or relationships represented in number form.

Dictionary

Number Sequence: A list of numbers that follow a certain sequence or pattern. Example: 3, 6, 9, 12, 15, ... starts at 3 and adds 3 every time
Variable: A letter that represents an unknown number value.
Example: a + 5 = 8...........therefore a = 3.

24

Introduction

1, 3, 5, 7, 9 ...
Describe the rule of this number sequence in **words**.

Adding 2 to the previous term.

What does the rule $2n - 1$ mean in the number sequence 1, 3, 5, 7, 9, ...?

Position in sequence	1	2	3	4	5	n
Value of term	1	3	5	7	9	
	1st term: $2(1) - 1$	2nd term: $2(2) - 1$	3rd term: $2(3) - 1$	4th term: $2(4) - 1$	5th term: $2(5) - 1$	nth term: $2(n) - 1$

What is the rule as an **expression**? $2(n) - 1$

Q1

Describe the following in words.

Example: 4, 8, 12, 16, 20, ...
Adding 4 to the previous pattern

Answers:

a. 3; 6; 9; 12; ...
Adding 3 to the previous pattern

b. 10; 20; 30; 40; ...
Adding 10 to the previous pattern

c. 7; 14; 21; 28; ...
Adding 7 to the previous pattern

d. 6; 12; 18; 24; ...
Adding 6 to the previous pattern

e. 8; 16; 24; 32; ...
Adding 8 to the previous pattern

f. 5; 10; 15; 20; ...
Adding 5 to the previous pattern

Q2

Describe the following sequence using an expression.

Example: 4, 8, 12, 16, 20, ... **First term: 4(1) + 1** **The n^{th} term is 4 (n).**

Position in sequence	1	2	3	4	5	n
Value of term	4	8	12	16	20	

Answers:

a. 6; 11; 16; 21; ... First term: 5(1) + 1 The n^{th} term is 5(n) + 1

Position in sequence	1	2	3	4	5	n
Value of term	6	11	16	21	26	$5n + 1$

75 Algebraic expressions *continued*

Topic: Algebraic expressions **Content links:** 74, 76, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

b. 3; 5; 7; 9; 11; ... First term: 2(1) + 1 The nth term is $2(n) + 1$

Position in sequence	1	2	3	4	5	n
Value of term	3	5	7	9	11	$2(n) + 1$

c. 9; 15; 21; 27; ... First term: 6(1) + 3 The nth term is 6(n) + 3

Position in sequence	1	2	3	4	5	n
Value of term	9	15	21	27	33	$6(n) + 3$

Q3

What does the rule mean? Use the same values for position as in the example.

Example: The rule 2n – 1 means the following number sequence: 1, 3, 5, 7, 9 ...

Position in sequence	1	2	3	4	5	n
Value of term	1	3	5	7	9	

Answers:

a. Rule $3n - 1$

Position in sequence	1	2	3	4	5	n
Value of term	2	5	8	11	14	$3(n) - 1$

b. Rule $4n - 3$

Position in sequence	1	2	3	4	5	n
Value of term	1	5	9	13	17	$4(n) - 3$

c. Rule $6n - 2$

Position in sequence	1	2	3	4	5	n
Value of term	4	10	16	22	28	$6(n) - 2$

d. Rule $5n - 5$

Position in sequence	1	2	3	4	5	n
Value of term	0	5	10	15	20	$5(n) - 5$

e. Rule $7n - 4$

Position in sequence	1	2	3	4	5	n
Value of term	3	10	17	24	31	$7(n) - 4$

25

Problem solving

Write an algebraic expression for the following: Sipho built 3 times more puzzles than I did last holiday.

Answer: $3n$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

76 More algebraic expressions

Topic: Algebraic expressions **Content links:** 74-75, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

Objectives

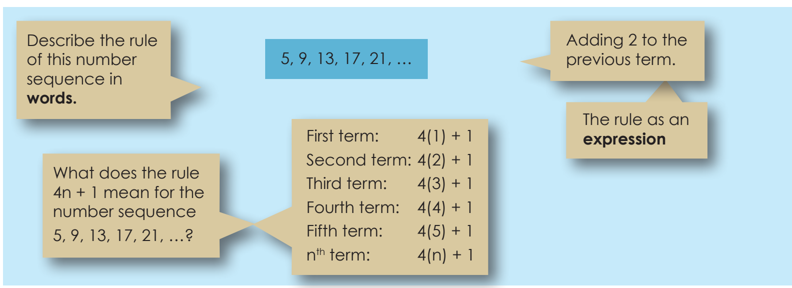
- Identify variables and constant in given formulae or equations
- Recognize and interpret rules or relationships represented in number form.

Dictionary

Number Sequence: A list of numbers that follow a certain sequence or pattern.Example: 3, 6, 9, 12, 15, ... starts at 3 and jumps 3 every time
Variable: A letter that represents an unknown number value.
Example: a + 5 = 8...........therefore a = 3.

26

Introduction



Q1

Describe the following in words.

Example: 2, 6, 10, 14, 18, ...
Adding 4 to the previous number

Answers:

a. 3; 5; 7; 9; ...
Adding 2 to the previous number

b. 5; 10; 15; 20; ...
Adding 5 to the previous number

c. 21; 18; 15; 12; ...
Subtracting 3 from the previous number

d. 99; 98; 97; 96; ...
Subtracting 2 from the previous number

e. 4; 8; 12; 16; ...
Adding 4 to the previous number

f. 7; 14; 21; 28; ...
Adding 7 to the previous number

Q2

Describe the following sequence using an expression.

Example: 2, 6, 10, 14, 18, ...
4 (n)–2 since 1st term: 4(1) –2; 2nd term: 4(2) – 2; Third term 4(3) – 2; ...

a. 2; 4; 6; 8; 10; ...

$2(n)$ since 1^{st} term: $2(1)$;
2^{nd} term: $2(2)$;
3^{rd} term: $2(3)$; ...

b. 3; 5; 7; 9; 11; ...

$2(n)+1$ since 1^{st} term: $2(1)+1$;
2^{nd} term: $2(2)+1$;
3^{rd} term: $2(3)+1$; ...

More algebraic expressions *cont...*

Topic: Algebraic expressions **Content links:** 74-75, 120-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

c. 8; 16; 24; 32; …

$8(n)$ since 1st term: $8(1)$; 2nd term: $8(2)$; 3rd term: $8(3)$; ...

d. 5; 10; 15; 20; …

$5(n)$ since 1st term: $5(1)$; 2nd term: $5(2)$; 3rd term: $5(3)$; ...

Q3

If the rule is ___, what could the sequence be? Create five possible answers for each.

Answers:

a. "Adding 7"

$1 + 7 = 8$
$2 + 7 = 9$
$3 + 7 = 10$
$4 + 7 = 11$
$5 + 7 = 12$

b. "Subtracting 9"

$10 - 9 = 1$
$20 - 9 = 11$
$30 - 9 = 21$
$40 - 9 = 31$
$50 - 9 = 41$

c. "Adding 5"

$1 + 5 = 6$
$2 + 5 = 7$
$3 + 5 = 8$
$4 + 5 = 9$
$5 + 5 = 10$

d. "Subtracting 8"

$12 - 8 = 4$
$13 - 8 = 5$
$14 - 8 = 6$
$15 - 8 = 7$

e. "Adding 3" "Subtracting 4"

$10 + 3 - 4 = 9$
$11 + 3 - 4 = 10$
$12 + 3 - 4 = 11$
$13 + 3 - 4 = 12$

27

Problem solving

If the rule is "adding $\frac{1}{4}$", what could the sequence be? Create five possible answers.

Answer: accept different answers given by learners.
Possible answers:

1. $1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}, 2$
2. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}$
3. $5, 5\frac{1}{4}, 5\frac{1}{2}, 5\frac{3}{4}, 6$
4. $-8, -7\frac{3}{4}, -7\frac{2}{4}, -7\frac{2}{4}, -7$
5. $-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}$

Reflection questions

Did learners meet the objectives?

77 Algebraic equations

Topic: Algebraic equations **Content links:** 78-79, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Solve and complete number sentences by inspection and trial and improvement
- Analyse and interpret number sentences that describe a given situation.

Dictionary

Variable: A letter that represents an unknown number value.
Example: a + 5 = 8............therefore a ,the variable = 3.
Operation: Calculation by mathematical methods.

28

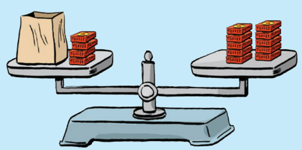
Introduction

Look at and describe:

variable, constants

$x + 23 = 45$

operation, equal sign



Read and answer:

Imagine that on the right–hand side of this balance scale there are 10 objects of equal mass, and on the left–hand side there are 4 similar objects and an unknown number of other objects in a bag. The scale is balanced; therefore, we know that there must be an equal mass on each side of the scale.

Explain how you would find out how many objects there are in the bag.

Q1 Solve for x.

Example:
$x + 5 = 9$
$x + 5 - 5 = 9 - 5$
$x = 4$

Answers:

a. $x + 12 = 30$
$x + 12 - 12 = 30 - 12$
$x = 18$

b. $x + 8 = 14$
$x + 8 - 8 = 14 - 8$
$x = 6$

c. $x + 17 = 38$
$x + 17 - 17 = 38 - 17$
$x = 21$

d. $x + 20 = 55$
$x + 20 - 20 = 55 - 20$
$x = 35$

e. $x + 25 = 30$
$x + 25 - 25 = 30 - 25$
$x = 5$

f. $x + 18 = 26$
$x + 18 - 18 = 26 - 18$
$x = 8$

Q2 Solve for x.

Example:
$x - 5 = 2$
$x - 5 + 5 = 2 + 5$
$x = 7$

Topic: Algebraic equations **Content links:** 78-79, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Answers:

a. $x - 7 = 5$
$x - 7 + 7 = 5 + 7$
$x = 12$

b. $x - 3 = 1$
$x - 3 + 3 = 1 + 3$
$x = 4$

c. $x - 15 = 12$
$x - 15 + 15 = 12 + 15$
$x = 27$

d. $x - 17 = 15$
$x - 17 + 17 = 15 + 17$
$x = 32$

e. $x - 23 = 20$
$x - 23 + 23 = 20 + 23$
$x = 43$

f. $x - 28 = 13$
$x - 28 + 28 = 13 + 28$
$x = 41$

Q3

Solve for x.

Example: $x + 4 = -7$
$x + 4 - 4 = -7 - 4$
$x = -11$

Answers:

a. $x + 3 = -15$
$x + 3 - 3 = -15 - 3$
$x = -18$

b. $x + 7 = -12$
$x + 7 - 7 = -12 - 7$
$x = -19$

c. $x + 2 = -5$
$x + 2 - 2 = -5 - 2$
$x = -7$

d. $x + 5 = -15$
$x + 5 - 5 = -15 - 5$
$x = -20$

e. $x + 12 = -20$
$x + 12 - 12 = -20 - 12$
$x = -32$

f. $x + 10 = -25$
$x + 10 - 10 = -25 - 10$
$x = -35$

29

Problem solving

Write an equation for the following and solve it.

Jason read 7 books and Gugu read 11 books. How many books did they read altogether?

Answer: $7 + 11 = 18$

Rebecca and her friend read 29 books altogether. Rebecca read 14 books. How many books did her friend read?

Answer: $29 - 14 = 15$

Bongani buys 12 new CDs and Sizwe buys 14. How many CDs did they buy together?

Answer: $12 + 14 = 26$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

78 More algebraic equations

Topic: Algebraic equations **Content links:** 77, 79, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Solve and complete number sentences by inspection and trial and improvement
- Analyse and interpret number sentences that describe a given situation.

Dictionary

Variable: A letter that represents an unknown number value.
Example: a + 5 = 8...........therefore a ,the variable = 3.
Inverse Operation: An opposite method of calculation.
Example: inverse operation of adding is subtracting.

30

Introduction

$2x = 30$

What does $2x$ mean?

(2x means 2 multiplied by x)

What is the inverse operation of multiplication?

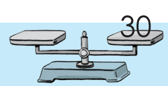
Division

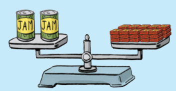
We need to divide $2x$ by 2 to solve for x.

$\frac{2x}{2} = \frac{30}{2}$

$x = 15$

Remember you need to keep the two sides of the equation balanced. What you do on the one side of the equal sign, you must do on the other side as well.





Q1

Solve for x.

Example: $3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

Answers:

a. $5x = 20$

$\frac{5x}{5} = \frac{20}{5}$

$x = 4$

b. $2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

c. $2x = 18$

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

d. $4x = 48$

$\frac{4x}{4} = \frac{48}{4}$

$x = 12$

e. $3x = 27$

$\frac{3x}{3} = \frac{27}{3}$

$x = 9$

f. $5x = 30$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

g. $10x = 100$

$\frac{10x}{10} = \frac{100}{10}$

$x = 10$

h. $9x = 81$

$\frac{9x}{9} = \frac{81}{9}$

$x = 9$

i. $15x = 45$

$\frac{15x}{15} = \frac{45}{15}$

$x = 3$

j. $7x = 14$

$\frac{7x}{7} = \frac{14}{7}$

$x = 2$

78 More algebraic equations *continued*

Topic: Algebraic equations **Content links:** 77, 79, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Q2

Solve for $\boldsymbol{x}$. Answers:

a. $7x - 2 = 12$
$7x - 2 + 2 = 12 + 2$
$7x = 14$
$\frac{7x}{7} = \frac{14}{7}$
$x = 2$

b. $4x - 4 = 12$
$4x - 4 + 4 = 12 + 4$
$4x = 16$
$\frac{4x}{4} = \frac{16}{6}$
$x = 4$

c. $3x - 1 = 2$
$3x - 1 + 1 = 2 + 1$
$3x = 3$
$\frac{3x}{3} = \frac{3}{3}$
$x = 1$

d. $2x - 1 = 7$
$2x - 1 + 1 = 7 + 1$
$2x = 8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$

e. $5x - 3 = 17$
$5x - 3 + 3 = 17 + 3$
$5x = 20$
$\frac{5x}{5} = \frac{20}{5}$
$x = 4$

f. $5x - 7 = 13$
$5x - 7 + 7 = 13 + 7$
$5x = 20$
$\frac{5x}{5} = \frac{20}{5}$
$x = 4$

g. $6x - 5 = 25$
$6x - 5 + 5 = 25 + 5$
$6x = 30$
$\frac{6x}{6} = \frac{30}{6}$
$x = 5$

h. $9x - 8 = 82$
$9x - 8 + 8 = 82 + 8$
$9x = 90$
$\frac{9x}{9} = \frac{90}{9}$
$x = 10$

i. $8x - 7 = 49$
$8x - 7 + 7 = 49 + 7 + 2$
$8x = 56$
$\frac{8x}{8} = \frac{56}{8}$
$x = 7$

j. $3x - 2 = 16$
$3x - 2 + 2 = 16 + 2$
$3x = 18$
$\frac{3x}{3} = \frac{18}{3}$
$x = 6$

31

Problem solving

Create an equation and solve it. How fast can you do it? Answers:

Two times y equals sixteen.

a. $2y = 16$
$\frac{2y}{2} = \frac{16}{2}$
$y = 8$

Sixteen times b equals four.

d. $16b = 4$
$\frac{16b}{16} = \frac{4}{16}$
$b = \frac{1}{4}$

Nine times q equals eighty-one.

g. $9q = 81$
$\frac{9q}{9} = \frac{81}{9}$
$q = 9$

Five times c equals sixty-five.

b. $5c = 65$
$\frac{5c}{5} = \frac{65}{5}$
$c = \frac{13}{5}$

Eight times t equals eighty.

e. $8t = 80$
$\frac{8t}{t} = \frac{80}{0}$
$t = 10$

Five times y equals one-hundred.

h. $5y = 100$
$\frac{5y}{5} = \frac{100}{5}$
$y = 20$

Eight times x equals sixteen.

c. $8x = 16$
$\frac{8x}{8} = \frac{16}{8}$
$x = 2$

Three times d equals thirty-nine.

f. $3d = 39$
$\frac{3d}{3} = \frac{39}{3}$
$d = 13$

Seven times a equals twenty-one.

i. $7a = 21$
$\frac{7a}{7} = \frac{21}{3}$
$a = 3$

Reflection questions
Did learners meet the objectives?

79 Algebraic equations in context

Topic: Algebraic equations **Content links:** 77-78, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Write a number sentence to describe a problem situation
- Analyse and interpret number sentences that describe a given situation

Dictionary

Perimeter: Distance right around an object or shape.
Area: The space an object occupies determined by multiplying two of the object's characteristics such as length and breadth.

32

Introduction

What do the following equations mean?

$P = 4l$ — The perimeter of a square is 4 times the length.

$P = 2l + 2b$ — The perimeter of a rectangle is 2 times the length plus 2 times the breadth.

$A = l^2$ — The area of a square is the length squared.

$A = l \times b$ — The area of a rectangle is length times breadth.

Note that you did perimeter and area in the first and second terms of grade 7.

Q1

Substitute and calculate.

Example: If $y = x^2 + 2$, calculate y when $x = 4$

$y = 4^2 + 2$

$y = 16 + 2$

$y = 18$

a. $y = x^2 + 2; x = 4$
$y = a^2 + 2$
$= (4)^2 + 2$
$= 16 + 2$
$= 18$

b. $y = b^2 + 10; b = 1$
$y = b^2 + 10$
$= (1)^2 + 10$
$= 1 + 10$
$= 11$

c. $y = a^2 + 4; a = 4$
$y = a^2 + 4$
$= (4)^2 + 4$
$= 16 + 4$
$= 20$

d. $y = r^2 + 3; r = 5$
$y = r^2 + 3$
$= (5)^2 + 3$
$= 25 + 3$
$= 28$

e. $y = p^2 + 7; p = 6$
$y = p^2 + 7$
$= (6)^2 + 7$
$= 36 + 7$
$= 43$

f. $y = c^2 + 7; c = 7$
$y = c^2 + 7$
$= (7)^2 + 7$
$= 49 + 7$
$= 56$

Q2

Calculate the following:

Example: What is the perimeter of a rectangle if the length is 2 cm and the breadth is 1,5 cm?

P = 2l + 2b

P = 2(2 cm) + 2(1,5 cm)

P = 4 cm + 3 cm

P = 7 cm

Answers:

a. The perimeter of a rectangle where the breath equals 2,2 cm and the length equals 2,5 cm.
P $= 2l + 2b$
$= 2(2{,}5 \text{ cm}) + 2(2{,}2 \text{ cm}) = 9{,}4 \text{ cm}$

b. The area of a square if the breath equals 3,5 cm.
$A = l^2$
$= (3{,}5 \text{ cm})^2 = 12{,}25 \text{ cm}^2$

79 Algebraic equation in context *cont...*

Topic: Algebraic equations **Content links:** 77-78, 123-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

c. The perimeter of a square if the breath equals 4,2 cm.
$P = 4\,l$
$= 4(4,2 \text{ cm})$
$= 16,8 \text{ cm}$

d. The area of a rectangle if the length = 3,5 cm and breadth = 2,5 cm.
$P = l \times b$
$= 3,5 \text{ cm} \times 2,5 \text{ cm}$
$= 8,75 \text{ cm}$

e. The area of a square if the length = 5 cm.
$P = l^2$
$= (5 \text{ cm})^2$
$= 25 \text{ cm}^2$

f. The perimeter of a rectangle if the breadth = 4,3 cm and length = 8,2 cm.
$P = 2l + 2b$
$= 2(8,2 \text{ cm}) + 2(4,3 \text{ cm})$
$= 16,4 \text{ cm} + 8,6 \text{ cm} = 25 \text{ cm}$

g. The perimeter of a square if the length = 2,6 cm
$P = 4l$
$= 4\ (2,6 \text{ cm})$
$= 10,4 \text{ cm}$

h. The perimeter of a rectangle if the breath = 8,5 and the length = 12,4
$P = 2l + 2b$
$= 2(12,4 \text{ cm}) + 2(8,5 \text{ cm})$
$= 24,8 \text{ cm} + 17 \text{ cm}$
$= 41,8 \text{ cm}$

i. The area of a rectangle if the breath = 10,5 and length = 15,5.
$A = l \times b$
$= 15,5 \text{ cm} \times 10,5 \text{ cm}$
$= 162,75 \text{ cm}^2$

33

j. The perimeter of a rectangle if the breadth is 3,5 cm and the length is 6,7 cm.
$P = 2l + 2b$
$= 2(6,7) + 2(3,5)$
$= 13,4 + 7 = 20,4 \text{ cm}$

Problem Solving

Write an equation and then solve it for each of these:

- What is the perimeter of a rectangular swimming pool if the breadth is 12 m and the length is 16 m?
- Work out the area of a square if one side is equal to 5,2 cm.
- What is the perimeter of a rectangle if the length is 5,1 cm and the breadth is 4,9 cm.
- Establish the area of your rectangular bedroom floor for new tiles is the length is 4,5 m and the breadth is 2,8 m.

Answers:

a. $P = 2l + 2b$
$= 2(16 \text{ m}) + 2(12 \text{ m})$
$= 32 \text{ m} + 24 \text{ m} = 56 \text{ m}$

b. $A = l^2$
$= (5,2 \text{ cm})^2$
$= 27,04 \text{ cm}^2$

c. $P = 2l + 2b$
$= 2(5,1 \text{ cm}) + 2(4,9 \text{ cm})$
$= 10,2 \text{ cm} + 9,8 \text{ cm} = 20 \text{ cm}$

d. $A = l \times b$
$= 4,5 \text{ m} \times 2,8 \text{ m}$
$= 12,6 \text{ m}^2$

Topic: Graphs **Content links:** 81-85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

80 Interpreting graphs: temperature and time graphs

Objectives

- Analyse and interpret graphs of problem situations, with a special focus on the following trends and features:
  - – Constant, increasing or decreasing.
  - – Linear or non-linear

Dictionary

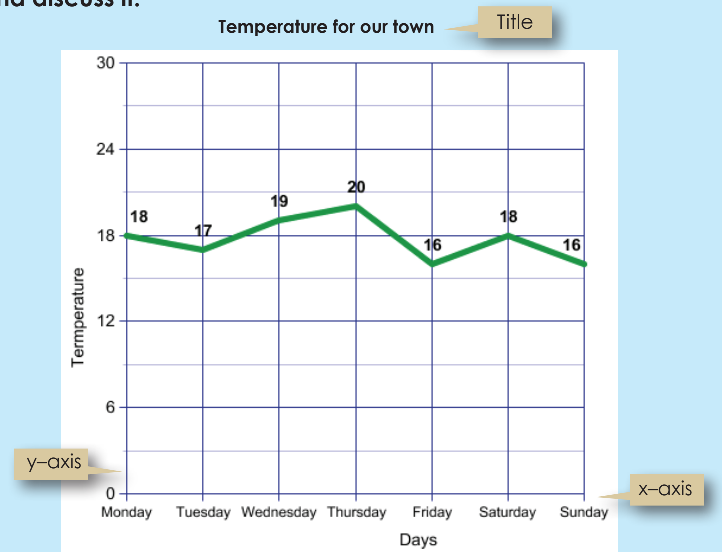
Linear: a graph that is a straight line
Non-linear: a graph with a curve(s)
Increasing: a graph that slopes upwards from left to right (e.g. from (2,3) to (5,7))
Decreasing: a graph that slopes downwards from left to right (e.g. from(2,3) to (5,7))
Maximum: point on a graph where the graph changes from increasing to decreasing (highest point on the graph)
Minimum: point on the graph where the graph changes from decreasing to increasing (lowest point on the graph)
X-axis: the horizontal line on the graph (left-right) through zero
Y-axis: the vertical line on the graph (top-bottom) through zero

34

Introduction
Ask the learners if they would make any changes or add anything to the graph.

Look at the graph and discuss it.

Would you make any changes or add anything to the graph?



80 Interpreting graphs: temperature and time graphs *continued*

Q1

a.i. 19,5 ^{0}C
ii. 23 ^{0}C
iii. 24 ^{0}C
iv. 32 ^{0}C
v. 34 ^{0}C
b. 27 ^{0}C
c. The temperature drops ∴ It is colder the fewer chirps the lower the temperature.
d. If the temperature increases the chirping increases. If the chirping decreases the temperature also decrease.

Q2

Average temperature per annum for Johannesburg, Cape Town and Durban.

Answers:

a. i. 22 ^{0}C
ii. 16 ^{0}C
iii. 22 ^{0}C
iv. 22 ^{0}C
v. 18 ^{0}C

b. i. 10 ^{0}C
ii. 11 ^{0}C
iii. 9 ^{0}C
iv. 20 ^{0}C
v. 7 ^{0}C

c. i. 26 ^{0}C – 22 ^{0}C = 4 ^{0}C
ii. 24 ^{0}C – 21^{0}C = 3 ^{0}C
iii. 18 ^{0}C – 18 ^{0}C = 0 ^{0}C
iv. 24 ^{0}C – 23 ^{0}C = 1 ^{0}C
v. 22^{0}C – 21^{0}C = 1 ^{0}C
d. During the summer months the temperature increases and during winter months the temperature decreases.

35

Problem solving

What is the difference between the minimum and maximum temperatures of Durban, Cape Town and Johannesburg in December? Which province would you most like to visit in December. Why?

Answer:
Johannesburg: December min: 14 ^{0}C, max: 26 ^{0}C, difference 12^{0}C
Durban: December min: 7 ^{0}C, max: 26 ^{0}C, difference 19 ^{0}C
Cape Town: December min: 14 ^{0}C, max: 24 ^{0}C, difference 10 ^{0}C
Durban: The temperature is more constant, minimum is warmer

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Graphs **Content links:** 80, 82-85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

81 Interpreting graphs: rainfall and time graphs

Objectives

- Analyse and interpret graphs of problem situations, with a special focus on the following trends and features:
  – Constant, increasing or decreasing
  – Linear or non-linear

Dictionary

Linear: a graph that is a straight line
Non-linear: a graph with a curve(s)
Increasing: a graph that slopes upwards from left to right (e.g. from (2,3) to (5,7))
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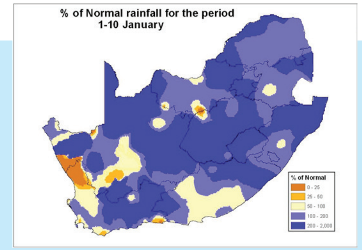
X-axis: the horizontal line on the graph (left-right) through zero
Y-axis: the vertical line on the graph (top-bottom) through zero

38

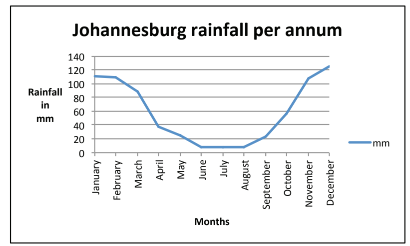
Introduction

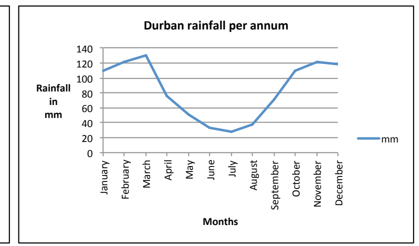
How do you read information from and interpret the graphs on this page.

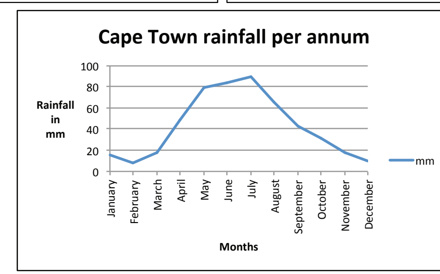


Q1

Look at the graphs and answer the following questions:







Topic: Graphs **Content links:** 80, 82-85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

81 Interpreting graphs: rainfall and time graphs *continued*

Answers:

a. Rainfall per annum
b. Months of the year
c. Rain in mm
d. Durban 117 mm
e. Johannesburg 39 mm
f. Cape Town, smaller chance of rain
g. Johannesburg, the city has a high rainfall in December
h. Cape Town. Winter months rainfall between 60 and 90 mm
i. Johannesburg and Durban high rainfall average during summer months
j. Johannesburg: There is an increase in the rainfall during summer and a decrease in winter.
Durban: There is an increase in rain during summer and decrease during winter months.
Cape Town: Has a increase in rain during the winter months and a decrease during summer.

Oral questions

After the learners have completed Question 1 ask them the following questions:

- What do you think linear means?
- Are the graphs on page 38 linear or non-linear?
- What does increasing or decreasing mean on these three graphs?

Q2 Use the graphs to complete the following tables. Answers:

Months	Average rainfall		
	Johannesburg	Durban	Cape Town
January	110 mm	110 mm	15 mm
February	100 mm	120 mm	9 mm
March	80 mm	130 mm	19 mm
April	40 mm	80 mm	45 mm
May	25 mm	55 mm	80 mm
June	10 mm	35 mm	85 mm
July	10 mm	30 mm	90 mm
August	10 mm	40 mm	65 mm
September	20 mm	70 mm	40 mm
October	60 mm	110 mm	30 mm
November	110 mm	120 mm	20 mm
December	125 mm	120 mm	10 mm

39

Investigate the rainfall in your area.

What is the highest rainfall per year for your town? Which month? Keep a record during a rainy month and draw a graph to represent the data.

Answers: Example of possible answers:

1. 125 mm
2. December
3. Practical activity for learner

82 Drawing graphs

Topic: Graphs **Content links:** 80-81, 83-85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

Objectives

- Analyse and interpret graphs of problem situations, with a special focus on the following trends and features
  – Constant, increasing or decreasing
  – Linear or non-linear

Dictionary

Linear: a graph that is a straight line
Non-linear: a graph with a curve(s)
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Minimum: point on the graph where the graph changes from decreasing to increasing (lowest point on the graph)
X-axis: the horizontal line on the graph (left-right) through zero
Y-axis: the vertical line on the graph (top-bottom) through zero

40

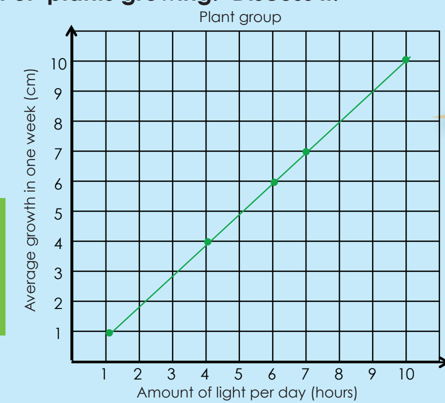
Introduction

Sam kept this record of plants growing. Discuss it.

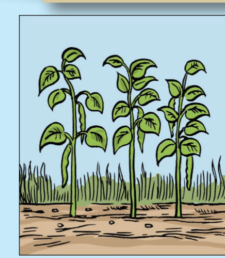
Would you make any changes or add anything to the graph?

DEFINITION

Linear equation: The graph from a linear equation is a straight line.



Is this graph a decreasing or increasing graph?



Ask the learners:

- Is this graph linear or non-linear?
- Is the graph increasing or decreasing?

Explain why.

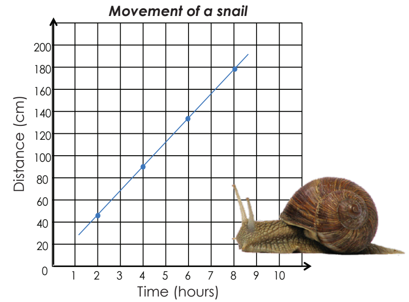
- Where is the x-axis?
- Where is the y-axis?

82 Drawing graphs *continued*

Topic: Graphs **Content links:** 80-81, 83-85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

Q1

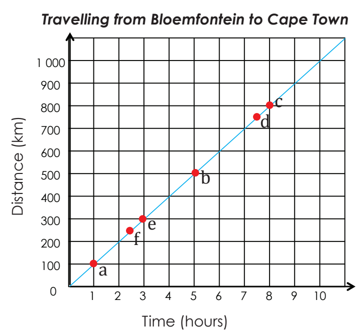
Use the graph to answer the following questions on the movement of a snail.
Answers:



a. 180 cm
b. 90 cm: Half way between 80 and 100 on the y-axis = 90 cm
c. 135 cm: Three-quarters of the way between 120 and 140 on the y-axis = 135 cm
d. 45 cm: One-quarter of the way between 40 and 60 on the y-axis = 45 cm
e. 200 cm: Extended the line which crossed the y-axis at 200 cm
f. Because if forms a straight line
g. Increasing

Q2

The graph shows the distances travelled by car from Bloemfontein to Cape Town.
Answers:



a. 100 km	b. 500 km	c. 800 km
It took the person 1 hr to travel 100 km. We write it as 100 km/hr.	It took the person 5 hrs to travel 500 km. We write it as 500 km/5 hrs.	It took the person 8 hrs to travel 800 km. We write it as 800km/8hrs.
d. 750 km	**e. 300 km**	**f. 250 km**
It took the person $7\frac{1}{2}$hrs to travel 750 km. We write it as 750 km/7,5 hrs.	It took the person 3 hrs to travel 300 km. We write it as 300 km/3 hrs.	It took the person $2\frac{1}{2}$ hrs to travel 100 km. We write it as 100 km/hr.

Q3

How far did the person travel in:

a. 1 hour	b. 1 hour 30 minutes	c. 3 hours
100 km	150 km	300 km
d. 4 hours 30 minutes	**e. 5 hours**	**f. 2 hours 30 minutes**
450 km	500 km	250 km

41

How long did you travel?

Use the graph on "Travelling from Bloemfontein to Cape Town" to work out how long it will take to travel 275 km.

Answer: 100 km = 1 hour; 75 km = 45 minutes
275 km = 2 hours 45 minutes (or 2,75 hours)

Reflection questions

Did learners meet the objectives?

83	Drawing more graphs

Topic: Graphs **Content links:** 80-82, 84-85
Grade 8 links: 114-120 **Grade 9 links:** 88-89

Objectives

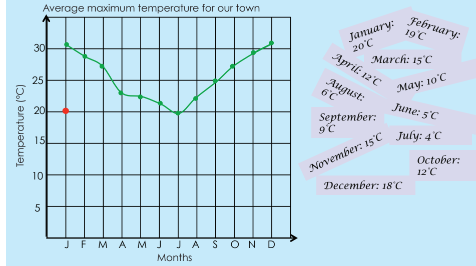
- Draw graphs from given descriptions of a problem situation, identifying the following features:
  - – Linear and non-linear graphs
  - – Increasing and decreasing
  - – Maximum and minimum

Dictionary

Linear: a graph that is a straight line
Non-linear: a graph with a curve(s)
Increasing: a graph that slopes upwards from left to right (e.g. from (2,3) to (5,7))
Decreasing: a graph that slopes downwards from left to right (e.g. from(2,3) to (5,7))
Maximum: point on a graph where the graph changes from increasing to decreasing (highest point on the graph)
Minimum: point on the graph where the graph changes from decreasing to increasing (lowest point on the graph)
X-axis: the horizontal line on the graph (left-right) through zero
Y-axis: the vertical line on the graph (top-bottom) through zero
Average: to calculate a "central" or "mean" value of a set of numbers by adding up all the numbers, then dividing the total by the amount of numbers there are, e.g. the average of this set of numbers (1, 2, 3, 4, 5, 6, 7) is (1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 ÷ 7 = 4)

42

You kept this record but forgot to plot the minimum temperature. Plot it using the information from your notes.



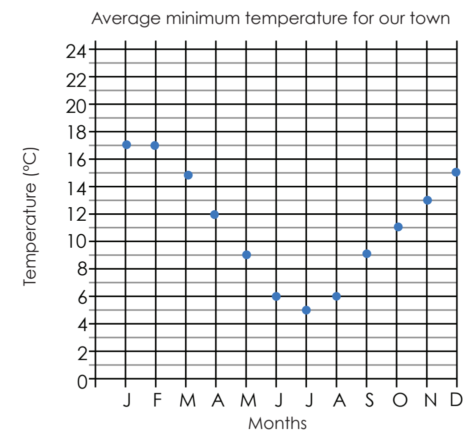
Ask the learners:

- Is this graph linear or non-linear?
- Is the graph increasing or decreasing? Explain why.
- Where is the x-axis? What does it represent?
- Where is the y-axis? What does it represent?

Q1

Answer the questions on the graph.

a. Average minimum temperature for our town.
b. Months of the year
c. Temperature ^{0}C in multiples of 2 ^{0}C
d. The month of the year
e. The temperature in 0 ^{0}C
f. The average minimum temperature (which we read on the y-axis) for a specific month (which we read on the x-axis)



83 Drawing more graphs *continued*

Q2

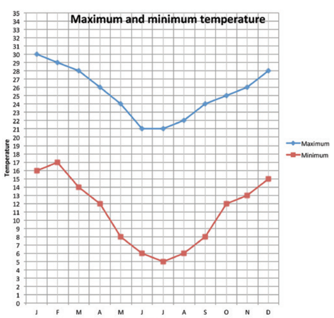
44

Use the grid paper on page 45 to draw a graph for this table.

Month	Maximum	Minimum
J	30	16
F	29	17
M	28	14
A	26	12
M	24	8
J	21	6
J	21	5
A	22	6
S	24	8
O	25	12
N	26	13
D	28	15

a. Months of the year
b. Temperature in ^{0}C
c. Temperature in multiples of 1 ^{0}C
d. Average temperature throughout the year
e. Minimum and maximum temperature for each month in ^{0}C
f. Temperatures decrease during the months of February to June and the increase during the months of August to January; non-linear graph

Q3



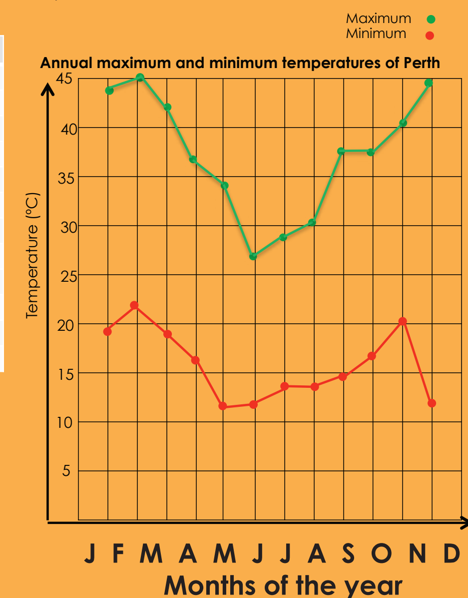
45

Research

Draw a graph showing the monthly maximum and minimum temperatures for any country other than South Africa, for one year.

Possible answer:
The table on the previous page shows the maximum and minimum temperatures of Perth (Australia)

Month	Maximum	Minimum
January	43	19
February	45	22
March	42	18
April	36	16
May	34	11
June	26	11
July	26	13
August	28	13
September	30	14
October	37	16
November	40	20
December	44	11



84 Drawing graphs again

Topic: Graphs **Content links:** 80-83, 85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

Objectives

- Draw graphs from given descriptions of a problem situation

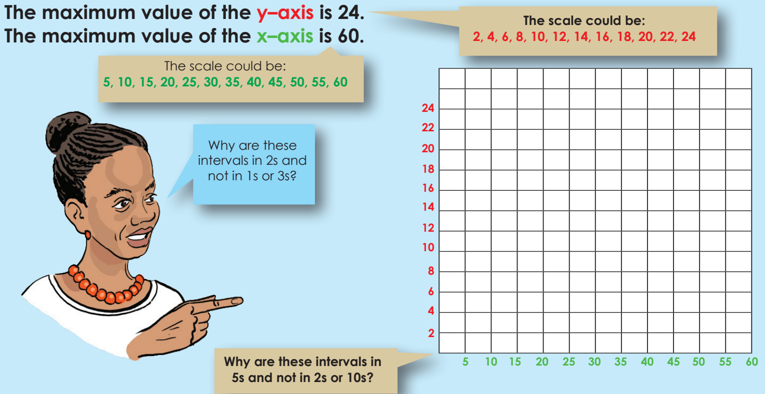
Dictionary

X-axis: the horizontal line on the graph (left-right) through zero
Y-axis: the vertical line on the graph (top-bottom) through zero
Intervals: amount of time or space between things or the numbers in-between two specific values

46

Introduction

You have to draw a graph with the following values. How will you do it?
The maximum value of the y–axis is 24.
The maximum value of the x–axis is 60.

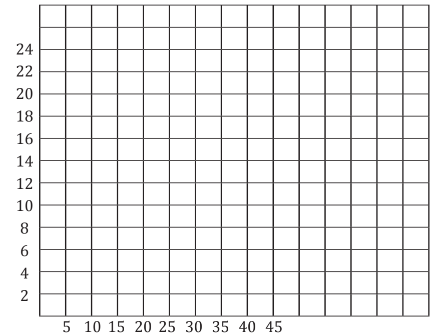


Q1

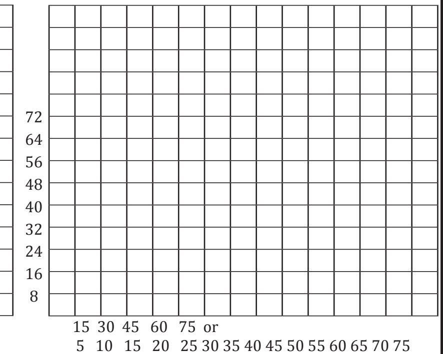
In this activity you should use the grid paper to draw the scales of your graph. Determine the scale for the y-axis and x-axis.

Answers: These are possible answers.

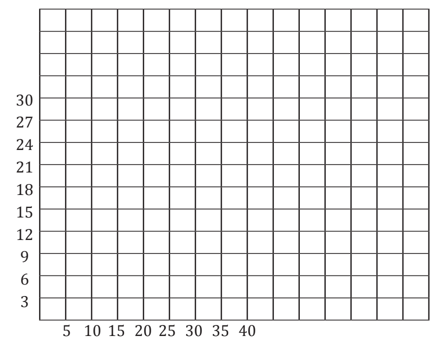
a. x-axis is 45 and y-axis is 24



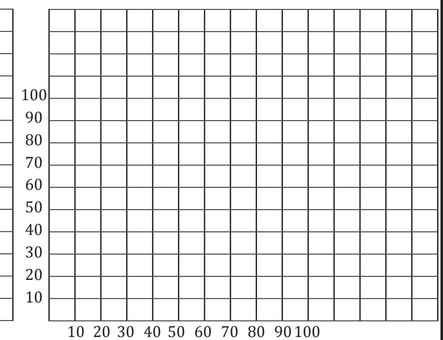
b. x-axis is 75 and y-axis is 72



c. x-axis is 40 and y-axis is 30



d. x- axis is 100 and y-axis is 100



84 Drawing graphs again *continued*

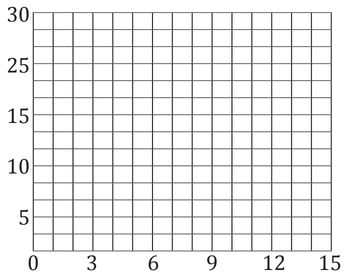
Topic: Graphs **Content links:** 80-83, 85
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

Q2

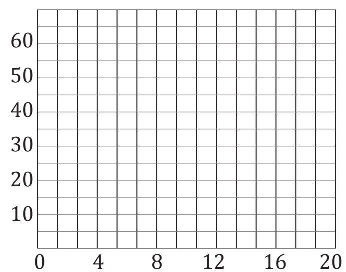
Draw the scales for the following graphs.

Answers: (Various options are possible.)

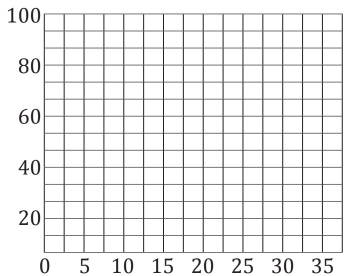
a. x-axis: 0, 3, 6, 9, 12, 15 and
y-axis: 0, 5, 10, 15, 20, 25, 30



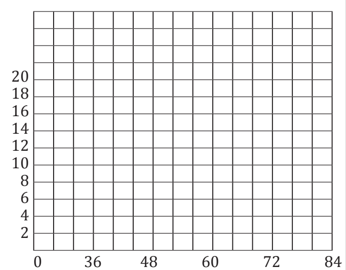
b. x-axis: 0, 4, 8, 12 and
y-axis: 0, 10, 20, 30, 40, 50, 60



c. x-axis: 0, 5, 10, 15, 20, 25, 30, 35, 40 and
y axis: 0, = 20, 40, 60, 80, 100



d. x-axis: 36, 48, 60, 72, 84 and
y-axis: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20



Q3

Cut and paste a graph from a newspaper. Describe the intervals.

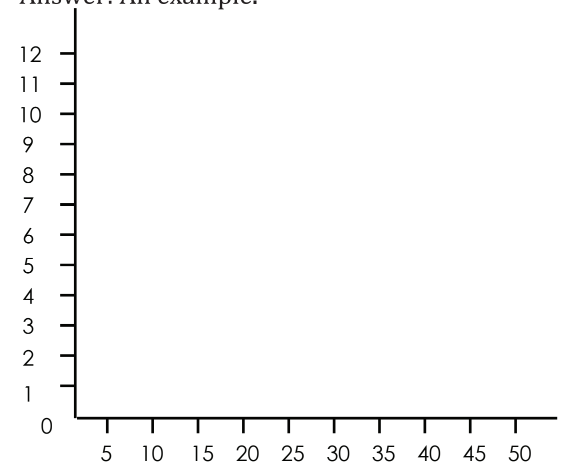
Answer: Learner's own answer

47

Drawing graphs

Draw a graph with 10 intervals on the x–axis and 12 intervals on the y–axis. You can use any multiples to label it.

Answer: An example.



Reflection questions

Did learners meet the objectives?

85 Drawing even more graphs

Topic: Graphs **Content links:** 80-84
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89

Objectives

Draw graphs from given descriptions of a problem situation, identifying the following features:

- Linear and non-linear graphs
- Increasing and decreasing

Dictionary

Linear: A graph that is a straight line
Non-linear: A graph with a curve(s)
Increasing: A graph that slopes upwards from left to right (e.g. from (2,3) to (5,7))
Decreasing: A graph that slopes downward from left to right (e.g. from (2,3) to (5,7))
X-axis: The horizontal line on the graph (left-right) through zero.
Y-axis: The vertical line on the graph (top-bottom) through zero

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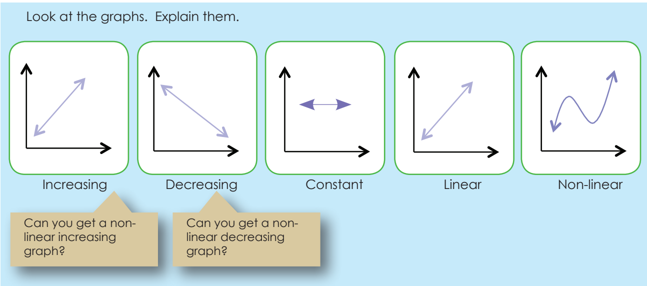
Introduction

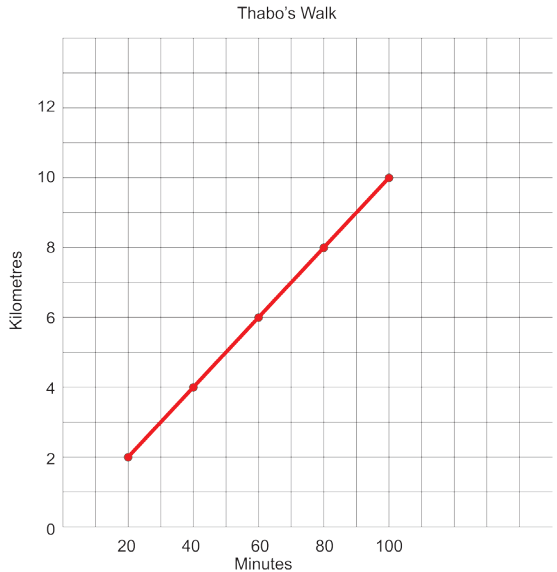


Q1

Draw graphs using data from the following tables. Describe each graph using the words increasing, decreasing, constant, linear and non-linear.

Answers: (Possible answers)

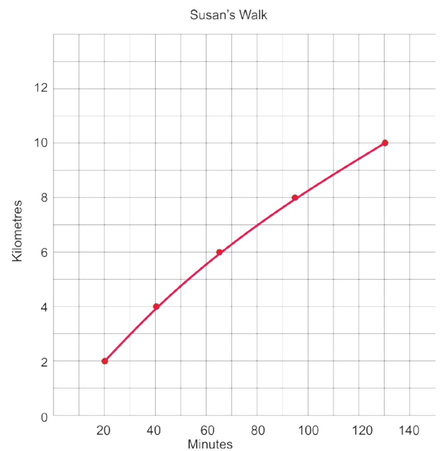
a. Thabo's brisk walking results. The time walked was recorded after 2, 4, 6, 8 and 10 km.



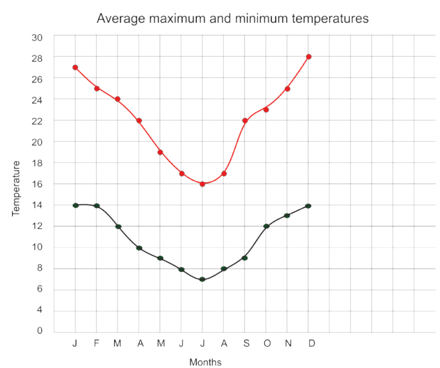
This is a linear graph with a constant increase in distance and corresponding increase in time.

85 Drawing even more graphs *continued*

Topic: Graphs **Content links:** 80-84
Grade 8 links: R9, 114-120 **Grade 9 links:** R9, 88-89



This is a non-linear graph that is increasing and with time increasing faster than distance.



This is a non-linear graph which shows the increase and decrease of temperature during the various months of the year.

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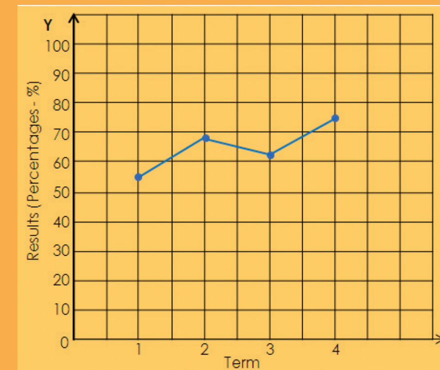
Be creative

Create your own table, draw a graph and describe it.

Be creative: Create your own table, draw a graph and describe it.
Possible answer:
My grade 6 results for each term

Term	Results (%)
1	55
2	68
3	62
4	75



The graph is a non-linear graph. There is an increase of the results for the 2nd and 4th terms but a decrease in the 3rd term.

Reflection questions

Did learners meet the objectives?

86 Transformations

Topic: Transformations **Content links:** R11, 87-94
Grade 8 links: R12, 121-126 **Grade 9 links:** R12, 105-113

Objectives

- Recognise, describe and perform translations, reflections, and rotations with geometric figures and shapes

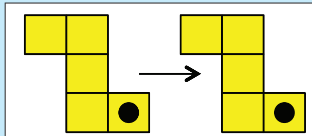
Dictionary

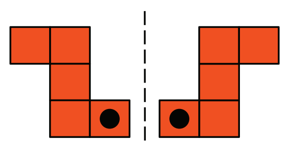
Translations: the movement of geometric figures/object from one point to another without changing its shape, size or orientation
Reflection: the change of geometric figures' form in an identical but opposite form, a transformation that has the same effect as a mirror
Rotation: the movement of geometric figures when they turn on one fixed point
Origin: a starting point or where the two axes cross in a graph or the point about which figures rotate in a rotation transformation

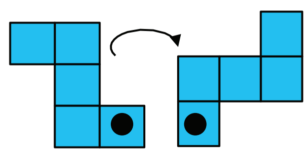
50 **Introduction**

Can you still remember these ?

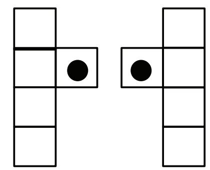
Explain each transformation

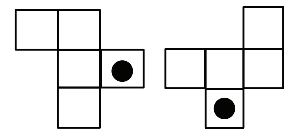

Translation

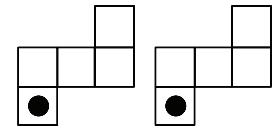

Reflection


Rotation

Q1 Say how each figure was moved. Write translation, rotation, or reflection.
Answers:
a. Reflection b. Rotation c. Translation

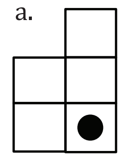


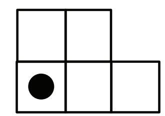


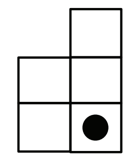


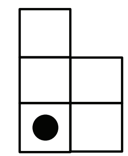
Q2 Label each shape as a translation, a reflection or a rotation of the original shape. Answers:

a.

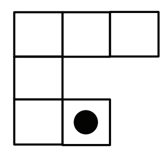

Origin

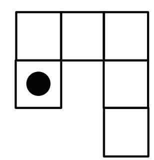

Rotation

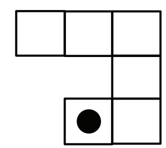

Translation

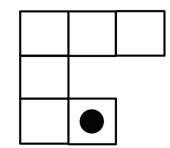

Reflection

b.


Origin


Rotation


Reflection

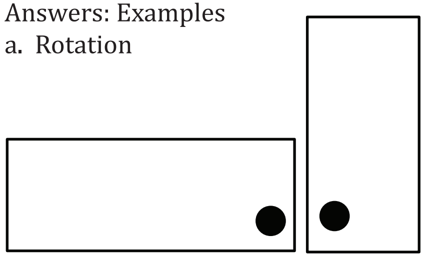

Translation

Transformations *continued*

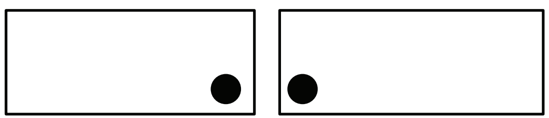
Topic: Transformations **Content links:** R11, 87-94
Grade 8 links: R12, 121-126 **Grade 9 links:** R12, 105-113

Create diagrams to show:
Answers: Examples

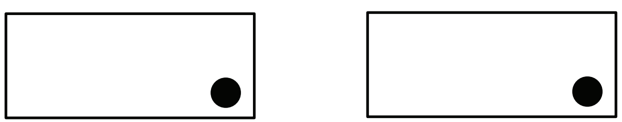
a. Rotation



b. Reflection



c. Translation



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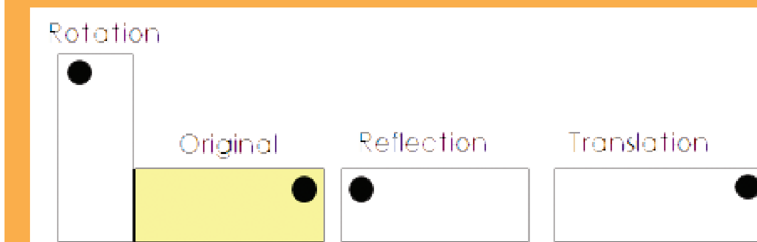
Create diagrams using reflection, rotation and translation.
Answer: Example of an answer.



Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

87 Rotation

Topic: Transformations **Content links:** 86
Grade 8 links: 124 **Grade 9 links:** R12, 109

Objectives

- Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes

Dictionary

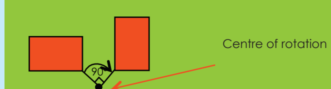
Rotation: the movement of geometric figures when they turn on one fixed point

Rotational symmetry: the symmetry of a shape which may be turned around a fixed point a certain distance and still look the same in the new position

52

Introduction

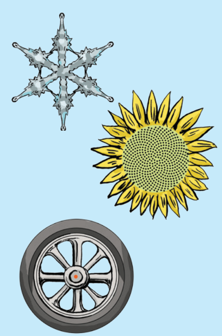
Rotation: A rotation is a transformation that moves points so that they stay the same distance from a fixed point, the centre of rotation.



Rotational symmetry: A figure has rotational symmetry if an outline of the turning figure matches its original shape.

Order of symmetry: This is how many times an outline matches the original in one full rotation.

Rotation in nature and machines.

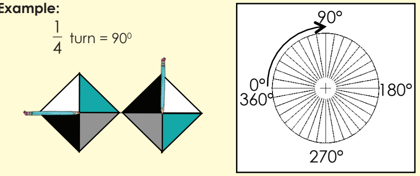


Use any **recycled material** to demonstrate the difference between rotation and rotational symmetry.

Q1 Look at the diagrams and explain them in your own words.

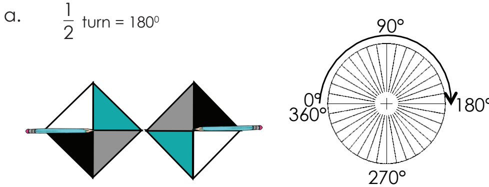
Example:

$\frac{1}{4}$ turn = 90°



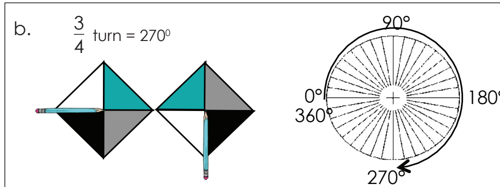
The paper rotated a quarter turn, which is the same as 90°. We can show this on a circular protractor.

a. $\frac{1}{2}$ turn = 180°



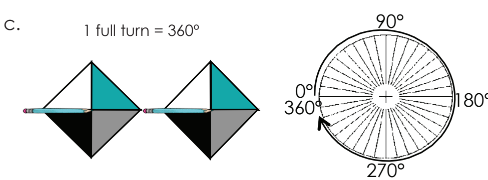
Answers:
The paper has rotated half a turn which is the same as a 180°

b. $\frac{3}{4}$ turn = 270°



The paper has rotated three quarter of a turn which is the same as a 270°

c. 1 full turn = 360°



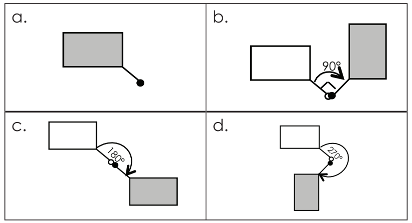
The paper has rotated a full turn which is the same as a 360°

87 Rotation *continued*

Topic: Transformations **Content links:** 86
Grade 8 links: 124 **Grade 9 links:** R12, 109

Q2

Look at the drawings below and explain them. Answers:

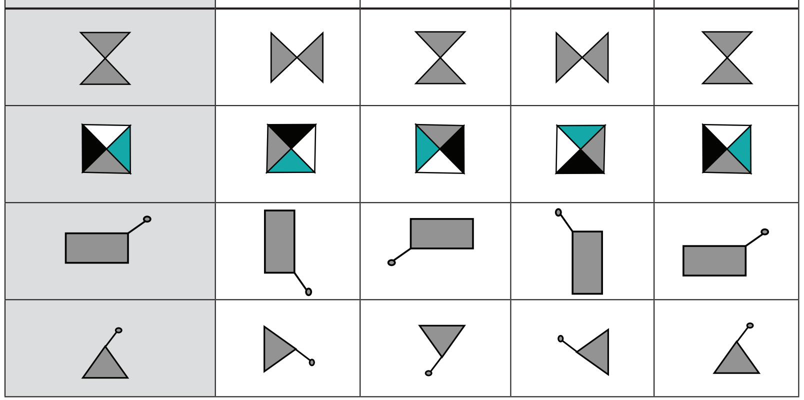


Full rotation (of 360^0)	Quarter rotation (of 90^0)
Half a rotation (of 180^0)	Three quarter rotation (of 270^0)

Q3

Complete the table below by rotating each shape and draw the rotated shape.

	90°	180°	270°	360°



Problem solving

Make up your own rotations, with the centre of rotation outside the shape.

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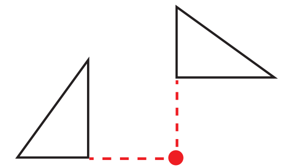
Answer: Learner's own answers



Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

88 Translation

Topic: Transformations **Content links:** 86
Grade 8 links: 124 **Grade 9 links:** R12, 109

Objectives

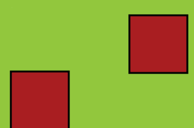
- Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper

Dictionary

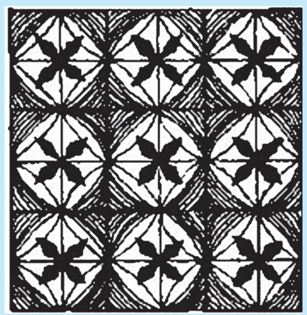
Translations: the movement of geometric figures/object from one point to another without changing its shape, size or orientation

Introduction

A translation is the movement of an object to a new position without changing its shape, size or orientation.



When a shape is transformed by sliding it to a new position, without turning, it is said to have been translated.



Q1

54

Explain each translation in your own words. The original shape is shaded.

Answers:

a. $\xrightarrow[4]{}$ $\uparrow 4$ 4 units to the right, 4 units up

b. $\xleftarrow[4]{}$ $\downarrow 4$ 4 units to the left, 4 units down

c. $\xrightarrow[2]{}$ $\downarrow 4$ 2 units to the right, 4 units down

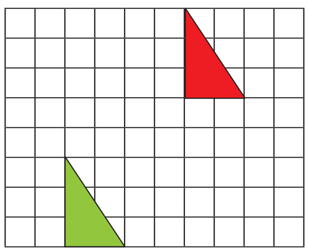
d. $\xrightarrow[3]{}$ $\downarrow 3$ 3 units to the right, 3 units down

Q2

Show the following translations on a grid board.

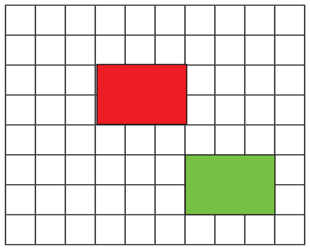
Answers:

a. Each point of the triangle is translated four squares to the right and five squares up.



origin
translated

b. Each point of the rectangle is translated three squares to the left and three squares up.

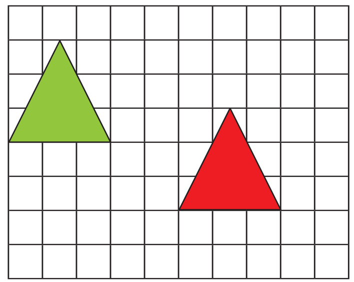


88 Translation *continued*

Topic: Transformations **Content links:** 86
Grade 8 links: 124 **Grade 9 links:** R12, 109

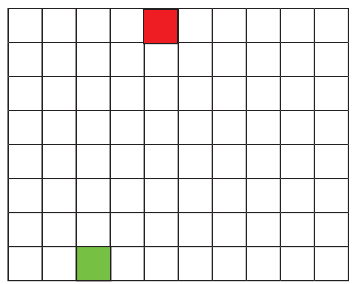
Q2

c. Each point of the triangle is translated five squares to the right and two squares down.



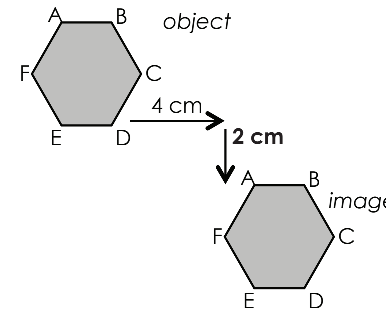
origin
translated

d. Each point of the square is translated two squares to the right and seven squares up.



Q3

In mathematics, the translation of an object is called its image. Describe the translation below.



Answers:
Each point of the object (hexagon) has translated 4 cm to the right and 2 cm down.

55

Problem solving

Find a translated pattern in nature and explain it in words.

Answer: Learners' own answers

Each point of the object (hexagon) has translated 4cm to the right and 3cm down.

Reflection questions

Did learners meet the objectives?

89 Reflection and reflective symmetry

Topic: Transformations **Content links:** 86
Grade 8 links: 122 **Grade 9 links:** R12, 105-107

Objectives

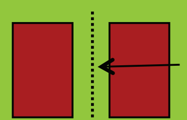
- Identify and draw lines of symmetry in geometric figures
- Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper.

Dictionary

Reflection: a transformation that has the same effect as a mirror
Reflective symmetry: a transformation that has the same effect as a mirror where one image is a mirror image of the other

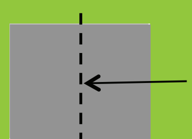
Introduction

Reflection: a reflection is a transformation that has the same effect as a mirror image.



Line of reflection

Reflective symmetry
An object is symmetrical when one half is a mirror image of the other half.

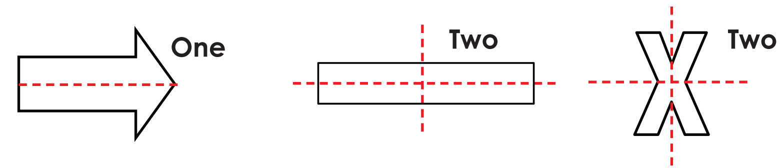


Line of symmetry

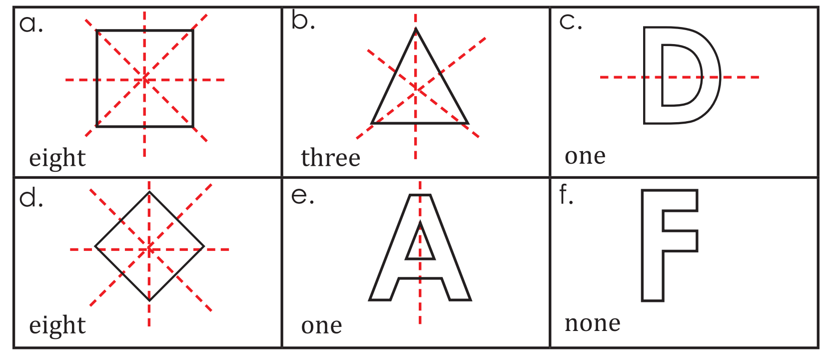
Look at the photograph.
What do you see?

56

Q1 How many lines of symmetry does each have? Draw them.
Answers:



Q2 Draw all the lines of symmetry for each figure, where applicable.
Answers:



Reflection and reflective symmetry *cont...*

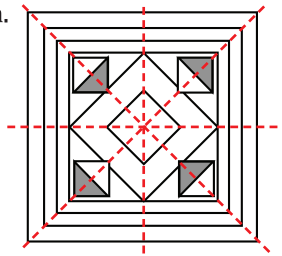
Topic: Transformations **Content links:** 86
Grade 8 links: 122 **Grade 9 links:** R12, 105-107

Q3

The following design uses reflective symmetry. One half is a reflection of the other half. The two halves are exactly alike and fit perfectly on top of each other when the design is folded correctly. How many lines of symmetry are there?

Answers:

a.



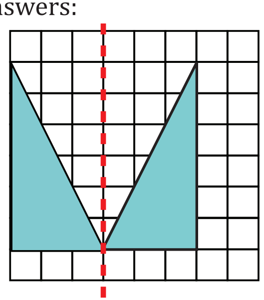
Four lines of symmetry

Q4

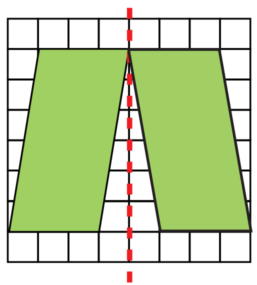
Show reflection using the geometric figure given. Remember to show the line of reflection.

Answers:

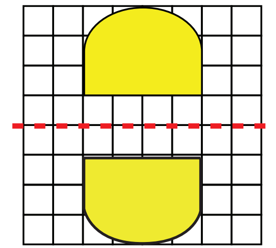
a.



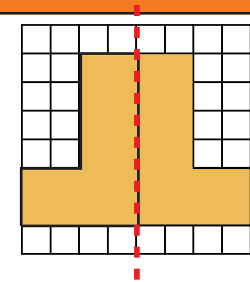
b.



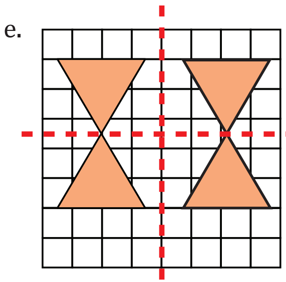
c.



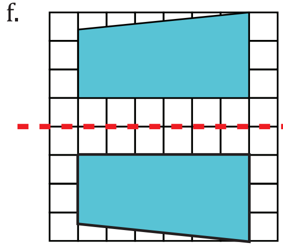
d.



e.



f.



Q5

Look at the reflections and describe them.

Answers:

a. This is a vertical reflection, in other words it is reflected on the y-axis

b. This is a horizontal reflection, in other words it is reflected on the x-axis

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Problem solving

Find a photograph of reflection in nature. Answer: Learner's own photograph.

90 Transformations again

Topic: Transformations **Content links:** 86
Grade 8 links: 121 **Grade 9 links:** R12, 110-111

Objectives

Recognize, describe and perform translations, reflection's and rotations with geometric figures and shapes on squared paper

Dictionary

Translation: the movement of a geometric figure or object from one point to another without changing its shape, size or orientation (SLIDE)
Reflection: the change of geometric figures' form in an identical but opposite form, a transformation that has the same effect as a mirror (FLIP)
Rotation: the movement of geometric figures when they turn on one fixed point (TURN)

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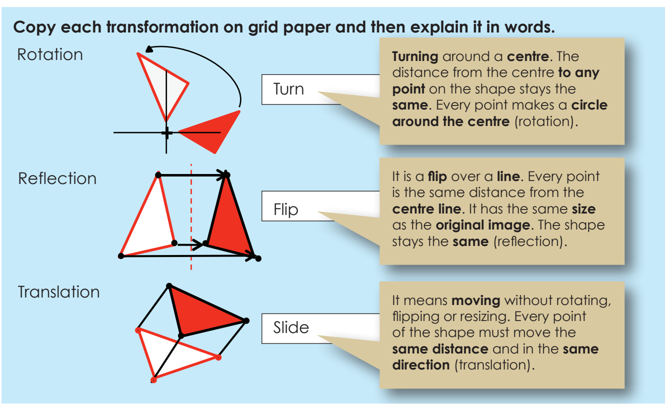
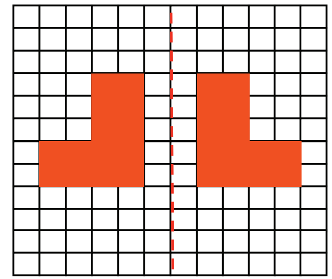
Introduction

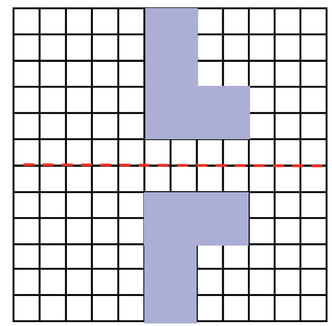


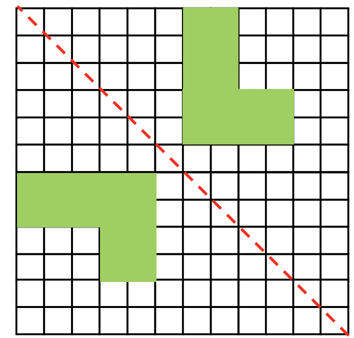
Q1 Describe each diagram. Make use of words such as mirror, shape, original shape, line of reflection and vertical.

Answers:



a. The original shape is mirrored on a vertical line creating a reflection.





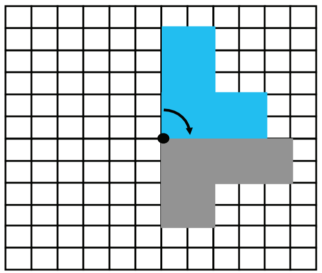
b. This is a reflection on a horizontal line.

c. This is a reflection of an original shape using a diagonal line creating a mirrored shape.

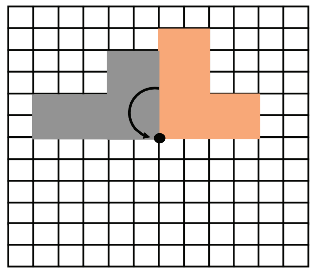
Topic: Transformations **Content links:** 86
Grade 8 links: 121 **Grade 9 links:** R12, 110-111

Rotation

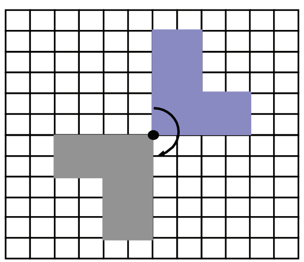
Make use of words such as rotated or turned, clockwise, anti-clockwise, point of rotation and distance.



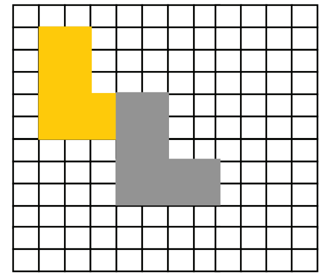
d. This is a clockwise rotation going a quarter distance (a 90^0 clockwise rotation).



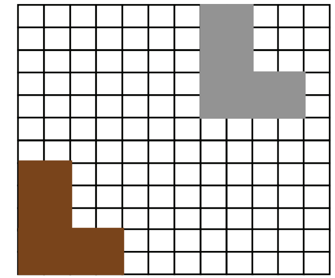
e. This an anti-clockwise rotation going a quarter distance (90^0 rotation).



f. This a rotation going in a clockwise direction halfway around a point of rotation (a 180^0 rotation).



g. This shape is slided from one place to another using right and down translations



h. This shape is transleted using up and right translations sliding it from one place to another.

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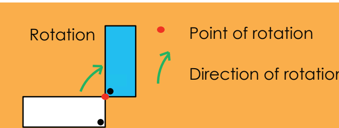
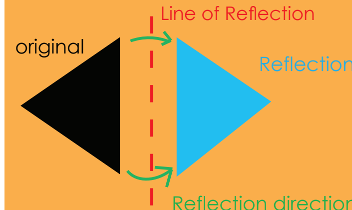
Share with your family

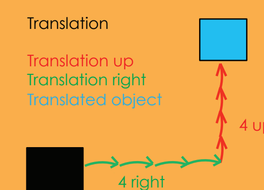
Draw any shape and then do the following and describe the transformation:

- reflection
- rotation
- translation

Answers: Here is a an example of an answer.







91 Investigation

Topic: Transformations **Content links:** 86
Grade 8 links: None **Grade 9 links:** None

Objectives

Recognize, describe and perform translations, reflection's and rotations with geometric figures and shapes on squared paper.

Dictionary

Translations: Is moving without resizing, rotating or flipping-all points move the same distance and direction.
Reflection: Is a flip over a line.
Rotation: Is turning around a centre.

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Introduction

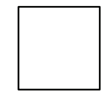
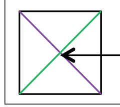
When we do an investigation we should:

- spend enough time exploring problems in depth
- find more than one solution to many problems
- develop your own strategies and approaches, based on your knowledge and understanding of mathematical relationships
- choose from a variety of concrete materials and appropriate resources
- express your mathematical thinking through drawing, writing and talking.

Q1

Prove that the diagonal of a square is not equal to the length of any of its sides.

a. Make a drawing to show each of the following:

What transformation is (rotation, reflection, and translation)	What a square is 
What diagonal lines of a square are 	That all the sides of a square are equal in length Has four lines of symmetry

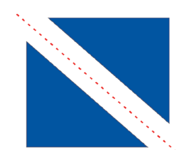
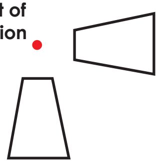
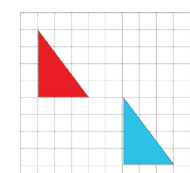
b. What do I want?

To compare the length of a side of a square with the length of a diagonal.

I can/must use rotation, translation and/or reflection.

c. What do I need to introduce? Make a drawing of each.

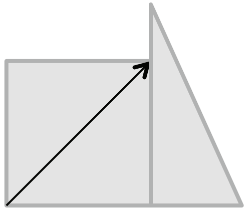
Note that sometimes we think of something later on; we don't always think of everything at the beginning. Therefore people will have different answers here.

A line of reflection	A point of rotation	A grid on which to measure translation
		

91 Investigation *continued*

Topic: Transformations **Content links:** 86
Grade 8 links: None **Grade 9 links:** None

d. Attack
We often get "stuck" and are tempted to give up. However, this is the exact point at which it is important for you to use the time and space to get through the point of frustration and look for alternative ideas. This is the phase when we make conjectures, collect data, discover patterns and try to justify our answers.
Answer: Cut a square along the diagonal. Fit diagonal on 4 sides and compare length. The diagonal is longer than all 4 sides since it overlaps when placed side by side.



e. Review
Check your conclusions or resolutions, reflect on what you did – the key ideas and key moments.

Answer: The square's diagonal is longer than any side since if cut and placed side by side it overlaps. Even if rotated all the sides become shorter than the diagonal.

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Family time

Share this investigation with a family member.

Answer: Encourage learners to share their work with their family.

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

92 Enlargement and reduction

Topic: Transformations **Content links:** R11, 93-94
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Objectives

- Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

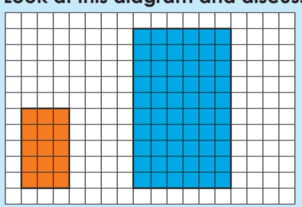
Dictionary

Resizing: changing the size of an object or geometric shape but retaining its aspect ratio.
Ratio: a relationship between two numbers of the same kind (that is, for every amount of one thing, how much is there of another thing)
Aspect ratio: the relation between the length and width of a geometric shape, e.g. a square has an aspect ratio of 1:1
Enlargement: making an object bigger than the original size
Reduction: making an object smaller than the original size

Introduction

62

Look at this diagram and discuss it.



Orange rectangle

The length = 5 The width = 3

Blue rectangle

The length = 10 The width = 6

The length of the **blue rectangle** is two times/twice the length of the **orange rectangle**.

The width of the **blue rectangle** is two times/twice the width of the **orange rectangle**.

The **orange rectangle** is enlarged twice/two times.

Q1

Use the diagrams to answer the questions.

Answers: a.

Blue square	Red square	Green square
Length = 2	Length = 4	Length = 9
Width = 2	Width = 4	Width = 9

b. The length of the red square is **2** times the length of the blue square. The width of the red square is **2** times the width of the blue square. The red square is **2** times enlarged.

c. The length of the green square is **3** times the length of the red square rectangle. The width of the green square is **3** times the width of the red square. The green square is **3** times enlarged.

d. The length of the green square is **6** times the length of the blue square. The width of the green square is **6** times the width of the blue square. The blue square is **6** times reduced.

Enlargement and reduction *continued*

Topic: Transformations **Content links:** R11, 93-94
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Q2

Use the diagrams to answer the questions.
Answers:

Blue rectangle:
The length = 3 cm
The width = 1 cm

Red rectangle:
The length = 6 cm
The width = 2 cm

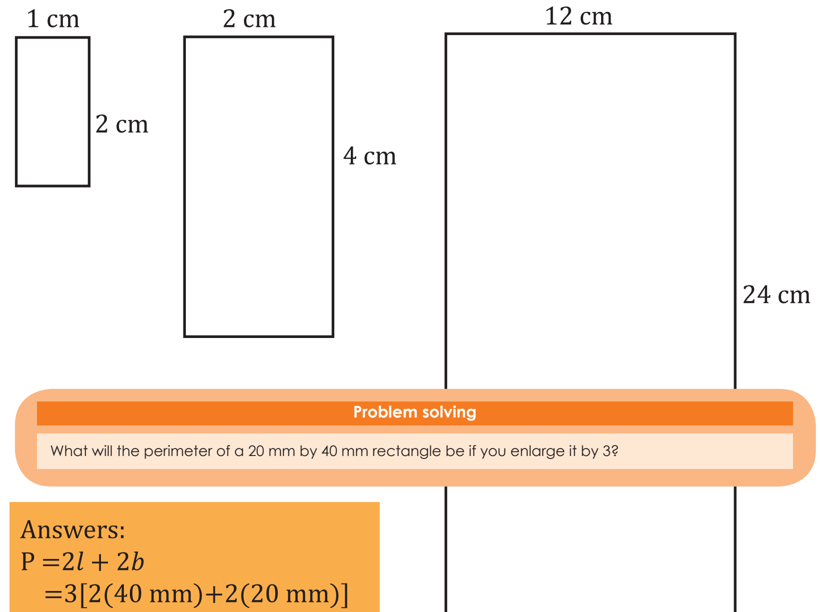
Green rectangle:
The length = 24 cm
The width = 8 cm

Compare to the:

a. Red rectangle, the blue rectangle is ***2*** times reduced.
b. Green rectangle, the blue rectangle is ***8*** times reduced.
c. Blue rectangle, the red rectangle is ***2*** times enlarged.
d. Green rectangle, the red rectangle is ***4*** times reduced.
e. Blue rectangle, the green rectangle is ***8*** times enlarged.
f. Red rectangle, the green rectangle is ***4*** times enlarged.

Q3

Draw a 1cm by 2cm rectangle. Enlarge it twice and then enlarge the second rectangle six times. Make a drawing to show your answer.
Answers:



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Problem solving

What will the perimeter of a 20 mm by 40 mm rectangle be if you enlarge it by 3?

Answers:
P $=2l + 2b$
$=3[2(40 \text{ mm})+2(20 \text{ mm})]$
$=3[80 \text{ mm} + 40 \text{ mm}]$
$=3[120 \text{ mm}]$
$=360 \text{ mm}$

Reflection questions
Did learners meet the objectives?

93 More enlargement and reduction

Topic: Transformations **Content links:** 92, 94
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Objectives

- Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

Dictionary

Enlargement: making an object bigger than the original size
Reduction: making an object smaller than the original size
Scale factor: the value of the multiplier or divisor used to make an enlargement or reduction in the size of a shape

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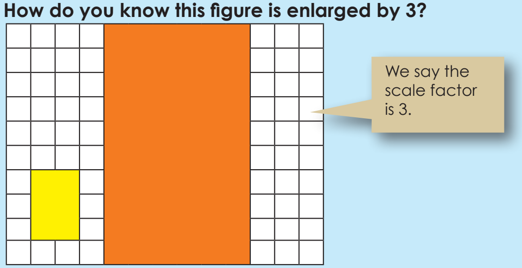
Introduction

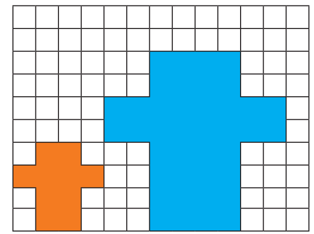
How do you know this figure is enlarged by 3?



The scale factor from small to large is 3.
The scale factor from large to small is 3.

Q1

By what is this shape enlarged? Write down all the steps.
Answers: Scale factor 2

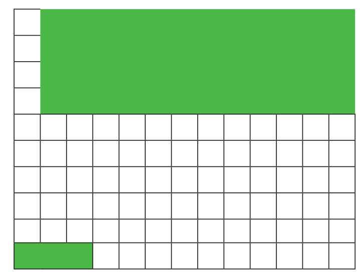


Horizontal length of red figure is 4 units
Horizontal length of blue figure is 8 units
Therefore scale factor is $\frac{8}{4} = 2$

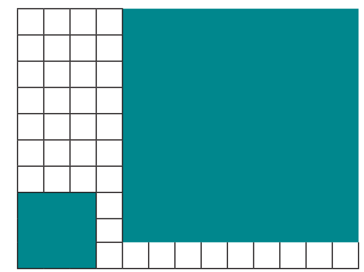
Q2

Enlarge the rectangle by:
Answers:

a. scale factor 4



b. scale factor 3



93 More enlargement and reduction *cont...*

Topic: Transformations **Content links:** 92, 94
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Q3

Complete the table. Start with the original geometric figure every time. Answers:

b. 5 cm x 1 cm 5 cm, 1 cm $= 5$ cm^2	10 cm x 2 cm 10 cm, 2 cm $= 20$ cm^2	25 cm x 5 cm 25 cm, 5 cm $= 125$ cm^2	50 cm x 10 cm 50 cm, 10 cm $= 500$ cm^2
c. 4 cm x 2 cm 4 cm, 2 cm $= 8$ cm^2	8 cm x 4 cm 8 cm, 4 cm $= 32$ cm^2	20 cm x 10 cm 20 cm, 10 cm $= 200$ cm^2	40 cm x 20 cm 40 cm, 20 cm $= 800$ cm^2
d. 8 cm x 3 cm 8 cm, 3 cm $= 24$ cm^2	16 cm x 9 cm 16 cm, 9 cm $= 96$ cm^2	40 cm x 15 cm 40 cm, 15 cm $= 600$ cm^2	80 cm x 30 cm 80 cm, 30 cm $= 2400$ cm^2
e. 1,5 cm x 2 cm 1,5 cm, 2 cm $= 3$ cm^2	3 cm x 4 cm 3 cm, 4 cm $= 12$ cm^2	7,5 cm x 10 cm 7,5 cm, 10 cm $= 75$ cm^2	18 cm x 20 cm 18 cm, 20 cm $= 300$ cm^2

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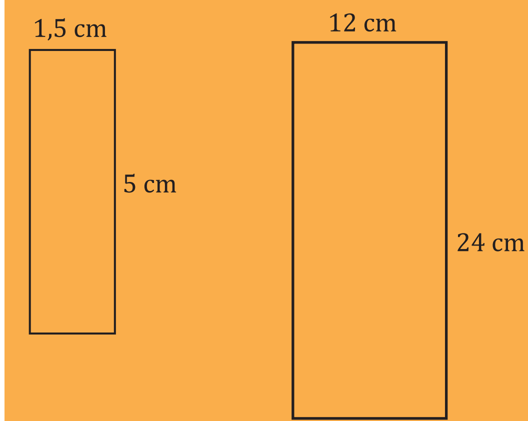
Problem solving

Enlarge a 1,5 cm by 5 cm geometric figure by scale factor 3.

Answers:



Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

94 Enlargements and reductions

Topic: Transformations **Content links:** 92-93
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Objectives

- Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

Dictionary

Enlargement: making an object bigger than the original size
Reduction: making an object smaller than the original size
Scale Factor: the value of the multiplier or divisor used to make an enlargement or reduction in the size of a shape

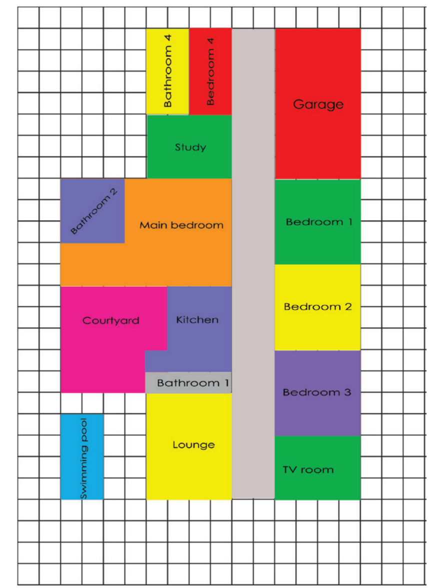
66

Introduction

Use the knowledge you gained in the previous two worksheets. You might need to revise the following words:

- enlargement
- reduction
- scale factor

A client asked you to make the following amendments to the house plan.



Q1 Enlarge the following by scale factor 2.
a. Garage
b. Bedroom 3

Q2 Join bedrooms 1 and 2 and reduce by scale factor 2.

Q3 Replace bedroom 3 with a bathroom double the size of bathroom 1.

Q4 Enlarge the TV room into a very large entertainment room by scale factor 3.

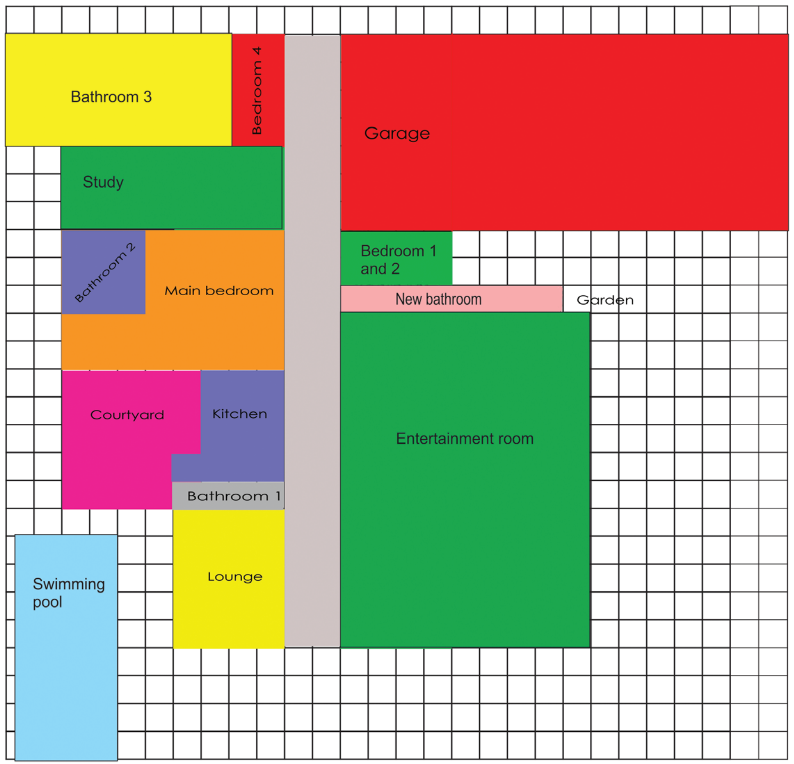
Q5 Double the size of the study.

Q6 Enlarge the swimming pool by scale factor 2.

Enlargements and reductions *continued*

Topic: Transformations **Content links:** 92-93
Grade 8 links: 125-126 **Grade 9 links:** R12, 112-113

Answers:



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Answer: A possible answer.
Learners must note that "double the size" is not the same as enlarge by a scale factor of". Thus doubling the size of the study means doubling the area (from 9 square units to 18) not enlarging the area by a scale factor of 2 (from 9 square units to 36 in this example).

Problem solving

Design your dream house. Enlarge it by scale factor 2.

Learners' own designs and enlargements

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

95 Prisms and pyramids

Topic: 3-D objects **Content links:** R10, 96-104
Grade 8 links: R13, 127-134 **Grade 9 links:** R13, 114-122

Objectives

- Describe, sort and compare polyhedra in terms of:
  – Number of edges
  – Number of vertices
  – Shape and number of faces
- Revise using nets to create models of geometric solids, including cubes, prisms and pyramids

Dictionary

Polygon: a plane 2-D shape enclosed by a number or straight lines (edges) joined together at vertices (corners)
Polyhedron: a 3-D object which consists of a collection of polygons, usually joined at their edges
Pyramid: This is a 3-D object that has a polygon as a base. The base of the pyramid give the pyramid its name. The other faces are all triangles which meet at the top (the apex).
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are always rectangles or parallelograms.
Edge: a straight line where two surfaces are joined
Vertice: a point where three or more surfaces meet (corner)
Face: a surface of a solid object

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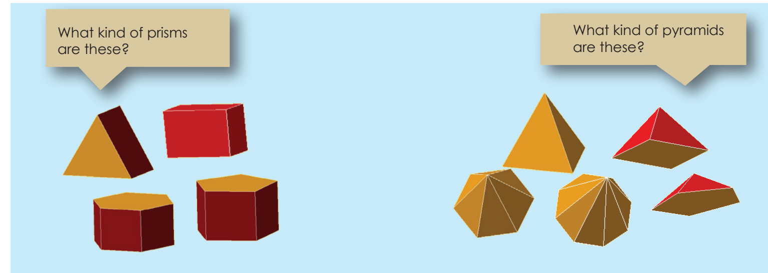
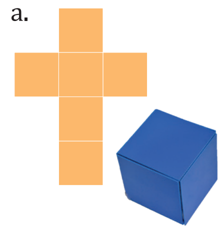
Introduction



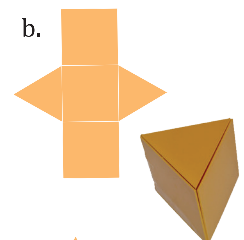
Q1 Make the following geometric objects using the nets below. Enlarge the nets by scale factor of 2. You will need some grid paper, a ruler, sticky tape and a pair of scissors.

Answers:

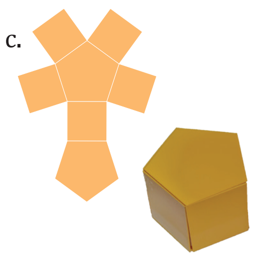
a.



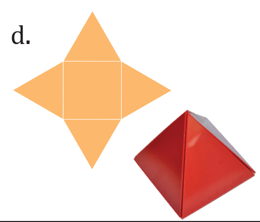
b.



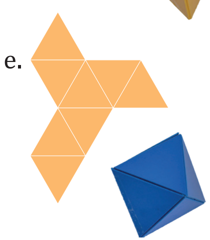
c.



d.



e.



[Note that e. is not a prism or pyramid but a more complex figure (two triangular pyramids joined together)]

95 Prisms and pyramids *continued*

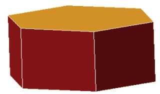
Topic: 3-D objects **Content links:** R10, 96-104
Grade 8 links: R13, 127-134 **Grade 9 links:** R13, 114-122

Q2

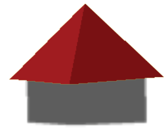
Identify and name all the geometric solids (3-D objects).
Answers:

a.


Triangular prism and rectangular prism

b.


Hexagonal prism

c.


Square pyramid and square-based prism (a cube)

Q3

Identify, name and label as many pyramids and prisms as you can in these photos. Answers:

a.


Rectangular prism
Triangular prism

b.


Rectangular prisms
Triangular prisms
Cubes
Square Prisms

c.


Rectangular prism
Triangular prisms
Square-based prism

Note: If learners identify other 3-D objects, ask them to explain them.

Q4

Compare prisms and pyramids.
Answers:

Prisms	Pyramids
Top and bottom faces are identical polygons The other faces are rectangles	Just one base that is a polygon The other faces are triangles

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Problem solving

Name five pairs of a pyramid and a prism that will exactly fit on top of each other, and say why.

Answer: (Possible answers)
Triangular prism and Triangular pyramid
Cube and Square pyramid
Rectangular prism and rectangular pyramid
Pentagonal prism and Pentagonal pyramid
Hexagonal prism and hexagonal pyramid
Octagonal prism and Octagonal pyramid

Each pyramid has one or more faces that fits exactly onto one or more of the same sized faces of the prism.

Reflection questions

Did learners meet the objectives?

96 3-D objects

Topic: 3-D objects **Content links:** RR10, 95, 97-104
Grade 8 links: R13, 127-134 **Grade 9 links:** R13, 114-122

Objectives

- Describe, sort and compare polyhedra in terms of:
  – Number of edges
  – Number of vertices
  – Shape and number of faces
- Revise using nets to create models of geometric solids, including cubes, prisms and pyramids
- Describe, sort and compare polyhedra in terms of:
  – Number of edges
  – Shape and number of faces
  – Number of vertices

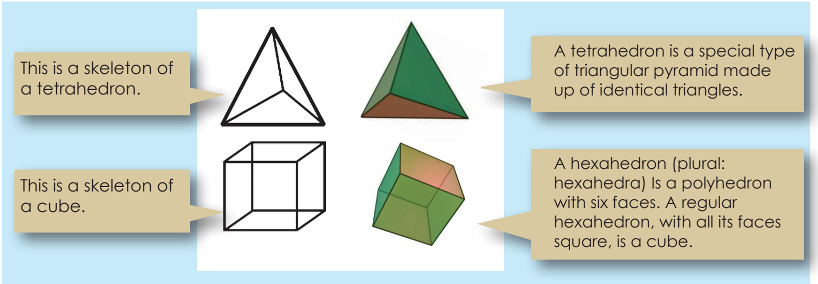
Dictionary

Polygon: a plane 2-D shape enclosed by a number or straight lines (edges) joined together at vertices (corners)
Polyhedron: a 3-D object which consists of a collection of polygons, usually joined at their edges
Pyramid: This is a 3-D object that has a polygon as a base. The base of the pyramid give the pyramid its name. The other faces are all triangles which meet at the top (the apex).
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are always rectangles or parallelograms.
Edge: a straight line where two surfaces are joined
Vertice: a point where three or more surfaces meet (corner)
Face: a surface of a solid object
Hexahedron: a 3-D object with 6 faces, e.g. cube
Cube: a 3-D geometric figure with six square faces
Tetrahedron: a 3-D object that has four equilateral (identical) triangles, e.g. a triangular pyramid

Introduction

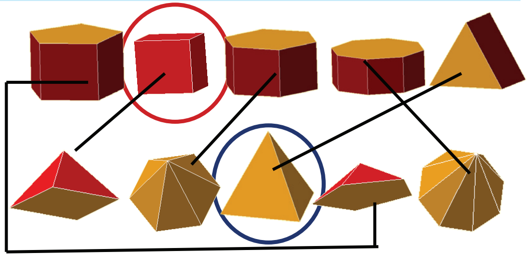
Ask the learners if they would make any changes or add anything to the graphic.

70



Q1

Which pyramid will fit exactly onto each prism? Draw lines to show it. Also circle the tetrahedron in blue and the hexahedron in red.
Answers:



96 3-D objects *continued*

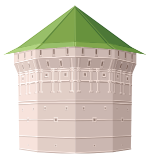
Topic: 3-D objects **Content links:** RR10, 95, 97-104
Grade 8 links: R13, 127-134 **Grade 9 links:** R13, 114-122

Q2

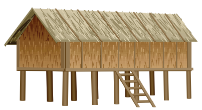
Describe the prism and pyramid in these pictures.

Answers:

Rectangular prism – walls, windows
Square prism – walls, windows
Triangular prism – roof
Hexagonal Prism – walls of "tower"
Hexagonal pyramid - roof of "tower"



Octagonal prism – the walls
Octagonal pyramid - roof

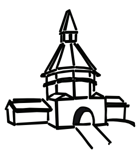


Rectangular prism – the walls
Triangular prism – the roof

Q3

Your friend made this drawing of a building she saw. Identify and name the solids.

Answers:



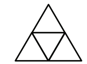
Rectangular prisms (walls)
triangular prisms (lower roofs)
cube (tower walls)
square pyramid (tower roof)
Octagonal prism (tower walls)
Octagonal pyramid (tower roof)

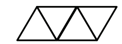
Q4

Draw the nets for the following:

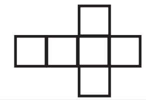
Answers:

Tetrahedron





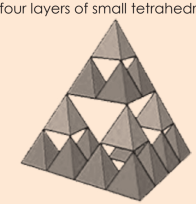
Hexahedron



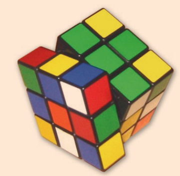
71

Problem solving

How many tetrahedrons do you need to complete the big tetrahedron made of four layers of small tetrahedrons?



How would you use the word hexahedron to describe this Rubic cube?



Answer:
Tetrahedron: you need 14 more small tetrahedrons to complete the 30 tetrahedrons making up the big tetrahedron
Rubic cube: one large hexahedron that consists of 27 smaller hexahedrons (each with six square faces)

97 Building 3–D models

Topic: 3-D objects **Content links:** 103-104
Grade 8 links: 127, 129, 132-133 **Grade 9 links:** 114, 121-122

Objectives

- Revise using nets to make models of geometric solids, including cubes, prisms and pyramids

Dictionary

Polygon: a plane 2-D shape enclosed by a number or straight lines (edges) joined together at vertices (corners)
Polyhedron: a 3-D object which consists of a collection of polygons, usually joined at their edges
Pyramid: This is a 3-D object that has a polygon as a base. The base of the pyramid gives the pyramid its name. The other faces are all triangles which meet at the top (the apex).
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are rectangles.
Hexahedron: a 3-D object with six faces, e.g. a cube
Cube: a 3-D geometric figure with six square faces
Tetrahedron: a 3-D object that has four equilateral (identical) triangles, e.g. a triangular pyramid
Edges: a straight line, where two surfaces are joined
Vertices: a point where three or more surfaces meet (corner)

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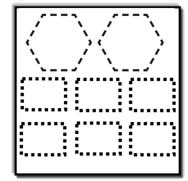
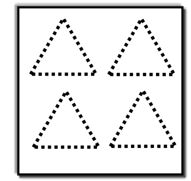
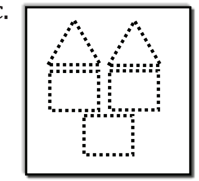
Introduction

Q1 Which geometric solid can be made of these geometric figures.
Answers:

a. 

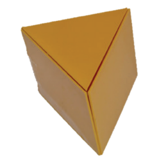
b. 

c. 

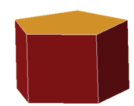
Hexahedron/ Hexagonal prism

Tetrahedron/ Triangular pyramid

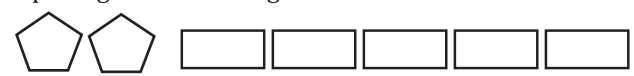
Triangular prism

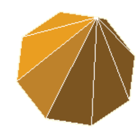


Q2 Identify all the geometric figures in the solids and make a drawing of all the shapes.
Answers:

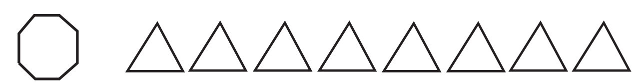
a. 

2 pentagons 5 rectangles



b. 

1 octagon 8 triangles



97 Building 3–D models *continued*

Topic: 3-D objects **Content links:** 103-104
Grade 8 links: 127, 129, 132-133 **Grade 9 links:** 114, 121-122

Q3

a. Use waste products to make these geometric solids:
- prisms (triangular prism, cube, rectangular, pentagonal, hexagonal and octagonal)
- pyramids (triangular, tetrahedron, rectangular, pentagonal, hexagonal and octagonal)

Q4

Use the geometric solids to create 'buildings of the future'.

Answer: Learners' own constructions

a. Write down how you created each polyhedron, focusing on the shapes of the faces and how you joined them. You may include drawings.
b. Write a description of how you put the geometric solids together to create your "buildings of the future". Say why you use certain solids for certain buildings.
c. Present your work to the class.

Answer: Learners' own answers and presentations

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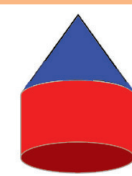
Some presentation guidelines

When presenting you should:
- Think about what you want to communicate and organise your presentation well
- Stand up straight and confidently with both feet firmly on the ground
- Start by explaining what the content of presentation is about
- Explain all points thoroughly
- Maintain the interest level of the class by:
  - Making eye contact with different people throughout the presentation
  - Using natural hand gestures to demonstrate
  - Using visual aids to enhance the presentation
- Demonstrate a strong positive feeling about the topic during the entire presentation
- Stay within the required time limits

Problem solving

Fit two geometric solids on top of each other. Where they touch the faces should be the same. The two geometric solids cannot be prisms or pyramids.

Answer: Here is one possible answer, a cone on top of a cylinder



98 Visualising 3–D objects/playing a game

Topic: 3-D objects **Content links:** 102
Grade 8 links: 134 **Grade 9 links:** 118-120

Objectives
- Visualise 3-D objects
- Recognise 3-D objects from different views

Dictionary
Polygon: a plane 2-D shape enclosed by a number or straight lines (edges) joined together at vertices (corners)
Polyhedron: a 3-D object which consists of a collection of polygons, usually joined at their edges
Pyramid: This is a 3-D object that has a polygon as a base. The base of the pyramid gives the pyramid its name. The other faces are all triangles which meet at the top (the apex).
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are rectangles.
Edges: a straight line , where two surfaces are joined
Vertices: a point where three or more surfaces meet (corner)
Faces: surfaces of a solid object

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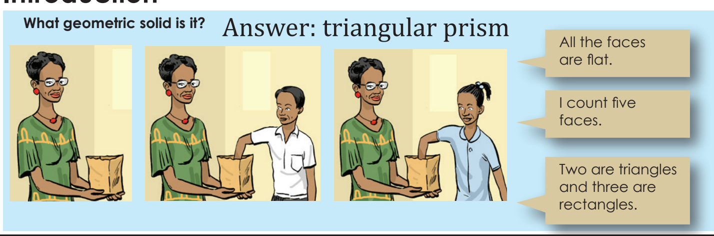
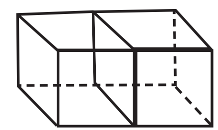
Introduction



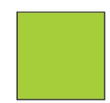
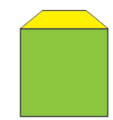
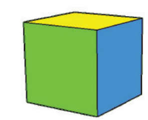
Answer: triangular prism

Q1

Ask your friend to close his or her eyes. Then ask him or her the following questions:
a. Draw, name and describe the new solid.
Answers:

Draw:	**Name:**	**Describe:**
	rectangular (square) prism	Each cube is made of six identical squares

b. Draw, name and describe the solid from different views.

Seeing one square of the cube from the front 	Seeing two squares of the cube, the front and the top 	Seeing three squares of the cube, front, top and one side 

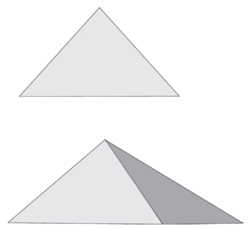
c. What type of pyramid (geometric objects) will we most likely find in Egypt?
Answer: Square pyramid

Visualising 3–D objects/playing a game *cont...*

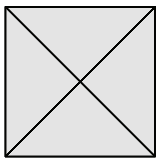
Topic: 3-D objects **Content links:** 102
Grade 8 links: 134 **Grade 9 links:** 118-120

d. Name and describe the solid from different views.
After imagining the object, draw it.

Views from the ground



Aerial view



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Problem solving

Describe a geometric solid to your family and ask them to imagine it.

Answer:
Imagine you have an object that has 9 flat faces. The shape only has one base that has 8 equal sides. There are 8 faces connected to the base and the faces forms a sharp point called an apex. What shape is it?
Octagonal Pyramid.

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

99 Faces, vertices and edges

Topic: 3-D objects **Content links:** 100-101
Grade 8 links: 130 **Grade 9 links:** 118-120

Objectives

- Describe, sort and compare polyhedra in terms of:
  – Number of edges
  – Shape and number of faces
  – Number of vertices

Dictionary

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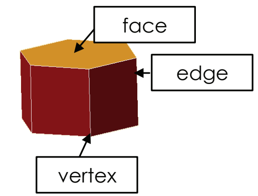
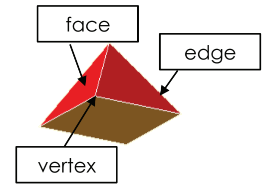
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are rectangles.
Edges: a straight line , where two surfaces are joined
Vertices: a point where three or more surfaces meet (corner)
Faces: surfaces of a solid object
Net: a flat diagram that can be folded to create a 3-D solid
Skeleton: a diagram that shows the framework of a 3-D object and shows its edges and vertices

Q1 76

Label the following using the words: surface (face), edge and vertex.



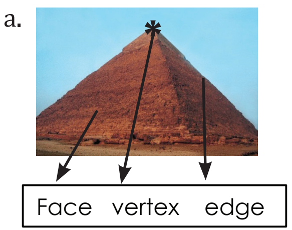


Q2

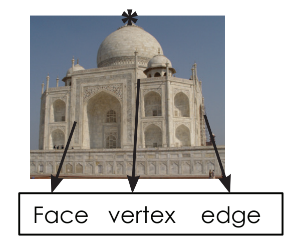
Label the surfaces, vertices and edges on each photograph.

Answers:

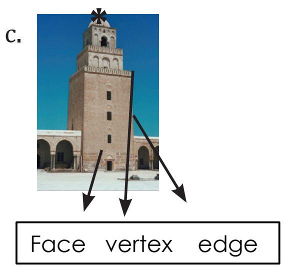
a.



b.



c.



d. An apex is the highest point of a geometric solid with respect to a line or plane chosen as base.

99 Faces, vertices and edges *continued*

Topic: 3-D objects **Content links:** 100-101
Grade 8 links: 130 **Grade 9 links:** 118-120

Q3

What do all these objects have in common? When closed, they all have:

Answers:

a.


- 6 faces
- 12 edges
- 8 vertices

b.


- 6 faces
- 12 edges
- 8 vertices

c.


- 6 faces
- 12 edges
- 8 vertices

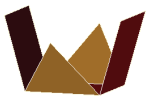
d.


- 6 faces
- 12 edges
- 8 vertices

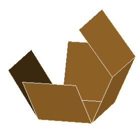
Q4

Label the following using the words: surface (face), edge and vertex. Also say which geometric object each one will form.

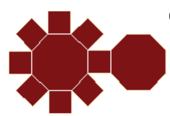
Answers:

a.


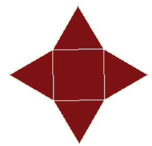
Triangular prism
- 9 edges
- 6 vertices
- 5 faces

b.


Cube
- 12 edges
- 8 vertices
- 6 faces

c.


Octagonal prism
- 9 edges
- 6 vertices
- 5 faces

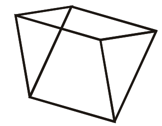
d.


Square pyramid
- 9 edges
- 6 vertices
- 5 faces

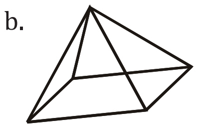
Q5

Look at these skeletons. Say how many vertices and edges you see in each structure.

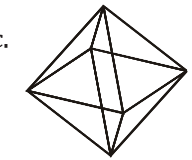
Answers:

a.


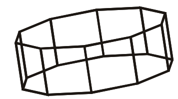
6 vertices 9 edges

b.


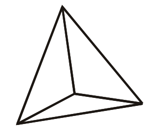
5 vertices 8 edges

c.


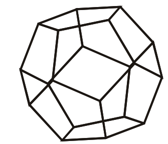
6 vertices 12 edges

d.


20 vertices 30 edges

e.


3 vertices 6 edges

f.


20 vertices 30 edges

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Problem solving

a. Can a prism have an odd number of vertices?
b. Can a pyramid have an odd number of vertices?
c. How many more faces does an octagonal pyramid have than a heptagonal pyramid?

Answer:
a. No b. Yes c. 1 face

100 More faces, vertices and edges

Topic: 3-D objects **Content links:** 99, 101
Grade 8 links: 130 **Grade 9 links:** 118-120

Objectives

- Describe, sort and compare polyhedra in term of:
  – Number of edges
  – Shape and number of faces
  – Number of vertices

Dictionary

Polyhedron: In geometry, a polyhedron (plural polyhedra or polyhedrons) is a geometric solid in three dimension with flat faces and straight edges. The word polyhedron comes from Classic Greek words: poly = many + hedron = base or face
Net: a flat diagram that can be folded to create a 3-D solid

Introduction

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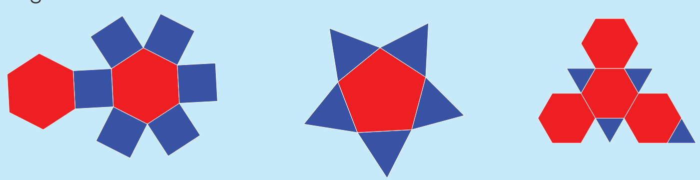
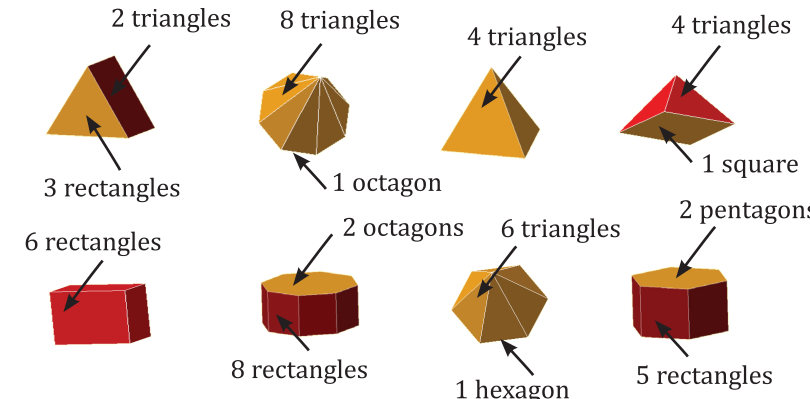
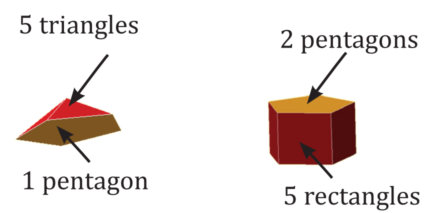
Think! Look at these **nets** of geometric solids. How many surfaces, vertices and edges does each solid have?



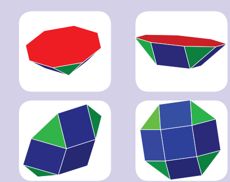
Q1

Write labels with arrows pointing to the geometrical figures which you can see in each object, and write down how many of each the object contains. Answers:





Identify all the geometric figures in this geometric solid. We provide you with four views of the geometric solid to help you.

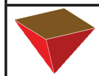
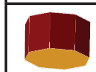
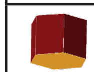
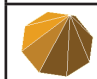


Answer: 1 octagon, 5 squares
4 triangles

100 More faces, vertices and edges *continued*

Topic: 3-D objects **Content links:** 99, 101
Grade 8 links: 130 **Grade 9 links:** 118-120

Q2

	Name of solid	Shapes made of	No. of edges	No. of vertices	No. of surfaces
	Tetrahedron (triangular pyramid)	4 triangles	6	4	4
	Square pyramid	1 square 4 triangles	8	5	5
	Octagonal prism	2 octagons 8 rectangles	24	26	10
	Hexagonal prism	2 hexagons 6 rectangles	18	12	8
	Hexagonal pyramid	1 hexagon 6 triangles	12	7	7
	Octagonal pyramid	2 octagons 8 triangles	16	9	9

Q3

Look at the table above.
a. Compare a triangular pyramid and a square pyramid. Describe the *similarities* and *differences* between them.

Similar: Both have 4 triangles
Different: Base of triangular pyramid is a triangle, of square pyramid is a square; triangular pyramid has four faces, square pyramid has five faces.

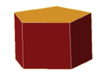
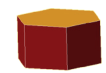
b. Describe the differences between a hexagonal prism and an octagonal prism.

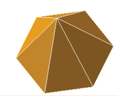
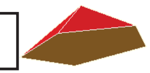
Hexagonal prism has 8 rectangles, 10 faces and the octagonal prism has 8 rectangles

c. Describe the differences between a hexagonal pyramid and an octagonal pyramid.

Hexagonal pyramid has 6 triangles, 7 faces and the octagonal pyramid has 8 triangles and 9 faces

d. What should you do to the geometric solid on the left to change it to the geometric solid on the right? Answers:

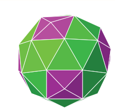
i.  Add one more rectangle 

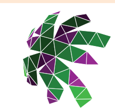
ii  Take away one triangle 

79

Solve this with a family member.

Describe the geometric solid using the words surfaces (faces), vertices and edges.
We give you the unfoldings to help you to solve this.





Answer: 20 faces, 12 vertices, 30 edges

Even more faces, vertices and edges

Topic: 3-D objects **Content links:** 99-100
Grade 8 links: 130 **Grade 9 links:** 118-120

Objectives

- Describe, sort and compare polyhedral in term of:
  - Number of edges
  - Shape and number of faces
  - Number of vertices

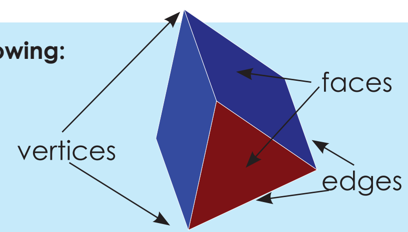
Dictionary

Prism and pyramid with the same name (hexagonal prism and hexagonal pyramid, for example) have a same type of the base, but differ in that the prism has two bases, and pyramid only one.

80 **Introduction**

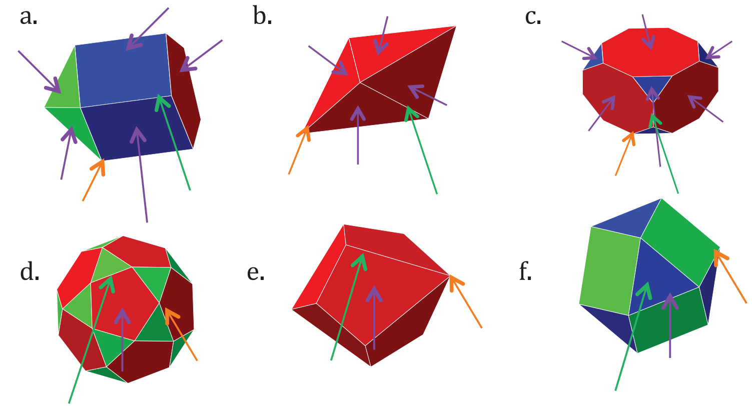
Revise the following:

- faces
- vertices
- edges



Identify the faces, vertices and edges in this photograph.

Q1 Look at the different polyhedra.
Identify the surfaces (faces), vertices and edges.
Answers:



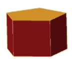
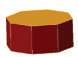
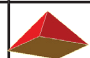
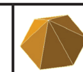
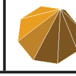
Faces/surfaces (Purple arrows)
Edges (Green arrows)
Vertices (orange arrows)

Q2 Visualise how many vertices a pentagonal prism has. 10
a. How many edges does it have? 15
b. How many faces? 7
c. What about a heptagonal prism? 21 edges and 9 faces
d. Or a heptagonal pyramid? 14 edges and 8 faces

101 Even more faces, vertices and edges *cont...*

Topic: 3-D objects **Content links:** 99-100
Grade 8 links: 130 **Grade 9 links:** 118-120

Q3

	Solid	Vertices	Edges	Faces	Calculate F – E + V for each geometric solid. F = faces, E = edges and V = vertices. What do you notice?
Triangular prism		6	9	5	$5 - 9 + 6 = 2$
Rectangular prism		8	12	6	$6 - 12 + 8 = 2$
Pentagonal prism		10	15	7	$7 - 15 + 10 = 2$
Hexagonal prism		12	18	8	$8 - 18 + 12 = 2$
Octagonal prism		16	24	10	$10 - 24 + 16 = 2$
Triangular pyramid		4	6	4	$4 - 6 + 4 = 2$
Square pyramid		5	8	5	$5 - 8 + 5 = 2$
Pentagonal pyramid		6	10	6	$6 - 10 + 6 = 2$
Hexagonal pyramid		7	12	7	$7 - 12 + 7 = 2$
Octagonal pyramid		9	16	9	$9 - 16 + 9 = 2$

Problem solving

Look at Euler's formula. This equations shows us the number of faces, edges and vertices 8 – 7 + 1 = 2. Is this a polyhedron. Why or why not?

81

Answer: No. If there are 8 faces there cannot be only 7 edges and only one vertex. Every 2 faces joined creates an edge and there is no 3-D solid with 7 edges.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

102 Views

Topic: 3-D objects **Content links:** None
Grade 8 links: 134 **Grade 9 links:** 118-120

Objectives
Revise using nets to make models of a geometric solid: cube

Dictionary
Cube: In geometry , a cube is a three–dimensional solid object which has six square faces (or sides), eight vertices and twelve edges. The cube can also be called a regular hexahedron and is one of the five Platonic solids.
Net: a flat diagram that can be folded to create a 3-D solid

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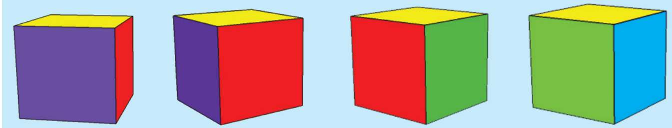
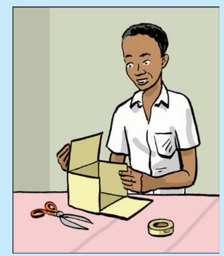
Introduction

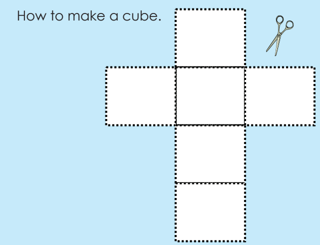
Did you know ?
A cube has 11 nets: there are 11 ways to flatten a hollow cube by cutting seven edges.

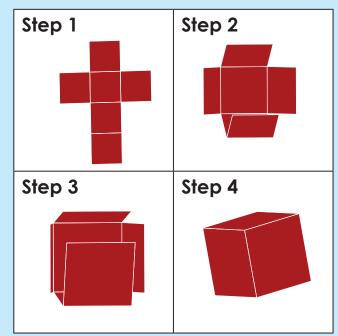
You can do this activity with those learners that are done with their work.

Make a cube and put it in front of you. Turn it to look at different views.









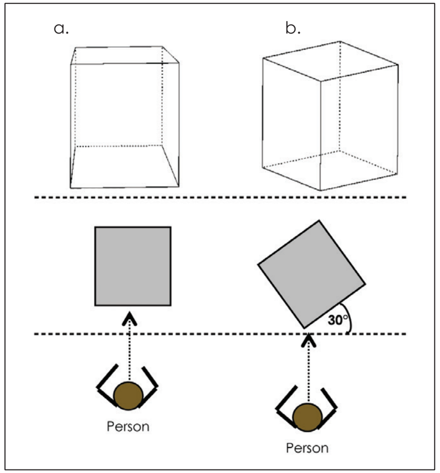
102 Views *continued*

Topic: 3-D objects **Content links:** None
Grade 8 links: 134 **Grade 9 links:** 118-120

Q1

Look at the drawings below. Explain them.
See if you can draw a cube at an angle of 30° as below in b, without a protractor.

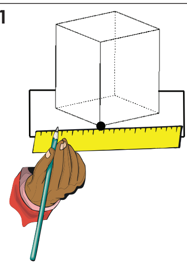
Answer: Drawing a. is a view from the front where the person can see mainly the square facing and the top. Drawing b. is also from the font but looking down a bit more where the person can see two sides and the top.
Learner's own drawing.



Q2

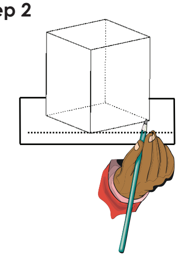
Now draw a cube using ruler and protractor by going through the following steps after first placing a cube on your desk on top of a piece of paper.

Step 1



Draw a line parallel to the side of the table. Then draw a line perpendicular to the vertex that touches the line.

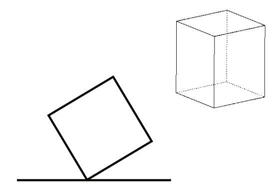
Step 2



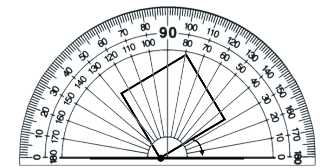
Place the cube on the line in the way you see it (approximately 30° turned).

Trace around the base of the cube.

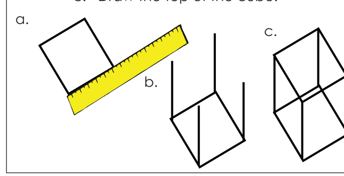
Step 3



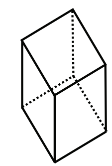
Step 4 Measure your angle to see how close your estimation was.



Step 5
a. Measure the length of the sides.
b. Draw lines showing the height of the cube of the same length.
c. Draw the top of the cube.



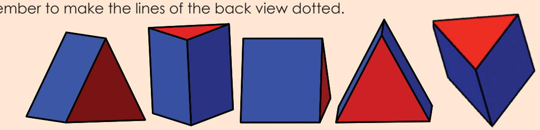
Step 6 It is important to use dotted lines to show the back of the cube (or any other geometric solid).



85

Problem solving

Sit at your desk, look at the sketches below and then place the geometric solid in the same position on your desk. Are all of the drawings possible? Make a drawing of any of these solids showing it in four steps. Remember to make the lines of the back view dotted.



Answer: Yes. All these drawing are of possible views.

103 Constructing a pyramid net

Topic: 3-D objects **Content links:** 95
Grade 8 links: None **Grade 9 links:** None

Objectives

- Construct a net of a pyramid

Dictionary

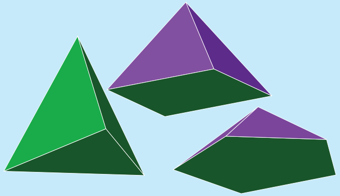
Pyramid: This is a 3-D object that has a polygon as a base. The base of the pyramid gives the pyramid its name. The other faces are all triangles which meet at the top (the apex).
Prism: This is a 3-D object that has two identical parallel faces that gives the shape its name. The rest of the faces are rectangles.

86

Introduction

Did you know?
The pyramid of Khufu (at Giza in Egypt) is one of Seven Wonders of the Ancient world, and it was originally 146,5 metres high when it was built about 4 575 years ago.

What is a pyramid? Look at the pictures and describe a pyramid.



Where do we find real pyramids?

Do we find pyramids only in Egypt?

Q1

Construct the net for a tetrahedron.

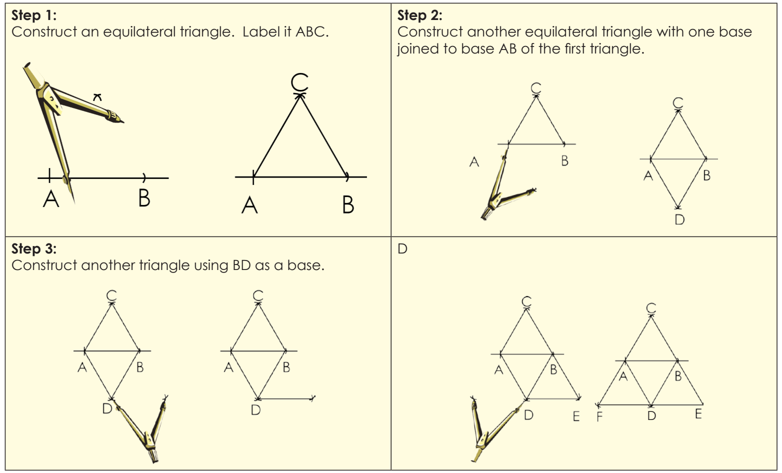
Answers:

Step 1:
Construct an equilateral triangle. Label it ABC.

Step 2:
Construct another equilateral triangle with one base joined to base AB of the first triangle.

Step 3:
Construct another triangle using BD as a base.

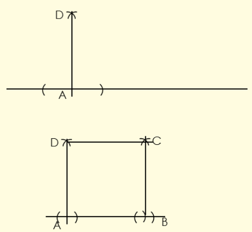
D

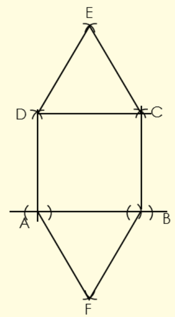


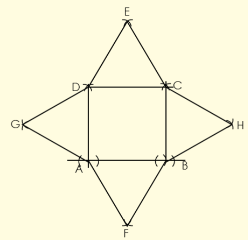
103 Constructing a pyramid net *continued*

Topic: 3-D objects **Content links:** 95
Grade 8 links: None **Grade 9 links:** None

Step 1:	**Step 2:**	**Step 3:**
Construct two perpendicular lines. The lengths of AD and AB should be the same. Use your pair of compasses to measure them. From there, construct square ABCD.	• Using AB as a base, construct a triangle. • Using DC as a base, construct a triangle.	• Using DA as a base, construct a triangle. • Using BC as a base, construct a triangle.







i) After you have constructed the square–based pyramid, answer the following questions:

- what difficulties did you have?

Answer: Learner's own answer

- what would you do differently next time?

Answer: Learner's own answer

ii) Now do the construction on cardboard, cut it out and make the square pyramid.

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Problem solving

Look at this gift box and make it yourself.

Answer: Learner's own answer

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

104 Construct a net of a prism

Topic: 3-D objects **Content links:** 95
Grade 8 links: None **Grade 9 links:** None

Objectives

- Construct a net of a triangular prism and a rectangular prism

Dictionary

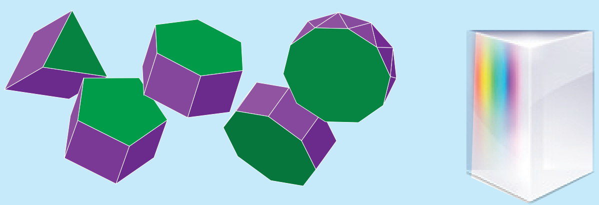
Prism: is a 3-D object that has two identical parallel faces (the ends or bases). These bases are used to name the prism, depending on their shape: triangular, rectangular, pentagonal, hexagonal, etc. The other faces are always rectangles or parallelograms. All its faces are flat.

A triangular prism is made up of two triangles as bases (ends) and three rectangles.

88

Introduction

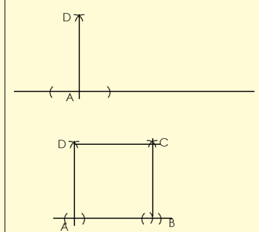
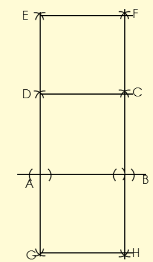
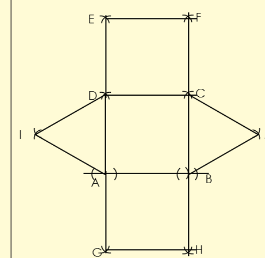
What is a prism? Look at the pictures and describe a prism.



Some people think a prism only takes on this shape. How can you find out if this is true?

Q1

Construct the net of a triangular prism.

Step 1:	Step 2:	Step 3:
Construct two perpendicular lines. The lengths of AD and AB could be the same or one could be longer to form a rectangle. Use your compasses to measure them). From there, construct rectangle ABCD.	• Using DC as a base, construct a square (or rectangle). • Using AB as a base, construct another square (or rectangle).	• Using DA as a base, construct a triangle. • Using BC as a base, construct a triangle.
		

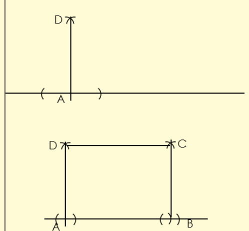
104 Construct a net of a prism *continued*

Topic: 3-D objects **Content links:** 95
Grade 8 links: None **Grade 9 links:** None

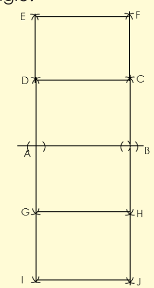
Q2

Construct a rectangular prism.

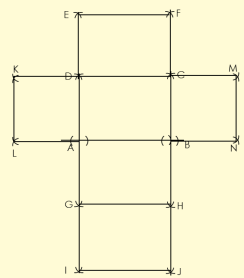
Step 1:
Construct two perpendicular lines. The length between A and B should be longer than that between D and A. Use your compass to measure them. From there, construct rectangle ABCD.



Step 2:
- Use DC as a base to construct another rectangle above.
- Use AB as a base to construct another rectangle below. Label the new points G and H.
- Use GH as a base to construct another rectangle.



Step 3:
- Use DA as a base to construct a square.
- Use CB as a base to construct a square.



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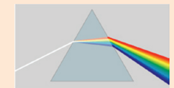
Problem solving

What does this prism show us?



Answer: This prism is showing us the refraction of light rays. As white light travels through a glass triangular prism it is broken up into light of the separate colours of the rainbow.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

105 Integers

Topic: Integers **Content links:** 106-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Objectives

- Recognize, order and compare integers
- Count forward and backwards in integers for any interval

Dictionary

Integer: any whole number, positive or negative and including zero

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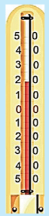
Introduction

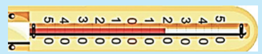


"What is the temperature on a hot, sunny day?"
Point out the degrees on this thermometer. What does it mean when the temperature is two degrees below zero? Show where this is on the thermometer.

You would use a negative sign to write this number since it is below zero.

–2

Where is five degrees below zero on the thermometer? Is this hotter or colder than two degrees below zero?



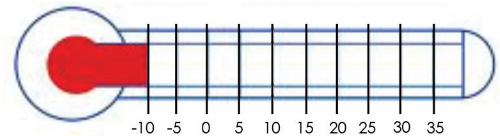
If you turn the thermometer sideways it looks like a number line and now you can see that the negative numbers are to the left of zero and the positive numbers are to the right of zero, with zero being neither positive nor negative.

Q1

Write the appropriate temperature for the stated weather condition. Answers: these are some possible answers

a. About 29 ^{0}C, at least above 25 ^{0}C
b. About 20 ^{0}C, betweeen 18 ^{0}C and 22 ^{0}C
c. About 1 ^{0}C, between 0 ^{0}C and 5 ^{0}C
d. – 8 ^{0}C
e. 10 below zero – the further it moves away from the zero (from right to left or from top to bottom on the thermometer scale) the colder it becomes

f.



Q2

Where will the money mentioned in each sentence go, in the negative or positive column? Answers:

Statement	Positive	Negative
a. Peter won R100 in the draw.	✔	
b. Peter gave his twin sister half of his prize		✔
c. Cindy lost her purse with R20 in it.		✔
d. David sold his cell phone for R200.	✔	
e. I bought airtime for R50 with some of my savings.		✔
f. We raised R500 during the school fetê.	✔	
g. We used R100 from the money raised to buy food for the party.		✔
h. My older brother earned R120 for the work he did.	✔	
i. We made R100 profit.	✔	
j. We made R200 loss.		✔

105 Integers *continued*

Topic: Integers **Content links:** 106-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Q3

Complete the questions below after completing the table in Question 2. Answers:

a. Words that should be circled are: won, gave, lost, sold, bought, raised, used, earned, profit, loss
b. When the money becomes more it is ticked off in the positive column.
c. When it becomes less it is ticked off in the negative column.
d. It includes positive and negative whole numbers and zero.
e. Some examples are:
Temperatures above (+)and below (–) zero
Checking bank accounts: sufficient funds (+) and overdrawn (–)
Business: Profit (+) and loss (–)
Healthy eating: Calories eaten (+) and calories burned (–)

Q4

Complete these number lines. Answers:

a. –5 –4 –3 –2 –1 0 1 2 3 4 5

b. –25 –20 –15 –10 –5 0 5 10 15 20 25

c. –16 –14 –12 –10 –8 –6 –4 –2 0 2 4

d. –50 –40 –30 –20 –10 0 10 20 30 40 50

e. –5 –4 –3 –2 –1 0 1 2 3 4 5

f. –9 –6 –3 0 3 6 9 12 15 18 21

Complete the following: Answers:

a. {3, 2, 1, 0, –1, –2, –3}
b. {–10, –9, –8, –7, –6, –5}
c. {8, 6, 4, 2, 0, –2, –4, –6}
d. {–9, –6, –3, 0, 3, 6}
e. {12, 8, 4, 0, –4, –8}

91

Problem solving

Take a newspaper and find five negative numbers in it.
a. Explain what each number tells us.
b. Write down the opposite numbers for the five numbers.

Answer: Here is an example.

a. – 5 ^{0}C : temperature of weather in Europe
– R50: price cut on dress
– 1%: percentage of share price
– 40: NIKKEI Commodities
– 5 kg: weight loss

b. – 5 ^{0}C: opposite number is 5 ^{0}C
– R50: opposite number is R50
– 1%: opposite number is 1%
– 40: opposite number is 40
– 5 kg: opposite number is 5 kg

Reflection questions

Did learners meet the objectives?

106 More integers

Topic: Integers **Content links:** 105, 107-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Objectives

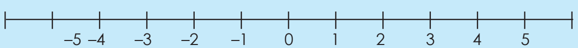
- Recognize, order and compare integers
- Count forward and backwards in integers for any interval

Dictionary

Integer: any whole number, positive or negative and including zero

92

Introduction



- What do we call the units to the right of the zero? (positive numbers or integers)
- What do we call the units to the left of the zero? (negative numbers or integers)
- What will five units to the left of 3 be?
- What will five units right of 3 be?
- What is the opposite of –4?
- What is the opposite of 4?
- What is three below zero?

Q1

Write an integer to represent each description.
Answers:

a. Five units to the left of 4 on a number line. –1
b. 20 below zero. –20
c. The opposite of 271. –271
d. Eight units to the left of –3 on a number line. –11
e. Eight units to the right of –3 on a number line. 5
f. 16 above zero. 16
g. 14 units to the right of –2 on a number line. 12
h. Seven units to the left of –8 on a number line. –15
i. The opposite of –108. 108
j. 15 below zero. –15

Q2

Order the integers from smallest to biggest. Answers:

a. –71; –66; –61; –51; –31; –5; 5; 21; 31; 39; 42; 66
b. –64; –20; 21; 42; 48; 72
c. –31; –30; –24; –14; –3; 4; 9; 15; 21; 26; 31; 44
d. –79; –41; –31; 54; 57
e. –31; –26; 10; 12; 23; 26; 31; 32
f. –56; –55; –54; –43; –39; –37; 18; 22; 43; 44; 52
g. –41; –31; –23; –21; 2; 4
h. –13; –12; –10; –6; –2; 4; 7; 9; 10; 12; 15
i. –25; –24; –15; –13; –12; –7; 2; 6; 11; 22
j. –44; –24; –20; –2; 5; 21; 41; 55; 73

106 More integers *continued*

Topic: Integers **Content links:** 105, 107-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Q3

Fill in <, > or =:

Answers:

a. $-2 < 2$ b. $-10 < 10$ c. $-5 < 0$
d. $-4 < -3$ e. $-9 < -6$ f. $-20 < -16$

Q4

Give five numbers smaller than and five numbers bigger than:

Answers: Possible answers are:

a. -2

Smaller	Bigger
−3	−1
−4	1
−5	2
−6	3
−7	8

b. -99

Smaller	Bigger
−100	−98
−140	−80
−145	−1
−160	3
−190	5

c. 1

Smaller	Bigger
0	2
−1	6
−3	10
−5	15
−9	101

93

Problem solving

Make your own word problem using a negative and a positive number

Answer: here is an example answer
In Bethlehem the temperature was $-12\ ^0\text{C}$ in the morning. If the temperature rose by $8\ ^0\text{C}$, what is the temperature now?

$-12\ ^0\text{C} + 8\ ^0\text{C} = 4\ ^0\text{C}$

The temperature is $4\ ^0\text{C}$ now.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

107 Calculate integers

Topic: Integers **Content links:** 105-106, 108-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Objectives

- Add and subtract with integers

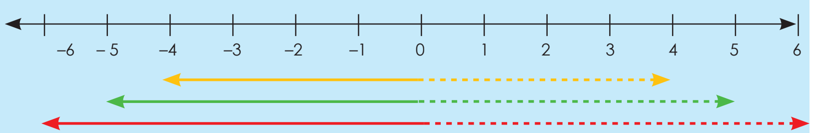
Dictionary

Integer: any whole number, positive or negative and including zero

94

Introduction

What is the opposite of –3? How many units are there from –3 to 3?



Explain the lines above.

Q1

We have learnt that two integers are opposites if they are each the same distance away from zero. Write down the opposite integers for the following:

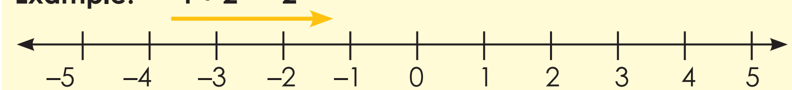
Answers:

a. –2 is 2	b. 3 is –3	c. –7 is 7
b. 8 is –8	e. –10 is 10	f. –15 is 15
g. 1 is –1	h. – 100 is 100	i. 75 is –75

Q2

Calculate the following.

Example: **$-4 + 2 = -2$**



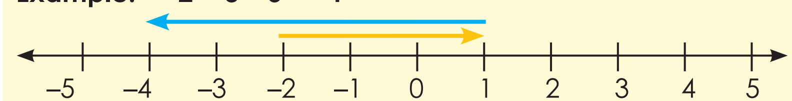
Answers:

a. $-5 + 5 = 0$	b. $-2 + 3 = 1$	c. $-7 + 8 = 1$
d. $2 - 3 = -1$	e. $+4 - 6 = -2$	f. $10 - 12 = -2$

Q3

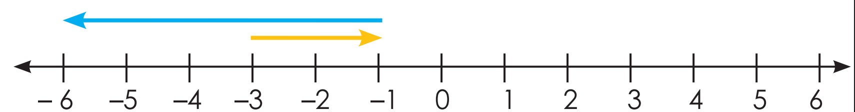
Calculate the following.

Example: **$-2 + 3 - 5 = -4$**



Answers:

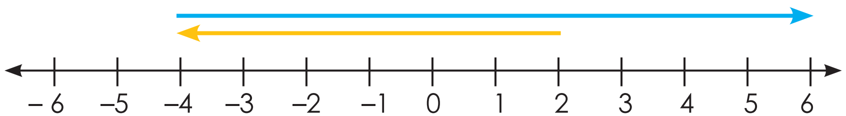
a. $-3 + 2 - 5 = -6$



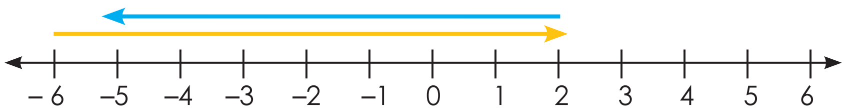
Calculate integers *continued*

Topic: Integers **Content links:** 105-106, 108-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

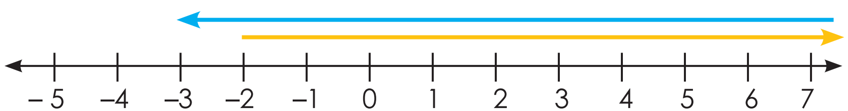
b. $2 - 6 + 10 = 6$



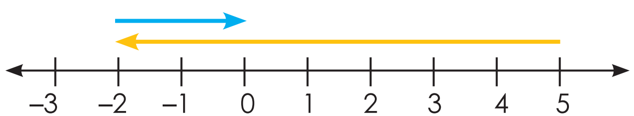
c. $-6 + 8 - 7 = -5$



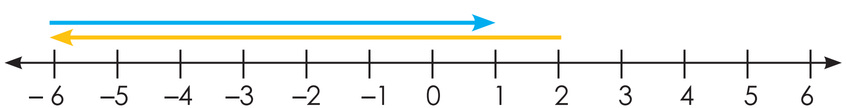
d. $-3 + 10 - 11 = -4$



e. $9 - 11 + 2 = 0$



f. $2 - 8 + 7 = 1$



Q4

Complete the following.

Answers:

a. Subtract 4 from – 3 is –7
b. Subtract 6 from – 8 is –14
c. Subtract 5 from 3 is –2
d. Subtract 9 from 7 is –2
e. Subtract 3 from – 2 is –5

95

Problem solving

What is:

- The sum of 10 and 8, and the sum of –9 and –8?
- The sum of 101 and 85, and the sum of –98 and –104?
- The sum of 19 and –8, and the sum of –19 and 8?
- The sum of –7 and –14, and the sum of –4 and 20?
- The sum of 100 and –50, and the sum of –100 and 50?

Answers:

$10 + 8 + (-9 - 8) = 18 - 17 = 1$
$101 + 85 + (-98 - 104) = 186 - 202 = -16$
$19 - 8 + (-19 + 8) = 11 + (-11) = 0$
$-7 - 14 + (-4 + 20) = -21 + 16 = -5$
$100 - 50 + (-100 + 50) = 50 - 50 = 0$

Reflection questions

Did learners meet the objectives?

108 Integer operations

Topic: Integers **Content links:** 105-107, 109-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Objectives
- Solve problems in context involving addition and subtraction with integers
- Add and subtract with integers

Dictionary
Integer: any whole number, positive or negative and including zero

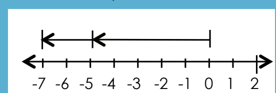
Introduction

96

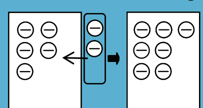
Discuss the following:

Add integers with the same sign
Find –5 + (–2).

Method 1: Use a number line.
- Start at zero.
- Move 5 units left.
- From there, move 2 units left.



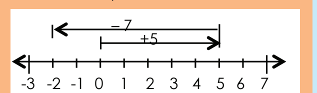
Method 2: Draw a diagram.



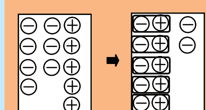
–5 + (–2) –5 + (–2) = – 7

Add integers with different signs
Find 5 + (–7).

Method 1: Use a number line.
- Start at zero.
- Move 5 units right.
- From there, move 7 units left.



Method 2: Draw a diagram.



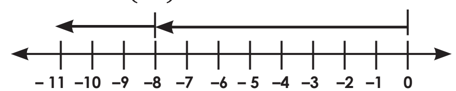
5 + (–7) 5 – 7 = – 2

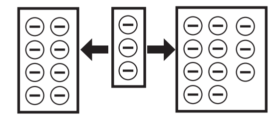
Q1

Complete the following.
- Number line method
- Drawing a diagram

Answers:

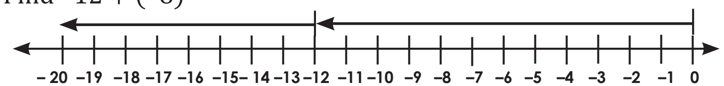
a. Find – 8 + (–3)

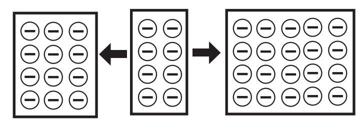




– 8 + (–3) = –11

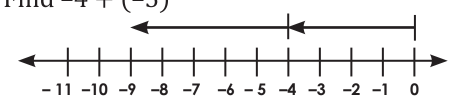
b. Find –12 + (–8)

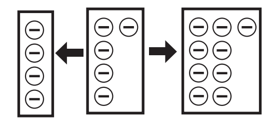




–12 + (–8) = –20

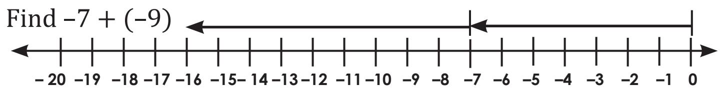
c. Find –4 + (–5)

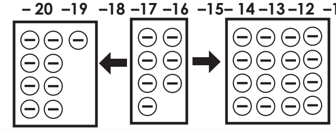




–4 + (–5) = –9

d. Find –7 + (–9)



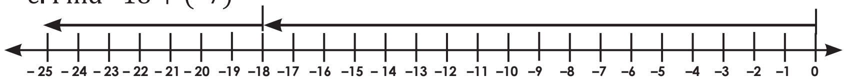


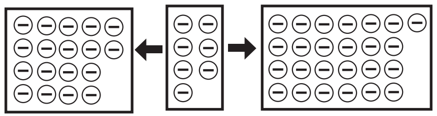
–7 + (–9) = –16

108 Integer operations *continued*

Topic: Integers **Content links:** 105-107, 109-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

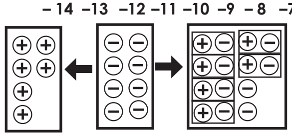
e. Find −18 + (−7)





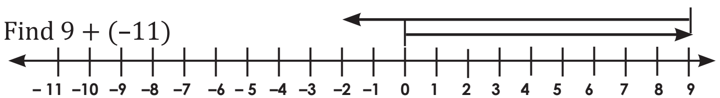
$-18 + (-7) = -25$

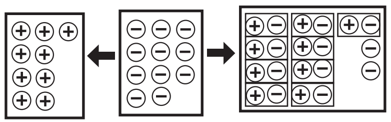
f. Find 6 + (−8)



$6 + (-8) = -2$

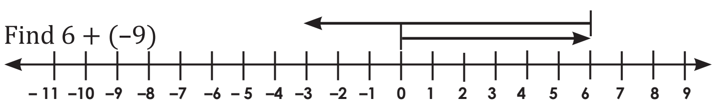
g. Find 9 + (−11)

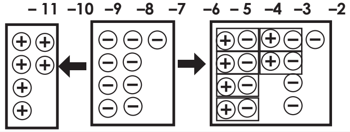




$9 + (-11) = -2$

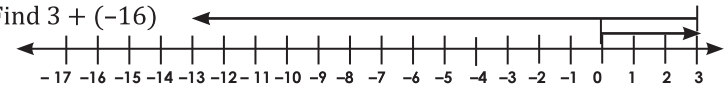
h. Find 6 + (−9)

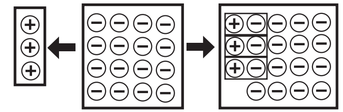




$6 + (-9) = -3$

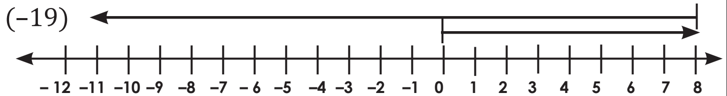
i. Find 3 + (−16)

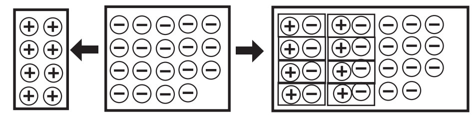




$3 + (-16) = -13$

j. Find 8 + (−19)





$8 + (-19) = -11$

Q2 Write sums for the following: Answers:

a. $-3 + (-2) = -5$

b. $+4 + (-6) = -2$

c. $-5 + (-4) = -7$

d. $+7 + (-10) = -3$

e. $-4 + (-2) = -6$

f. $-7 + (-3) = -10$

g. $-8 + 3 = -5$

97

Help a friend!

Write down step–by–step how you would explain integer operations to a friend who missed a day at school.

Answers: Here is a possible answer. I used number lines to do the calculations. When the number is positive I moved from the left (the zero) to the right and then, from the number where I stopped, I can move left (to subtract) or right (to add). I would show the friend how I did calculations with both positive and negative numbers.

109 Adding and subtracting integers

Objectives

- Add and subtract with integers

Dictionary

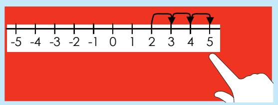
Integer: any whole number, positive or negative and including zero

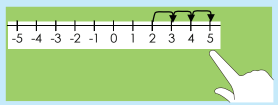
98

Introduction

Subtracting a negative number is just like adding a positive number. The two negatives cancel each other out. 2 + 3 = 2 – (–3)

If you are **adding** a **positive number**, move your finger to the **right** as many places as the value of that number. For example, if you are adding 3, move your finger three places to the right: 2 + 3 = 5

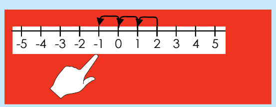


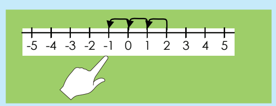


If you are **subtracting** a **negative number**, move your finger to the right as many places as the value of that number. For example, if you are subtracting –3, move your finger three places to the right: 2 – (–3) = 5

Adding a negative number is just like subtracting a positive number: 2+ (–3) = 2 – 3

If you are **adding** a **negative number**, move your finger to the **left** as many places as the value of that number. For example, if you are adding (–3), move your finger three places to the left: 2 + – 3 = –1



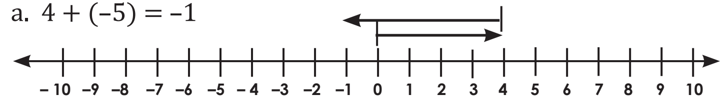


If you are **subtracting** a **positive number**, move your finger to the **left** as many places as the value of that number. For example, if you are subtracting 3, move your finger three places to the left: 2 – 3 = –1

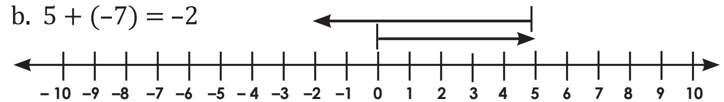
Q1

Calculate the following, make use of number lines. Answers:

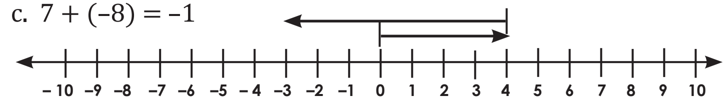
a. $4 + (-5) = -1$



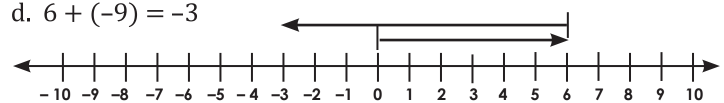
b. $5 + (-7) = -2$



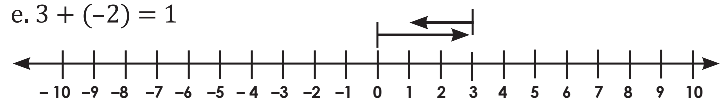
c. $7 + (-8) = -1$



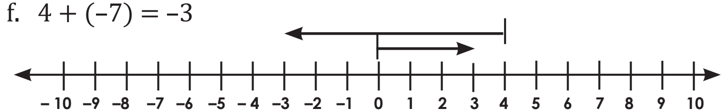
d. $6 + (-9) = -3$



e. $3 + (-2) = 1$



f. $4 + (-7) = -3$



Topic: Integers, Properties of numbers **Content links:** 105-108, 110-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

109 Adding and subtracting integers *continued*

Q2

Calculate the following:

Answers:

a. $4 - (-5) = 9$	b. $5 - (-7) = 12$	c. $5 - (-7) = 12$
d. $6 - (-9) = 15$	e. $3 - (-2) = 5$	f. $4 - (-7) = 11$
g. $5 - (-4) = 9$	h. $2 - (-1) = 3$	i. $3 - (-4) = 7$
j. $1 - (-3) = 4$	k. $2 - (-5) = 7$	l. $5 - (-11) = 16$
m. $7 - (-6) = 13$	n. $8 - (-12) = 20$	o. $5 - (-9) = 14$
p. $4 - (-4) = 8$	q. $3 - (-3) = 6$	r. $5 - (-12) = 17$
s. $2 - (-4) = 6$	t. $3 - (-6) = 9$	u. $5 - (-6) = 11$
v. $3 - (-8) = 11$	w. $7 - (-10) = 17$	x. $6 - (-6) = 12$
y. $4 - (-6) = 10$	z. $7 - (-14) = 21$	

Q3

Explain in your own words what you had to do to get to the answer:

Answers: Here are possible answers.

a. I used number lines to do the calculations. When the number is positive I moved from the left (the zero) to the right and then from the number where I stopped back (from right to left).

b. I used number lines to do the calculations. When the number is positive I moved from the left (the zero) to the right and then from the number where I stopped back (from right to left).

99

Problem solving

Make your own problem using integers.

Answers: Here is an example answer.
$9 - (-17) = 26$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

110 Integer calculations

Topic: Integers, Properties of numbers **Content links:** R9, 105-109, 111-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

Objectives

- Add and subtract with integers

Dictionary

Integer: any whole number, positive or negative and including zero

Introduction

Describe:				Give an example of each using symbols:
Positive number	+	Negative number	=	Positive answer Negative answer
Positive number	–	Negative number	=	Positive answer Negative answer
Negative number	+	Positive number	=	Positive answer Negative answer
Negative number	–	Positive number	=	Positive answer Negative answer

Q1

100

Calculate the following:

Answers:

a. $12 + (-31)$
$= 12 - 31$
$= -19$

b. $-28 + (-42)$
$= -28 - 42$
$= -70$

c. $7 + (-34)$
$= 7 - 34$
$= -27$

d. $33 + (-44)$
$= 33 - 44$
$= -11$

e. $5 + (-432)$
$= 5 - 432$
$= -427$

f. $-15 + (-20)$
$= (-15) - 20$
$= -35$

g. $-15 + 5$
$= -10$

h. $19 + 14$
$= 33$

i. $25 + 4$
$= 29$

j. $4 + 7 = 11$

Q2

Calculate the following:

Example: $-14 - -20$
$= -14 + 20$
$= 6$

Answers:

a. $7 - (-31)$
$= 7 + 31$
$= 38$

b. $35 - 31$
$= 4$

c. $(-17) - 8$
$= -25$

110 Integer calculations *continued*

Topic: Integers, Properties of numbers **Content links:** R9, 105-109, 111-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

d. $47 - (-46)$
$= 47 + 46$
$= 93$

e. $(-41) - 17$
$= -58$

f. $28 - (-46)$
$= 28 + 46$
$= 74$

g. $-47 - (-7)$
$= -47 + 7$
$= -40$

h. $-28 - 15$
$= -43$

i. $-15 - 3$
$= -18$

j. $5 - 31$
$= -26$

Q3

Solve the following:

Answers:

a. $-2 + 44 = 42$	b. $-14 + (-18) = -32$	c. $-9 + (-21) = -30$
d. $-3 + 36 = 33$	e. $14 + 2 = 16$	f. $14 + 49 = 63$
g. $42 + 23 = 65$	h. $-2 + (-10) = -12$	i. $38 + 27 = 65$
j. $-46 + (-26) = -72$	k. $2 + (-43) = -41$	l. $46 + (-16) = 30$
m. $-37 + (-44) = -81$	n. $37 + (-31) = 6$	o. $-4 + (-28) = -32$
p. $11 + (-30) = -19$	q. $-18 + 24 = 6$	r. $45 + 28 = 73$
s. $30 + (-29) = 1$	t. $12 + (-44) = -32$	u. $-44 + 29 = -15$
v. $-35 + 24 = -11$	w. $23 + 10 = 33$	x. $-31 + 49 = 18$
y. $22 + 4 = 26$	z. $41 + 19 = 60$	

101

Problem solving

a. Give three integers of which the sum is –9. Use two positive integers and one negative integer.

b. Give three integers of which the sum is –4. Use two negative integers and one positive integer.

c. Give four integers of which the sum is –11. Use two negative integers and two positive integers.

Answers:
a. $3 + 4 + (-16) = -9$
b. $-12 + (-4) + 12 = -4$
c. $-51 + (-3) + 40 + 3 = -11$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

111 Commutative property and integers

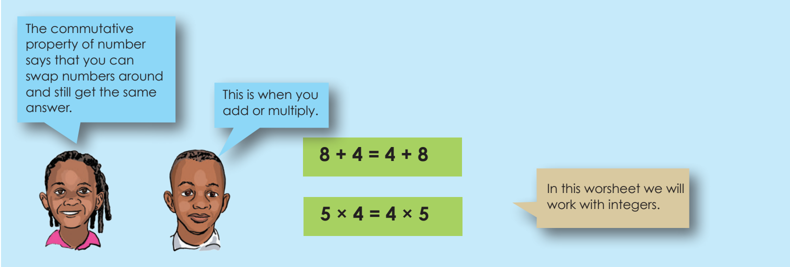
Objectives

- Recognize and use the commutative property of addition for integers

Dictionary

Integer: any whole number, positive or negative and including zero
Commutative: In any operation of addition or multiplication the order in which you combine numbers does not matter. Subtraction and division are not commutative.
E.g. $4 + 5 = 5 + 4$
$-4 + 5 = 5 + (-4)$

Introduction



Q1

102

Use the commutative property to change the following expressions to equations.

Example: $8 + (-3) = (-3) + 8 = 5$
$(-8) + 3 = 3 + (-8) = -5$

Answers:

a. $4 + (-5)$
$4 + (-5) = (-5) + 4$
$= -1$

b. $(-10) + 7$
$-(10) + 7 = 7 + (-10)$
$= -3$

c. $3 + (-9)$
$3 + (-9) = (-9) + 3$
$= 6$

d. $8 + (-11)$
$8 + (-11) = (-11) + 8$
$= -3$

e. $(-4) + 8$
$(-4) + 8 = 8 + (-4)$
$= 4$

f. $9 + (-2)$
$9 + (-2) = (-2) + 9$
$= 7$

Q2

Show that the communicative property holds for the addition of integers.

Example: $a = -2$ and $b = 3$
$a + b = b + a$
$(-2) + 3 = 3 + (-2)$
$1 = 1$

Topic: Integers, Properties of numbers **Content links:** R9, 1, 105-110, 112-113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

111 Commutative property and integers *continued*

a. $a+b=b+a$ *if* $a+4;\ b=-1$

$a+b=b+a$
$4+(-1)=(-1)+4$
$3=3$

b. $a+b=b+a$ *if* $a=-2;\ b=7$

$a+b=b+a$
$-2+7=7+(-2)$
$5=5$

c. $a+b=b+a$ *if* $a=2;\ b=7$

$a+b=b+a$
$2+7=7+2$
$9=9$

d. $x+y=y+x$ *if* $x=-1;\ y=13$

$x+y=y+x$
$-1+13=13+(-1)$
$12=12$

e. $x+y=y+x$ *if* $x=-5;\ y=9$

$x+y=y+x$
$-5+9=9+(-5)$
$4=4$

f. $d+e=e+d$ *if* $e=-12;\ d=7$

$d+c=c+d$
$7+(-12)=(-12)+7$
$-5=-5$

g. $t+s=s+t$ *if* $t=-4;\ s=10$

$t+5=5+t$
$-4+10=10+(-4)$
$6=6$

h. $a+b=b+a$ *if* $a=-10;\ b=7$

$a+b=b+a$
$-10+7=7+(-10)$
$-3=-3$

i. $y+z=z+y$ *if* $z=-8;\ y=2$

$y+z=z+y$
$2+(-8)=(-8)+2$
$-6=-6$

j. $k+m=m+k$ *if* $k=-13;\ m=20$

$k+m=m+k$
$-13+20=20+(-13)$
$7=7$

103

Problem solving

Use the commutative property to make your own equation and prove that it is satisfied using the numbers –8 and 21.

Answers: an example answer

$a+b=b+a$ *if* $a=-8$
$-8+21=21+(-8)$ $b=21$
$13=13$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

Topic: Integers, Properties of numbers **Content links:** R9, 2, 105-111, 113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

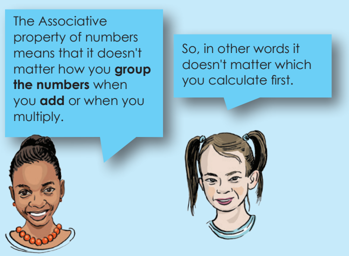
112 Associative property and integers

Objectives
- Recognise and use the associative property of addition for integers

Dictionary
Integer: any whole number, positive or negative and including zero
Associative: In any operation of addition or multiplication the manner in which pairs of numbers are grouped does not matter. In other words the order in which you add or multiply does not matter. Subtraction and division are not associative.
E.g. $(4 + 5) + 6 = (6 + 5) + 4$
$(4 \times 5) \times 6 = (6 \times 5) \times 4$

104

Introduction



Example addition:
$(2 + 3) + 5 = 2 + (3 + 5)$
Because $5 + 5 = 2 + 8 = 10$
Example multiplication:
$(2 \times 4) \times 3 = 2 \times (4 \times 3)$
$8 \times 3 = 2 \times 12 = 24$

In this worksheet we will look at integers and the associative property

Q1 Use the associative property to calculate the following.

Answers:

a. $[(-6) + (4 + 2)] = [(-6 + 4) + 2]$
$-6 + 6 = -2 + 2$
$0 = 0$

b. $[3 + 7 + (-5)] = [(3 + 7) + (-5)]$
$3 + 2 = 10 - 5$
$5 = 5$

c. $[(6 + 4) + (-2)] = [6 + (4 + (-2)]$
$10 - 2 = 6 + 2$
$8 = 8$

d. $[(-3) + (7 + 5)] = [(-3) + 7) + 5]$
$-3 + 12 = 4 + 5$
$9 = 9$

e. $[(-4) + (6 + 2)] = [(-4 + 6) + 2]$
$-4 + 8 = 2 + 2$
$4 = 4$

f. $[3 + ((-7) + 5)] = [(3 + (-7)) + 5]$
$3 + (-2) = (3 - 7) + 5$
$3 - 2 = -4 + 5$
$1 = 1$

Topic: Integers, Properties of numbers **Content links:** R9, 2, 105-111, 113
Grade 8 links: R4, 11-13 **Grade 9 links:** None

112 Associative property and integers *continued*

g. $[(-9) + (3 + 11)] = [(-9 + 3) + 11]$
$-9 + 14 = -6 + 11$
$5 = 5$

h. $[(12 + 13) + (-10)] = [12 + (12 + (-10)]$
$25 - 10 = 12 + 3$
$15 = 15$

i. $[(-3) + (9 + 11)] = [(-3) + 9 + 11]$
$-3 + 20 = 6 + 11$
$17 = 17$

j. $[(-12) + (13 + 10)] = [-12 + 13) + 10]$
$-12 + 23 = 1 + 10$
$11 = 11$

Q2 Show that the associative property for addition holds for integers.

Answers:

a. $(a + b) + c = a + (b + c)$
If: $a = 4$
$b = -5$
$c = 3$
$(4 + (-5)) + 3 = 4 + (-5 + 3)$
$-1 + 3 = 4 + (-2)$
$2 = 2$
$7 = 7$

b. $(a + b) + c = a + (b + c)$
If: $a = 2$
$b = 9$
$c = -4$
$(2 + 9) + (-4) = 2 + (9 + (-4))$
$11 - 4 = 2 + (9 - 4)$
$7 = 2 + 5$

c. $a + (b + c) = (a + b) + c$
If: $a = -8$
$b = 1$
$c = 2$
$-8 + (1 + 2) = (-8 + 1) + 2$
$-8 + 3 = -7 + 2$
$-5 = -5$

d. $a + (b + c) = (a + b) + c$
If: $a = -2$
$b = 11$
$c = 12$
$-2 + (11 + 12) = (-2 + 11) + 12$
$-2 + 23 = 9 + 12$
$21 = 21$

105

Problem solving

Use the associative property to make your own equation and prove that it is equal using the numbers -5, 17 and 12.

Answer: a possible answer:
$a + (b + c) = (a + b) + c$
$-5 + (17 + 12) = (-5 + 17) + 12$
$-5 + 29 = 12 + 12$
$24 = 24$

If $a = -5$
$b = 17$
$c = 12$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

113 Integers: distributive property and integers

Objectives

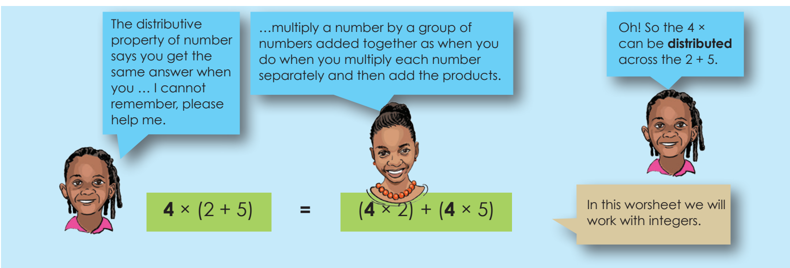
- Recognize and use the distributive property of integers

Dictionary

Integer: any whole number, positive or negative and including zero
Distributive property: the property of number that you get the same answer when you multiply a number by a group of numbers added together as when you do when you multiply each of those numbers separately and then add the products together

106

Introduction



Q1 Use the distributive property to calculate the sums. Before you calculate highlight or underline the distributed number.

Answers:

a. $-4 \times (2 + 1)$
$-4 \times 3 = (-4 \times 2) + (-4 \times 1)$
$-12 = -8 + -4$
$-12 = -12$

b. $-5 \times (3 + 6)$
$-5 \times 9 = (-5 \times 3) + (-5 \times 6)$
$-45 = -15 + -30$
$-45 = -45$

c. $4 \times (-2 + 1)$
$4 \times -1 = (4 \times -2) + (4 \times 1)$
$-4 = -8 + 4$
$-4 = -4$

d. $5 \times (-3 + 6)$
$5 \times (3) = (5 \times -3) + 5 \times 6)$
$15 = -15 + 30$
$15 = 15$

e. $4 \times (2 + -1)$
$4 \times 1 = (4 \times 2) + (4 \times - 1)\ 4$
$4 = 8 + -4$
$4 = 4$

f. $5 \times (3 + -6)$
$5 \times -3 = (5 \times 3) + (5 \times - 6)$
$-15 = 15 + -30$
$-15 = -15$

g. $(-3 \times 2) + (-3 \times 4)$
$-6 + (-12) = -3(2 + 4)$
$-6 - 12 = -3(6)$
$-18 = -18$

h. $(-7 \times 1) + (-7 \times 4)$
$-7 + (-28) = -7(1 + 4)$
$-7 - 28 = -7(5)$
$-35 = -35$

i. $(8 \times -4) + (8 \times 2)$
$-32 + 16 = 8(-4 + 2)$
$-16 = 8(-2)$
$-16 = -16$

113 Integers: distributive property and integers *continued*

Q2

Substitute and calculate. Answers:

a. $a \times (b + c)$ if $a = 2$, $b = -3$, $c = -5$

$a \times (b + c) = (a \times b) + (a \times c)$
$2 \times (-3 + (-5)) = (2 \times -3) + (2 \times -5)$
$-4 \times 4 = -6 + -10$
$-16 = -16$

b. $a \times (\ b + c)$ if $a = -7$, $b = 2$, $c = 3$

$a \times (b + c) = (a \times b) + (a \times c)$
$-7 \times (2 + 3) = (-7 \times 2) + (-7 \times 3)$
$-7 \times 5 = -14 + -21$
$-35 = -35$

c. $a \times (b + c)$ if $a = 1$, $b = -8$, $c = 2$

$a \times (b + c) = (a \times b) + (a \times c)$
$1 \times (-8 + 2) = (1 \times (-8)) + (1 \times 2)$
$1 \times -6 = -8 + 2$
$-6 = -6$

d. $(a \times b) + a + c)$ if $a = 3$, $b = -10$, $c = 5$

$a \times (b + c) = (a \times b) + (a \times c)$
$3 \times (-10 + 5) = (3 \times -10) + (3 \times 5)$
$3 \times -5 = -30 + 15$
$-15 = -15$

e. $m \times (n + p)$ if $m = 3$, $n = 2$, $p = -11$

$m \times (n + p) = (m \times n) + (m \times p)$
$3 \times (2 + -11) = (3 \times 2) + (3 \times -11)$
$3 \times -9 = 6 + (-33)$
$-27 = -27$

f. $(m \times n) + (m \times p)$ if $m = 7$, $n = 8$, $p = -9$

$(m \times n) + (m \times p) = m \times (n + p)$
$(7 \times 8) + (7 \times (-9)) = 7 \times (8 + (-9))$
$56 + (-63) = 7 \times (-1)$
$-7 = -7$

107

Problem solving

Make use of the distributive property to write your own equation for:
$a = -4$, $b = 5$ and $c = 11$

Answer: A possible answer.
$c \times (a + b)$
$= 11 \times (-4 + 5)$
$= (11 \times -4) + (11 \times 5)$
$= -44 + 55$
$= 11$

114 Number patterns: constant difference and ratio

Objectives

- Investigate and extend numeric and geometric patterns in physical or diagrammatic form in sequences involving a constant difference or ratio

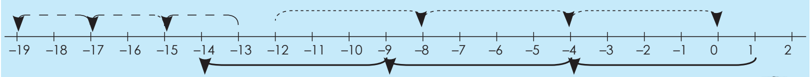
Dictionary

Number pattern: a list of numbers that follow a certain sequence or pattern, e.g. 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Difference: an equal difference between terms in a sequence. Example: 3, 6, 9, 12, 15, … (the constant difference is 3)
Constant Ratio: the value of the ratio between each pair of numbers in a sequence remains the same - constant, e.g. as in the geometrical sequence: 2, 4, 8, 16, ... (the ratio 2:4 = 4:8 = 8:16 is constant)

108

Introduction

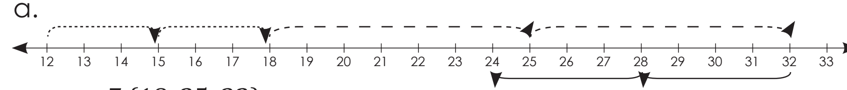
Describe the patterns using "adding" and "subtracting".



Subtracting 2: –13, –15, –17, –19
Adding 4: –12, –8, –4, 0
Subtracting 5: 1, –4, –9, –14

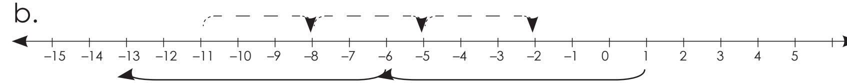
Q1 Describe each pattern.

a.



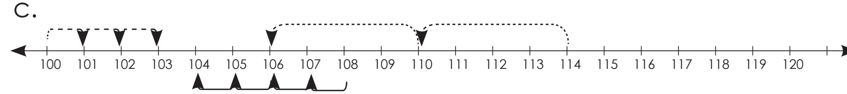
+7 {18, 25, 32}
+3 {12; 15; 18}
–4 {32; 28; 24}

b.



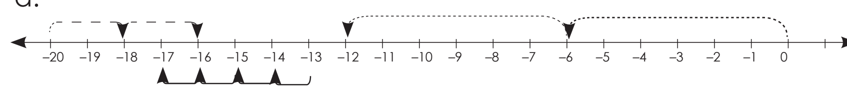
+3 {–11; –8; –5; –2}
–2 {0; +2; +4}
–7 {1; –6; –13}

c.



+1 {100;101; 102; 103}
–4 {114; 110; 106}
–1 {108; 107; 106; 105; 104}

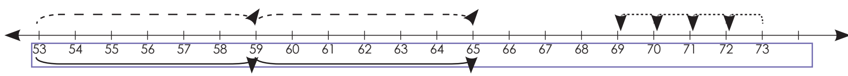
d.



+2 {–20; –18; –16}
–6 {0; –6; –12}
–1: {–13; –14; –15; –16; –17}

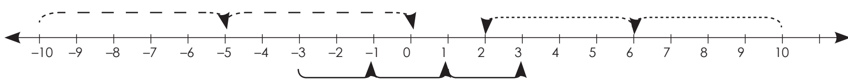
114 Number patterns: constant difference and ratio *continued*

e.



------- +6 {53; 59; 65}
.... −1 {73; 72; 71; 70}
——— +6 {53; 59; 65}

f.



------- +5 {−10; −5; 0}
.... −4 {10; 6; 2}
——— + 2 {−3; −1; 1; 3}

Q2

Describe the pattern.

Example: −12, −8, −4, 0
Adding 4

Answers:

a. 16, 11, 6, 1
Subtract 5

b. 25, 22, 19, 16
Subtract 3

c. −16, −8, 0, 8
Add 8

d. −4, −1, 2, 5
Add 3

e. −79, −69, −59, −49
Add 10

f. 58, 50, 42, 34
Subtract 8

Q3

Example: −12 , −48, −192, − 768 −12 x 4 = −48, −48 x 4 = −192, −192 x 4 = −768
Multiplying the previous number by 4

Answers:

a. −7, −21, −63, −189 Multiplying the previous number by −3

b. −4, −44, −484, −5 324 Multiplying the previous number by 11

c. −11, −66, −396, −2 376 Multiplying the previous number by 6

d. 2, −8, 32, −128 Multiplying the previous number by −4

e. 9, 72, 576, 4 608 Multiplying the previous number by 8

f. −5, −45, −405, −3 645 Multiplying the previous number by 9

109

Problem solving

Brenda collects shells. Every day she picks up double the amount of the previous day. On day 1 she picks up 8 shells. On day 2 she collects 16. How many shells will she pick up on day 3 if the pattern continues? Write down the rule.

Answers:
Answer: 8; 16; 32 ...
Rule: Double the previous term

Reflection questions

Did learners meet the objectives?

115 Number patterns: neither constant difference nor a constant ratio

Objectives

- Investigate and extend numeric patterns in physical or diagrammatic form in sequences involving a constant difference or ratio

Dictionary

Number pattern: a list of numbers that follow a certain sequence or pattern, e.g. 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... (starts at 2 and each following term is 2 times the term before)
Constant Difference: an equal difference between terms in a sequence. Example: 3, 6, 9, 12, 15, … (the constant difference is 3)
Constant Ratio: the value of the ratio between each pair of numbers in a sequence remains the same - constant, e.g. as in the geometrical sequence: 2, 4, 8, 16, ... (the ratio 2:4 = 4:8 = 8:16 is constant)
Neither a constant difference nor a constant ratio: there is no constant difference or constant ratio in a sequence, e.g. 2, 3, 5, 8, 11, ... (the difference increases by 1 each time: +1, +2, +3, +4, ...)

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Introduction

Describe the following: –1, –2, –4, –7, –11, –16, ...
What will the next three terms be, using the identified rule?
This pattern has neither a constant difference nor a constant ratio. It can be described in your own words as "increasing the difference between consecutive terms by 1 each time" or "subtracting 1 more than what was subtracted to get the previous term". Using this rule, the next three terms will be –22, –29, –37.

Take your time to describe the pattern in words.

Q1

Describe the pattern and make a diagram to show the value of each term.

Answers:

a. –4, 1, 5, 8, 10

$\boxed{-4}$ +5 $\boxed{1}$ +4 $\boxed{5}$ +3 $\boxed{8}$ +2 $\boxed{10}$

b. 8, 10, 13, 17, 22

$\boxed{8}$ +2 $\boxed{10}$ +3 $\boxed{13}$ +4 $\boxed{17}$ +5 $\boxed{22}$

c. 2, –2, –8, –16, -26

$\boxed{2}$ –4 $\boxed{-2}$ –6 $\boxed{-8}$ –8 $\boxed{-16}$ –10 $\boxed{-26}$

d. –11, –12, –10, –13, –9

$\boxed{-11}$ –1 $\boxed{-12}$ +2 $\boxed{-10}$ –3 $\boxed{-13}$ +4 $\boxed{-9}$

e. -7, -1, 11, 29, 53

$\boxed{-7}$ +6 $\boxed{-1}$ +12 $\boxed{11}$ +18 $\boxed{29}$ +24 $\boxed{53}$

f. 5, -3, -10, -16, -21

$\boxed{5}$ –8 $\boxed{-3}$ –7 $\boxed{-10}$ –6 $\boxed{-16}$ –5 $\boxed{-21}$

115 Number patterns: neither constant difference nor a constant ratio *cont...*

Q2

What will the value of the tenth pattern be?
Answers:
a. 102 (The sequence grows by adding the difference between the previous two terms plus 2.) [The rule is $n^2 + 2$.]

b. 110 (The sequence grows by adding the difference between the previous two terms plus 2.) [[The rule is $n^2 + n$ or $n(n + 1)$.]

c. 1 023 (The sequence grows by adding double the difference between the previous two terms to the previous term). [The rule is: $2^n - 1$.]

d. 55 (Square the position number and minus the previous term.) [The rule is $\frac{n(n+1)}{2}$.]

e. 919 (Cube the position number and minus the square of the previous position number.) [The rule is $n^3 - (n - 1)^2$.]

Q3

What will the value of the term be? Complete the table. Mark the one or ones where the sequence is neither a constant difference nor a constant ratio.
a. 500 (Add 25 to the previous term; Rule = $n \times 25$) [This is a constant difference.]
b. –416 (Add –4 to the previous term; Rule = $n \times -4$) [This is a constant difference.]
c. 205 379 (Rule = n^3) [THIS IS NOT A CONSTANT DIFFERENCE NOR CONSTANT RATIO.]
d. 468 (Add 13 to previous term; Rule = $n \times 13$) [This is a constant difference.]

e. 609 (Add 21 to previous term; Rule = $n \times 21$) [This is a constant difference.]

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Problem solving

a. Thabo builds a brick wall around the perimeter of his house. On the first day he uses 75 bricks, on the second day he uses 125 and on the third day he uses 175. How many bricks will he need on the fourth day? Write a rule for the pattern.

b. Ravi draws 2 figures on the first page, 4 figures on the second page, 8 figures on the third page, and 16 figures on the fourth page. If this pattern continues, how many figures will Ravi draw on the fifth page?

c. Lisa read 56 pages on Sunday, 66 pages on Monday, 76 pages on Tuesday, and 86 pages on Wednesday. If this pattern continued, how many pages would Lisa read on Thursday?

d. Thandi cut 1 rose from the first plant, 3 roses from the second plant, 7 roses from the third plant, and 13 roses from the fourth plant. If this pattern continued, how many rose would Thandi cut from the fifth plant?

Answers:
a. Thabo will need 225 bricks on day four as he increases the number of bricks by 50 each day. Rule for this is $n = (n \times 50) + 25$.
b. Ravi will draw 32 figures (each day he doubles the number of figures). [A rule for this is $n = 2^n$.]
c. Lisa will read 96 pages on Thursday (each day she reads an extra 10 pages). [Rules for this are n = $(n \times 10) + 46$ or $10n + 6$.]
d. Thandi will cut 21 roses from the fifth plant (she increases the number she cuts in multiples of 2 starting at 1). [A rule for this is $n = n^2 - 1 + 10$.]

Topic: Numeric and geometric patterns **Content links:** R9, 65-71, 114-115, 117
Grade 8 links: 27-28, 105, 107-108 **Grade 9 links:** 27-28, 65-69

116 Number sentences and words

Objectives

- Describe and justify the general rules for observed relationships between numbers in own words

Dictionary

Number Sentence: This is a mathematical sentence made up of numbers and symbols instead of words. The term is used in mathematics education as a way of asking students to write down a simple equation using numbers and mathematical symbols, so, e.g. 7 + 5 = 12 is a number sentence. It is not the same thing as a **word problem.**
Word problem: a mathematical problem expressed in words but which needs to be translated into mathematical numbers and symbols to be solved, e.g. Tasha had twenty more books than Ben, but six less than Mandla, who had thirty-two. How many books did Ben have?
This word problem is then translated into a number sentence [32 – 6 – 20 = 10].

Q1 Introduction

Look at the following sequences:
i. Calculate the 20th term using a number sentence.
ii. Describe the rule in your own words.

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Example: Number sentence: –6, –10, –14, –18
Rule in words: (–4 × the position of the term) – 2.

To make it easier to write down rules for number sequences we often use these abbreviations:
Tn is the term (the value of the term)
n is the term number (the position of the term)
Example:
For the rule for the number sequence {3, 5, 7, 9, ...} we would write:
$Tn = 2n + 1$
The value of the "5th term" would then be:
$T_5 = 2n + 1$
$T_5 = (2 \times 5) + 1 = 11$

a. Number sentence: 8, 14, 20, 26

i.
$T_n = 6(n) + 2$
$T_1 = 6(1) + 2 = 8$
$T_{20} = 6(n) + 2 = 6(20) + 2 = 122$

ii. (6 × position of term) plus 2 (or Add 6 to the value of the previous term)

b. Number sentence: 0, –3, –6, –9

i.
$T_n = -3n + 3$ $T = -3(1) + 3$
$T_1 = -3 + 3 = 0$
$T_{20} = -3n + 3$
$= (-3)(20) + 3 - 60 + 3 = -57$

ii. (3 × position of term) plus 3 (or Add – 3 to the value of the previous term)

c. Number sentence: –4, –5, –6, –7

i.
$T_n = -1n - 3$
$T_1 = -1(1) - 3 = -1 - 3 = -4$
$T_{20} = -1(20) - 3 = -20 - 3 = -23$

ii. (–1 × position of term) minus 3 (or Add – 1 to the value of the previous term)

d. Number sentence: –2, 3, 8, 13

i.
$T_n = 5n - 7$
$T_1 = 5(1) - 7 = -2$
$T_{20} = 5(20) - 7 = 100 - 7 = 93$

ii. (5 × position of term) minus 7 (or Add 5 to the value of the previous term)

Topic: Numeric and geometric patterns **Content links:** R9, 65-71, 114-115, 117
Grade 8 links: 27-28, 105, 107-108 **Grade 9 links:** 27-28, 65-69

116 Number sentences and words *continued*

e. Number sentence:
–2, –6, –10, –14

i.
$T_n = -4n + 2$
$T_1 = -4(1) + 2 = -2$
$T_{20} = -4(20) + 2 = -80 + 2 = -78$

ii. (–4 × position of term) plus 3 (or Add – 4 to the value of the previous term)

f. Number sentence:
–1, 6, 13, 21

$T_n = 7n - 8$
$T_1 = 7(1) - 8 = -1$
$T_{20} = 7(20) - 8 = 140 - 8 = 132$

(7 × position of term) minus 8 (or Add 7 to the value of the previous term)

g. Number sentence:
13, 21, 29, 37

i.
$T_n = 8n + 5$
$T_1 = 8(1) + 5 = 13$
$T_{20} = 8(20) + 5 = 160 + 5 = 165$

ii. (8 × position of term) plus 3 (or Add 8 to the value of the previous term)

h. Number sentence:
0, 1, 2, 3

$T_n = n - 1$
$T_1 = 1 - 1 = 0$
$T_{20} = 20 - 1 = 19$

(1 × position of term) minus 1 (or Add 1 to the value of the previous term)

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i. Number sentence:
7, 5, 3, 1

$T_n = -2n + 9$
$T_1 = -2(1) + 9 = 7$
$T_{20} = -40 + 9 = -31$

(–2 × position of term) plus 9 (or Add – 2 to the value of the previous term)

j. Number sentence:
2, 4, 6, 8

$T_n = 2n$
$T_1 = 2(1) = 2$
$T_{20} = 2(20) = 40$

2 × position of term (Add 2 to the value of the previous term)

Problem solving

Tshepo earns R25 per week for washing his father`s motor car. If he saves R5,50 the first week, R7,50 the second week and R9,50 the third week, how much will he save in the fourth week if the pattern continues?

Calculate the total amount he saves over 4 weeks. Write a rule for the number sequence.

Answer:
Week 4 = R11,50 (Rule is add R2 each week)
Four weeks total = 5,50 + 7,50 + 9,50 + 11,50 = R34

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

117 Number sequences: describe a pattern

Objectives

- Describe and justify the general rules for observed relationships between numbers in own words
- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns in tables

Dictionary

Number pattern: a list of numbers that follow a certain sequence or pattern, e.g. 3, 6, 9, 12, 15, ... (starts at 3 and adds 3 every time)
Arithmetic sequence: an arithmetic progression or arithmetic sequence where the sequence of numbers is such that the difference between the consecutive terms is constant, e.g in the sequence 1, 3, 5, 7, 9, ... the common difference is 2
Geometric sequence: a number sequence made by multiplying or dividing by the same value each time, e.g.: 2, 4, 8, 16, 32, 64, 128, 256, ... the difference between terms is not constant (starts at 2 and each following term is 2 times the term before)

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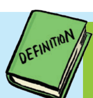
Introduction



A sequence is a list of numbers or objects which are in a special order.
Example:
Arithmetic number sequence: –2, –4, –6, –8
Geometric number sequence: –2, –4, –8, –16

What is the difference between an arithmetic number sequence and a geometric number sequence? Give one example of each.

Q1

Describe the sequence in different ways using the template provided.

a. –1, 2, 5, 8

i.

ii.

Position in the sequence	1	2	3	4
Term	–1	2	5	8

$3(1) - 4$ $3(2) - 4$ $3(3) - 4$ $3(4) - 4$

iii. Where n is the position of the term.
First term: $3(1) - 4 = -1$
Second term: $3(2) - 4 = 2$
Third term: $3(3) - 4 = 5$
Fourth term: $3(4) - 4 = 8$

n^{th} term: $3n - 4$

b. 3, 5, 7, 9

i.

ii.

Position in the sequence	1	2	3	4
Term	3	5	7	9

$2(1) + 1$ $2(2) + 1$ $2(3) + 1$ $2(4) + 1$

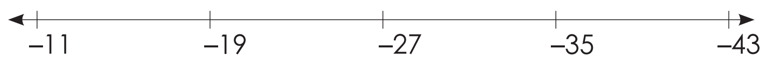
iii. Where n is the position of the term.
First term: $2(1) + 1 = 3$
Second term: $2(2) + 1 = 5$
Third term: $2(3) + 1 = 7$
Fourth term: $2(4) + 1 = 9$

n^{th} term: $2n + 1$

117 Number sequences: describe a pattern *continued*

c. –11, –19, –27, –35

i.



ii.

Position in the sequence	1	2	3	4
Term	–11	–19	–27	–35

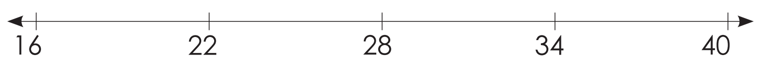
–8(1) – 3 | –8(2) – 3 | –8(3) – 3 | –8(4) – 3

iii. Where n is the position of the term.
First term: –8(1) – 3 = 5
Second term: –8(2) – 3 = 5
Third term: –8(3) – 3 = 5
Fourth term: –8(4) – 3 = 5

n^{th} term: $-8n - 3$

d. 16, 22, 28, 34

i.



ii.

Position in the sequence	1	2	3	4
Term	16	22	28	34

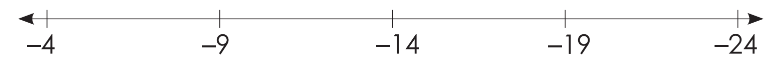
6(1) + 10 | 6(2) + 10 | 6(3) + 10 | 6(4) + 10

iii. Where n is the position of the term.
First term: 6(1) + 10 = 16
Second term: 6(2) + 10 = 22
Third term: 6(3) + 10 = 28
Fourth term: 6(4) + 10 = 34

n^{th} term: $6n - 10$

e. –4, –9, –14, –19

i.



ii.

Position in the sequence	1	2	3	4
Term	–4	–9	–14	–19

–5(1) + 1 | –5(2) + 1 | –5(3) + 1 | –5(4) + 1

iii. Where n is the position of the term.
First term: –5(1) + 1 = –4
Second term: –5(1) + 1 = –9
Third term: –5(1) + 1 = –14
Fourth term: –5(1) + 1 = –19

n^{th} term: $-5n + 1$

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Problem solving

Write the rule for the number sequence: –3, –5, –7, –9

Answer: Subtracting 2 from the previous term or rule $-2n - 1$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

118 Input and output values

Topic: Input and output values **Content links:** R9, 48-51, 72, 119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R8, 29-36, 70-80

Objectives

- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented in flow diagrams, by formulae, number sentences and verbally
- Determine input values, output values or rules for patterns and relationships using flow diagrams and formulae

Dictionary

Input values: A number that is inputted into the flow diagram that determines the output value. Example: 15 + 5 = 20. 15 is the input value.
Output values: A number value that is the result of the flow diagram's input and process. For Example see diagram on the right.

Introduction

118

Look and discuss.

a		b
1		–9
9		–81
11	$b = –a \times 9$	–99
25		–225
8		–72

The rule is: $b = –a \times 9$

$b = –a \times 9$. Look at the flow diagram. Which numbers can replace a?

- $b = –1 \times 9 = –9$
- $b = –9 \times 9 = –81$
- $b = –11 \times 9 = –99$
- $b = –25 \times 9 = –225$
- $b = –8 \times 9 = –72$

p		t
2		
8		
6	$t = –p \times 5 + 6$	
5		
3		

Calculate:

- $t = –2 \times 5 + 6 = –16$
- $t = –8 \times 5 + 6 = –46$
- $t = –6 \times 5 + 6 = –36$
- $t = –5 \times 5 + 6 = –31$
- $t = –3 \times 5 + 6 = –21$

Q1

Revision: complete the flow diagrams.

a.

p		t
1		–6
17		–102
12	$t = p \times (–6)$	–72
9		–54
36		–216

b.

p		t
23		69
16		48
28	$t = –p \times –3$	84
34		102
15		45

c.

p		t
19		–133
2		–14
11	$t = p \times –7$	–77
8		–56
17		–119

d.

p		t
–11,5		30
–1		9
+17	$t = –p \times 2 + 7$	–27
7.5		–8
–4		15

Q2

Use the given rule to calculate the value of b.
Answers:

a. Rule: $b = -a \times 6$

a	b
6	–36
15	–90
8	–48
2	–12
17	–102

$b = -a \times 6$
$b = -6 \times 6 = -36$
$b = -15 \times 6 = -90$
$b = -8 \times 6 = -48$
$b = -2 \times 6 = -12$
$b = -17 \times 6 = -102$

b. Rule: $b = a \times 15$

a	b
2	30
8	120
12	180
20	300
29	435

$b = a \times 15$
$b = 2 \times 15 = 30$
$b = 8 \times 15 = 120$
$b = 12 \times 15 = 180$
$b = 20 \times 15 = 300$
$b = 29 \times 15 = 435$

c. Rule: $y = -x + 9$

x	y
7	2
8	1
6	3
–2	11
–16	25

$y = -x + 9$
$y = -7 + 9 = 2$
$y = -8 + 9 = 1$
$y = -6 + 9 = 3$
$y = -2 + 9 = 11$
$y = -16 + 9 = 25$

d. Rule: $s = r + 11$

r	s
4	15
7	18
9	20
20	31
5	16

$s = r + 11$
$s = 4 + 11 = 15$
$s = 7 + 11 = 18$
$s = 9 + 11 = 20$
$s = 20 + 11 = 31$
$s = 5 + 11 = 16$

118 Input and output values *continued*

Topic: Input and output values **Content links:** R9, 48-51, 72, 119
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R8, 29-36, 70-80

Q3

Use the given rule to calculate the variable. Answers:

a.

b	rule	a
39		127
16		58
13	$a = b \times 3 + 10$	49
25		85
10		40

b.

g	rule	h
17		49
2		19
8	$g = h \times 2 + 15$	31
14		43
29		73

c.

y	rule	x
4		12
2		8
10	$x = y \times 2 + 4$	24
3		10
7		18

d.

n	rule	m
5		21
10		26
12	$m = n + 8 \times 2$	28
33		49
7		23

e.

b	rule	c
31		−56
28		−50
16	$c = b \times -2 + 6$	−26
9		−12
14		−22

f.

q	rule	p
2		40
4		64
6	$p = q \times 12 + 16$	88
8		112
10		136

Q4

Prepare one flow diagram to present to the class.
Answer: An example.

b	rule	c
10		30
20		60
30	$c = b \times 3$	90
40		120
50		150

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Problem solving

- Draw your own flow diagram where a = –c – 9.
- Draw your own flow diagram where a = c x 3 –7

Answer: Learner's own answer. Here are two examples.

a	rule	b
1		−10
2		−11
3	$b = -a - 9$	−12
4		−13

c	rule	d
1		−1
2		5
3	$d = c \times 3 - 7$	11
4		17

119 More input and output values

Topic: Input and output values **Content links:** R9, 48-51, 72, 118
Grade 8 links: R7, 28, 106, 109 **Grade 9 links:** R8, 29-36, 70-80

Objectives

- Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented by formulae, number sentences, in tables and verbally

Dictionary

Input values: A number that is inputted into the flow diagram that determines the output value. Example: 15 + 5 = 20. 15 is the input value.
Output values: A number value that is the result of the flow diagram's input and process. For Example see diagram on the right.

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Introduction

45

x	1	2	3	4		12	n
y	5	7	9	11		m	93

27

Why does n = 45 and m = 27?

To find m and n, you have to substitute the corresponding values for x or y into the rule and solve the equation by inspection.

The rule $y = 2x + 3$ describes the relationship between the given x and y values in the table.

But in tables such as this one, **more than one rule** might be possible to describe the relationship between x and y values.

Now try and find another rule.

Multiple rules are acceptable if they match the given input values with the corresponding output values

Q1

Solve m and n using the given rule.

a. $y = 3x - 1$

x	2	4	6	n	10	20
y	5	11	17	23	29	m

m = 59
n = 8

b. $y = -2x + 6$

x	1	2	3	4	5	n
y	4	2	0	m	–4	–174

m = –2
n = 8

c. $y = -4x - 2$

x	3	4	5	6	n	10	100
y	14	–18	–22	–26	–30	–42	m

m = –402
n = 7

d. $y = x + 2$

x	2	n	4	5	16	17
y	4	5	6	7	m	19

m = 18
n = 3

e. $t = -8s + 2$

s	1	2	3	n	5	6	7
t	–6	–14	–22	–30	m	– 46	–54

m = –38
n = 4

f. $q = 7p - 7$

p	1	5	10	20	n	100
q	0	28	m	133	168	693

$m = 63$

$n = 25$

Q2

What is the value of m and n?

a.

x	1	2	3	4		25	n	51
y	–2	–5	–8	–11		m	–95	–152

Rule: $-3n + 1$
$m = 74$
$n = 32$

b.

x	1	2	3	4		n	30	60
y	–3	2	7	12		27	m	292

Rule: $5n - 8$
$m = 74$
$n = 7$

c.

x	1	2	3	4		10	15	n
y	–9	–11	–13	–15		–27	m	–47

Rule: $-2n - 7$
$m = -67$
$n = 10$

d.

x	1	2	3	4		7	n	46
y	4	5	6	7		10	13	m

Rule: $n + 3$
$m = 49$
$n = 10$

e.

x	1	2	3	4		6	10	n
y	–1	–7	–13	–19		–31	m	–61

Rule: $-6n + 5$
$m = -55$
$n = 11$

f.

x	1	2	3	4		n	41	70
y	–12	–14	–16	–18		–70	m	–150

Rule: $-2n - 10$
$m = -92$
$n = 30$

123

Problem solving

a. What is the tenth term? 4 x –5, 5 x –5, 6 x –5

b. If $y = 5x - 8$ and $x = 2, 3, 4, \ldots$, draw a table to show it.

Answer:
a. 13 x – 5

b.

x	2	3	4	5		6	7
y	2	7	12	17		22	27

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

120 Algebraic expressions

Topic: Algebraic expressions **Content links:** 74-76, 121-122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

Objectives

- Identify variables and constants in given formulae or equations
- Recognize and interpret rules or relationships represented in symbolic form

Dictionary

Algebraic Expressions: An expression that contains variables in forms of algebra. Example: $2a + 8$

124

Introduction

Compare the two examples.

–5 + 4

–5 + 4 = –1

–5 + 4 is an **algebraic expression**

–5 + 4 = –1 is an **algebraic equation**

What is on the left–hand side of the equal sign?

What is on the right–hand side?

Q1

Say if it is an expression or an equation. Answers:

a. –4 + 8 — Expression

b. –9 + 7 = –2 — Equation

c. –5 + 10 — Expression

d. –8 + 4 = –4 — Equation

e. –7 + 5 — Expression

f. –15 + 5 –10 — Expression

Q2

Describe the following: Answer:

a. –8 + 2 = –6

–8 + 2 is an expression that is equal to the value on the right-hand side, –6.
–8 + 2 = –6 is an equation. The left-hand side of an equation equals the right hand side.

b. –15 + 9 = –6

–15 + 9 is an expression that is equal to the value on the right-hand side, –6.
–15 + 9 = –2 is an equation. The left-hand side of an equation equals the right hand side.

c. –11 + 9 = –2

–11 + 9 is an expression that is equal to the value on the right-hand side, –2.
–11 + 9 = –2 is an equation. The left-hand side of an equation equals the right hand side.

d. –5 + 3 = –2

–5 + 3 is an expression that is equal to the value on the right-hand side, –2.
–5 + 3 = –2 is an equation. The left-hand side of an equation equals the right hand side.

e. $-8 + 1 = -7$
$-8 + 1$ is an expression that is equal to the value on the right-hand side, -7.
$-8 + 1 = -7$ is an equation. The left-hand side of an equation equals the right hand side.

f. $-4 + 3 = -1$
$-4 + 3$ is an expression that is equal to the value on the right-hand side, -1.
$-4 + 3 = -1$ is an equation. The left-hand side of an equation equals the right hand side.

Q3 Make use of the variable "a" and integers to create 10 expressions of your own.

Answers: Examples of possible answers

$-12 + a$	$a + (-12)$	$a + 4$
$a - 8$	$-15 + a$	$a + 20$
$a + (-5)$	$a - 2$	$a + 13$

Q4 Make use of the variable "a" and integers to create 10 equations of your own.

Answers: Examples of possible answers

$a + 2 = 12$	$a - 7 = 15$	$a + 10 = 20$
$a - 5 = -15$	$a + 3 = 8$	$a + 7 = -10$
$a + 18 = -15$	$a + 7 = -8$	$a + 2 = -18$
$a - 3 = -9$		

Q5 Say if it is an expression or an equation.

Answers:

a. $-9 + a = -2$ — Equation

b. $-3 + a = -1$ — Equation

c. $-5 + a = -3$ — Equation

d. $-18 + a$ — Expression

e. $-12 + a = -3$ — Equation

f. $-7 + a$ — Expression

125 **Problem solving**

Create 10 examples of algebraic expressions with a variable and a constant. From these create algebraic equations and solve them.

Answers: Examples of possible answers

Expression	Equation	Solution
a + 7	a + 7 = 25	a = 25 – 7 = 18
a – 2	a – 2 = 18	a = 18 + 2 = 20
a – 17	a – 17 = 20	a = 20 + 17= 37
a + (–3)	a + (–3) = 8	a = 8 + 3 = 11
a –15	a –15 = 17	a = 17 + 15 = 32
a – 9	a – 9 = 22	a = 22 + 9= 31
a + 15	a + 15 = 30	a = 30 –15 = 15
a – 4	a – 4 = 7	a = 7 + 4 = 11
a + 18	a + 18 = 35	a = 35 – 18 = 17
a – 20	a – 20 = 25	a = 25 + 20 = 45

121 The rule as an expression

Topic: Algebraic expressions **Content links:** 74-76, 120, 122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

Objectives

- Identify variables and constants in given formulae or equations
- Recognize and interpret rules or relationships represented in symbolic form

Dictionary

Algebraic Expressions: An expression that contains variables in forms of algebra. Example: $2a + 8$

126

Introduction

The rule is $-2(n) + 1$

Position in sequence	1	2	3	4	5	n
Term	–1	–3	–5	–7	–9	

Write the rule as an expression.
First term: $-2(1) + 1 = -2 + 1 = -1$
Second term: $-2(2) + 1 = -4 + 1 = -3$
Third term: $-2(3) + 1 = -6 + 1 = -5$
Fourth term: $-2(4) + 1 = -8 + 1 = -7$
Fifth term: $-2(5) + 1 = -10 + 1 = -9$
n^{th} term: $-2(n) + 1 =$

Note: These expressions all have the same meaning:
$-2n + 1$
$-2 \times n + 1$
$-2.n + 1$
$-2(n) + 1$

Q1

Describe the following in words.
Answers:

a. 9; 6; 3; 0; –3;
Subtracting 3 from previous term

b. 4; 10; 16; 22; 28;
Adding 6 to the previous term

c. 7; 14; 21; 28; 35;
Adding 7 to the previous term

d. 12; 24; 36; 48; 60;
Adding 12 to the previous term

e. 8; 16; 24; 32;
Adding 8 to the previous term

f. 6; 16; 26; 36; 46;
Adding 10 to the previous term

Q2

Describe the following sequence using an expression.
Answers:

a. 6; 8; 10; 12; 14
$2n + 4$

b. 5; 11; 17; 23; 29
$6n - 1$

c. 4; 13; 22; 31; 40
$9n - 5$

d. 8; 16; 24; 32; 40
$8(n)$

e. 15; 25; 35; 45; 55
$10n + 5$

f. 4; 7; 10; 13; 16
$3(n) + 1$

121 The rule as an expression *cont...*

Topic: Algebraic expressions **Content links:** 74-76, 120, 122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

Q3

Show what the rule means by completing the table.
Answers:

Example: For the following number sequence the rule $-2n - 1$ means:

Position in sequence	1	2	3	4	5	n
Term	-3	-5	-7	-9	-11	$-2n - 1$

(-3 is the first term, -5 is the second term, -7 is the third term, etc.)

a.

Position in sequence	1	2	3	4	5	n
Term	10	13	16	19	22	$3n + 7$

b.

Position in sequence	1	2	3	4	5	n
Term	2	10	18	26	34	$8n - 6$

c.

Position in sequence	7	2	3	4	5	n
Term	2	9	16	23	30	$7n - 5$

d.

Position in sequence	7	2	3	4	5	n
Term	-1	1	3	4	7	$2n - 3$

e.

Position in sequence	1	2	3	4	5	n
Term	8	17	26	35	44	$9n - 1$

f.

Position in sequence	1	2	3	4	5	n
Term	24	37	50	63	76	$13n + 11$

127

Problem solving

Write a rule for the following:

On the first day I spend R15, on the second day I spend R30, on the third day I spend R45. How much money do I spend on the tenth if this pattern continues?

Answer: Rule is $15n$. Answer is R150.

I save R15 in January, R30 in February R45 in March. How much money will I save in September if the pattern continues?

Answer: Rule is $15n$. Answer is R135

Thabo sells one chocolate on Monday, three chocolates on Tuesday and five on Wednesday. How many chocolates will he sell on Friday if the pattern continues?

Answer: Rule is $2n - 1$. Answer is 9 chocolates.

A farmer plants 2 rows of maize on the first day, 6 rows on the second day and 11 rows on the third day. How many rows must will he plant on the 12th day if the pattern continues.

Answer: Rule is (value of the previous term) + (the difference between the previous two terms + 1). A more complex rule is $n^2 + (5n - 2)) \div 2$. Answer is 101 rows.

Bongi spends twenty minutes on the computer on day one, thirty minutes on day two and forty minutes on day three. How much time will she spend on the computer on day nine if the pattern continues?

Answer: Rule is $10n + 10$. Answer is $10(9) + 10 = 100 = 1$ hour 40 minutes.

Reflection questions

Did learners meet the objectives?

122 Sequences and algebraic expressions

Objectives

- Identify variables and constants in given formulae or equations
- Recognize and interpret rules or relationships represented in symbolic form

Dictionary

Number Sequence: A list of numbers that follow a certain sequence or pattern. Example: 3, 6, 9, 12, 15, ... starts at 3 and adds 3 every time
Algebraic expression: a group of numbers made up of constants and variables, linked by operators, e.g. $2a + 8$

128

Introduction

–5, –9, –13, –17, –21 ...
Describe the rule of this number sequence in **words**.
What does the rule $-4n + 1$ mean for the number sequence –3, –7, –11, –15, –19, ...
Write the rule as an **expression**.

Subtracting 4 from the previous term.

First term:	$-4(1) + 1 = -3$
Second term:	$-4(2) + 1 = -7$
Third term:	$-4(3) + 1 = -11$
Fourth term:	$-4(4) + 1 = -15$
Fifth term:	$-4(5) + 1 = -19$
n^{th} term:	$-4(n) + 1$

Some texts use various abbreviations for sequences and terms

n = the position of the term (1st, 2nd, etc.)

a = the sequence

So a_n refers to term n in the sequence a

Some people use these abbreviations:

T = the sequence

T_n = the term n in the sequence T

T_{n-1} = the previous term to term n

T_{n+1} = the term after term n

Q1

Describe the following in words.
Answers:

a. –3; –12; –21; –30; –39

Subtracting 9 from the previous term.

b. –6; –13; –20; –27; –34

Subtracting 7 from the previous term.

c. –3; –5; –7; –9; –11

Subtracting 2 from the previous term.

d. 6; –4; –14; –24; –34

Subtracting 10 from the previous term.

e. –7; –8; –9; –10; –11

Subtracting 1 from the previous term.

f. –8; –12; –16; –20; –24

Subtracting 4 from the previous term.

g. –14; –17; –20; –23; –26

Subtracting 3 from the previous term.

h. –19; –21; –23; –25; –27

Subtracting 2 from the previous term.

i. 9; –2; –13; –24; –35

Subtracting 11 from the previous term.

j. –1; –6; –11; –16; –21

Subtracting 5 from the previous term.

Topic: Algebraic expressions **Content links:** 74-76, 120, 122
Grade 8 links: R8, 29-36, 39-43 **Grade 9 links:** R8, 29-36, 70-80, 86-87

122 Sequences and algebraic expressions *continued*

Q2 Describe the following sequence using an expression.
Answer:

a. 2, 4, 6, 8, 10, …
$2n$

b. 3, 5, 7, 9, 11, …
$2n + 1$

c. –8; –20; –32; –44; –56
$-12n + 4$

d. –13; –17; –21; –25; –29
$-4n - 9$

e. –16; –22; –28; –34; –40
$-6n - 10$

f. 9; –2; –13; –24; –35
$-11n + 20$

g. 4; –4; –12; –20; –28
$-8n + 12$

h. –3; –12; –21; –30; –39
$-9n + 6$

i. –8; –18; –28; –38; –48
$-10n + 2$

j. 6; –1; –8; –15; –22
$-7n + 13$

127

Problem solving

Write three different rules for each of these:

a. 3; –3; –9; –15; –21

b. 5; 4; 3; 2; 1

c. –14; –22; –30; –38; –46

d. 19; 7; –5; –17; –29

e. –23; –30; –37; –44; –51

Answers:

3; –3; –9; –15; –21
- Add -6 to the value of the previous term
- $-6 \times$ the position of the term $+ 9 = -6n + 9$
- Value of first term $+ (-6(n - 1))$

5; 4; 3; 2; 1
- Add -1 to the value of the previous term
- $-1 \times$ position of the term $+ 6 = -n + 6$
- Value of first term $+ (-1(n - 1))$

–14; –22; –30; –38; –46
- Add -8 to the value of the previous term
- $-8 \times$ position of the term $- 6 = -8n - 6$
- Value of first term $+ (-8(n - 1))$

19; 7; –5; –17; –29
- Add -12 to the value of the previous term
- $-12 \times$ position of the term $+ 31 = -12n + 31$
- Value of first term $+ (-12(n - 1))$

–23; –30; –37; –44; –51
- Add -7 to the value of the previous term
- $-7 \times$ position of the term $- 16 = -7n - 16$
- Value of first term $+ (-7(n - 1))$

123 The algebraic equation

Topic: Algebraic equations **Content links:** 77-79, 124-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Solve and complete number sentences by inspection and trial and improvement
- Analyse and interpret number sentences to describe problem situation

Dictionary

Algebraic expression: a group of numbers made up of constants and variables, linked by operators, e.g. $x - 6 + 8$
Algebraic equation: a statement that two expressions (one of which may be a constant) have the same value, e.g. $x - 6 + 8 = 4$

130

Introduction

variable constants

$$x + -23 = -45$$

operation equal sign

Solving equations

Because an equation represents a balanced scale, it can also be manipulated like one.

Initial equation is $x - 2 = -5$

Add 2 to both sides
$x - 2 + 2 = -5 + 2$

Answer $x = -3$

Q1

Solve for x.

Example:
$x - 5 = -9$
$x - 5 + 5 = -9 + 5$
$x = -4$

Answers:

a. $x - 12 = -30$
$x - 12 + 12 = -30 + 12$
$x = -18$

b. $x - 8 = -14$
$x - 8 + 8 = -14 + 8$
$x = -7$

c. $x - 17 = -38$
$x - 17 + 17 = -38 + 17$
$x = -21$

d. $x - 20 = -55$
$x - 20 + 20 = -55 + 20$
$x = -30$

e. $x - 25 = -30$
$x - 25 + 25 = -30 + 25$
$x = -5$

f. $x - 18 = -26$
$x - 18 + 18 = -26 + 18$
$x = -8$

g. $x - 6 = -12$
$x - 6 + 6 = -12 + 6$
$x = -6$

h. $x - 34 = -41$
$x - 34 + 34 = -41 + 34$
$x = -7$

i. $x - 10 = -20$
$x - 10 + 10 = -20 + 10$
$x = -10$

j. $x - 25 = -33$
$x - 25 + 25 = -33 + 25$
$x = -8$

123 The algebraic equation *continued*

Topic: Algebraic equations **Content links:** 77-79, 124-125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Q2

Example: $x + 5 = -2$
$x + 5 - 5 = -2 - 5$
$x = -7$

Answer:

a. $x + 7 = -5$
$x + 7 - 7 = -5 - 7$
$x = -12$

b. $x + 3 = -1$
$x = -4$

c. $x + 15 = -12$
$x + 15 - 15 = -12 - 15$
$x = -27$

d. $x + 17 = -15$
$x + 17 - 17 = -15 - 17$
$x = -32$

e. $x + 23 = -20$
$x + 23 - 23 = -20 - 23$
$x = -43$

f. $x + 28 = -13$
$x + 28 - 28 = -13 - 28$
$x = -41$

g. $x + 10 = -2$
$x + 10 - 10 = -2 - 10$
$x = -12$

h. $x + 33 = -20$
$x + 33 - 33 = -20 - 33$
$x = -53$

i. $x + 5 = -10$
$x + 5 - 5 = -10 - 5$
$x = -15$

Q3

Solve for x.

Answer:

a. $x - 3 = -15$
$x - 3 + 3 = -15 + 3$
$x = -12$

b. $x - 7 = -12$
$x - 7 + 7 = -12 + 7$
$x = -5$

c. $x - 2 = -5$
$x - 2 + 2 = -5 + 2$
$x = -3$

d. $x - 5 = -15$
$x - 5 + 5 = -15 + 5$
$x = -10$

e. $x - 12 = -20$
$x - 12 + 12 = -20 + 12$
$x = -8$

f. $x - 10 = -25$
$x - 10 + 10 = -25 + 10$
$x = -15$

g. $x - 23 = -34$
$x - 23 + 23 = -34 + 23$
$x = -11$

h. $x - 2 = -7$
$x - 2 + 2 = -7 + 2$
$x = -5$

i. $x - 30 = -40$
$x - 30 + 30 = -40 + 30$
$x = -10$

131

Problem solving

Write an equation for the following and solve it:

Five times a certain number minus four equals ninety–five.

$5x - 4 = 95$
$5x - 4 + 4 = 95 + 4$
$5x = 99$
$x = \frac{99}{5}$ $x = 19\frac{4}{5}$

124 More on the algebraic equation

Topic: Algebraic equations **Content links:** 78-79, 123, 125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Solve and complete number sentences by inspection and trial and improvement
- Write a number sentence to describe a given situation

Dictionary

Algebraic expression: a group of numbers made up of constants and variables, linked by operators, e.g. $x - 6 + 8$
Algebraic equation: a statement that two expressions (one of which may be a constant) have the same value, e.g. $x - 6 + 8 = 4$

132

Introduction

$-2x = 30$

What does $2x$ mean?

$-2x$ means negative 2 multiplied by x

What is the inverse operation of multiplication?

division

We need to divide $-2x$ by -2 to solve for x.

$\frac{-2x}{-2} = \frac{30}{-2}$

$x = -15$

Remember you need to balance the scale. What you do on the one side of the equal sign, you must do on the other side as well.

Q1

Solve for x.
Answers:

a. $-5x = 60$
$\frac{-5x}{-5} = \frac{60}{-5}$
$x = -12$

b. $-2x = 24$
$\frac{-2x}{-2} = \frac{24}{-2}$
$x = -12$

c. $-12x = 48$
$\frac{-12x}{-12} = \frac{48}{-12}$
$x = -4$

d. $-7x = 21$
$\frac{-7x}{-7} = \frac{21}{-7}$
$x = -3$

e. $-15x = 60$
$\frac{-15x}{-15} = \frac{60}{-15}$
$x = -4$

f. $-9x = 54$
$\frac{-9x}{-9} = \frac{54}{-9}$
$x = -6$

g. $-5x = 10$
$\frac{-5x}{5} = \frac{10}{-5}$
$x = -2$

h. $-12x = 36$
$\frac{-12x}{-12} = \frac{36}{-12}$
$x = -3$

i. $-8x = 64$
$\frac{-8x}{-8} = \frac{64}{-8}$
$x = -8$

Q2

Solve for x.
Answers:

a. $-2x - 5 = 15$
$-2x - 5 + 5 = 15 + 5$
$\frac{-2x}{-2} = \frac{20}{-2}$
$x = -10$

b. $-9x - 4 = 32$
$-9x - 4 + 4 = 32 + 4$
$\frac{-9x}{-9} = \frac{36}{-2}$
$x = -4$

124 More on the algebraic equation *cont...*

Topic: Algebraic equations **Content links:** 78-79, 123, 125
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Q2

c. $-3x - 3 = 18$
$-3x - 3 + 3 = 18 + 3$
$\frac{-3x}{-3} = \frac{21}{-3}$
$x = -7$

d. $-3x - 2 = 22$
$-3x - 2 + 2 = 22 + 2$
$\frac{-3x}{-3} = \frac{21}{-3}$
$x = -8$

e. $-8x - 4 = 12$
$-8x - 4 + 4 = 12 + 4$
$\frac{-8x}{-8} = \frac{16}{-8}$
$x = -2$

f. $-20x - 5 = 95$
$-20x - 5 + 5 = 95 + 5$
$\frac{-20x}{-20} = \frac{100}{-20}$
$x = -5$

g. $-12x - 5 = 55$
$-12x - 5 + 5 = 55 + 5$
$\frac{-12x}{-12} = \frac{60}{-12}$
$x = -5$

h. $-7x - 3 = 25$
$-7x - 3 + 3 = 25 + 3$
$\frac{-7x}{-7} = \frac{28}{-7}$
$x = -4$

i. $-2x - 2 = 18$
$-2x - 2 + 2 = 18 + 2$
$\frac{-2x}{-2} = \frac{20}{-2}$
$x = -10$

133

Problem solving

Write an equation and solve it.

a. Negative two times y equals negative twelve.
b. Negative three times a equals negative ninety–nine.
c. Negative five times b equals negative sixty.
d. Negative four times d equals forty–four.
e. Negative three times x equals thirty.
f. Negative two times y equals sixty–four.
g. Negative nine times m equals one hundred and eight.
h. Negative six times a equals sixty–six.
i. Negative five times b equals fifteen.
j. Negative eight times c equals forty.

Answers:

a. $-2y = -12$
$\frac{-2y}{-2} = \frac{12}{-2}$
$y = 6$

b. $-3a = -99$
$\frac{-3a}{-3} = \frac{-99}{-3}$
$a = 33$

c. $-5b = -60$
$\frac{-5b}{-5} = \frac{-60}{-5}$
$b = 12$

d. $-4d = 44$
$\frac{-4d}{-4} = \frac{44}{-4}$
$d = -11$

e. $-3x = 30$
$\frac{-3x}{-3} = \frac{30}{-3}$
$x = -10$

f. $-2y = 64$
$\frac{-2y}{-2} = \frac{64}{-2}$
$y = -32$

g. $-9m = 108$
$m = \frac{108}{-9}$
$m = -12$

h. $-6a = 30$
$a = \frac{36}{-6}$
$x = -6$

i. $-5b = 15$
$b = \frac{15}{-5}$
$b = -3$

j. $-8c = 40$
$c = \frac{40}{-8}$
$c = -5$

125 More algebraic equations

Topic: Algebraic equations **Content links:** 78-79, 123-124
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Objectives

- Determine the numerical value of an expression by substitution

Dictionary

Algebraic expression: a group of numbers made up of constants and variables, linked by operators, e.g. $x - 6 + 8$

Algebraic equation: a statement that two expressions (one of which may be a constant) have the same value, e.g. $x - 6 + 8 = 4$

134

Introduction

If $y - y^2 + 1$; calculate y when $x = -3$

$y = (-3)^2 + 1$

$y = 9 + 1$

$y = 10$

Test

$y = x^2 + 1$

$10 = (-3)^2 + 1$

$10 = 9 + 1$

$10 = 10$

$(-3)^2$ is not the same as -3^2

Q1

Substitute.

Answers:

a. $y = x^2 + 3; x = 3$
$y = (3)^2 + 3$
$= 9 + 3$
$= 12$

Test:
$y = x^2 + 3$
$= (3)^2 + 3$
$12 = 12$

b. $y = b^2 + 3; b = 4$
$y = (4)^2 + 3$
$= 16 + 3$
$= 19$

Test:
$y = b^2 + 3$
$= (4)^2 + 3$
$19 = 19$

c. $y = b^2 + 2; x = 4$
$y = (4)^2 + 2$
$= 16 + 2$
$= 18$

Test:
$y = b^2 + 2$
$= 4^2 + 2$
$18 = 18$

d. $y = q^2 + 9; q = 5$
$y = (5)^2 + 9$
$= 25 + 9$
$= 34$

Test:
$y = q^2 + 9$
$= (5)^2 + 9$
$34 = 34$

e. $y = c^2 + 1; c = 7$
$y = (7)^2 + 1$
$= 49 + 1$
$= 50$

Test:
$y = c^2 + 1$
$= (7)^2 + 1$
$50 = 50$

f. $y = p^2 + 6; p = 2$
$y = (2)^2 + 6$
$= 4 + 6$
$= 10$

Test:
$y = p^2 + 6$
$= (2)^2 + 6$
$10 = 10$

g. $y = d^2 + 7; d = 9$
$y = (9)^2 + 7$
$= 81 + 7$
$= 88$

Test:
$y = d^2 + 7$
$= (9)^2 + 7$
$88 = 88$

h. $y = x^2 + 5; x = 3$
$y = (3)^2 + 5$
$= 9 + 5$
$= 14$

Test:
$y = x^2 + 5$
$= (3)^2 + 5$
$14 = 14$

i. $y = f^2 + 8; f = 10$
$y = (10)^2 + 8$
$= 100 + 8$
$= 108$

Test:
$y = f^2 + 8$
$= (10)^2 + 8$
$108 = 108$

j. $y = x^2 + 4; x = 12$
$y = (12)^2 + 4$
$= 144 + 4$
$= 148$

Test:
$y = x^2 + 4$
$= (12)^2 + 4$
$148 = 148$

125 More algebraic equations *continued*

Topic: Algebraic equations **Content links:** 78-79, 123-124
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

Q2 Substitute and calculate.

Answers:

Example: If $y = x^2 + \frac{2}{x}$; calculate y when $x = -4$

$y = (-4)^2 + \frac{2}{-4}$

$y = 16 + \frac{1}{-2}$

$y = 15\frac{1}{2}$

a. $y = x^2 + \frac{2}{x}$; $x = -4$

$y = (-4)^2 + \frac{2}{-4}$

$= 16 - \frac{1}{2}$

$= 15\frac{1}{2}$

b. $y = x^2 + \frac{10}{x}$; $x = 15$

$y = (15)^2 + \frac{10}{15}$

$= 225 + \frac{2}{3}$

$= 225\frac{2}{3}$

c. $y = x^2 + \frac{6}{x}$; $x = -6$

$y = (-6)^2 + \frac{6}{-6}$

$= 36 - 1$

$= 35$

d. $y = x^2 + \frac{5}{x}$; $x = -10$

$y = (-10)^2 + \frac{5}{-10}$

$= 100 - \frac{1}{2}$

$= 99\frac{1}{2}$

e. $y = x^2 + \frac{1}{x}$; $x = -2$

$y = (-2)^2 + \frac{1}{-2}$

$= 4 - \frac{1}{2}$

$= 3\frac{1}{2}$

f. $y = x^2 + \frac{4}{x}$; $x = -16$

$y = (-16)^2 + \frac{4}{-16}$

$= 256 - \frac{1}{4}$

$= 256\frac{3}{4}$

g. $y = x^2 + \frac{3}{x}$; $x = -9$

$y = (-9)^2 + \frac{3}{-9}$

$= 81 - \frac{1}{3}$

$= 80\frac{2}{3}$

h. $y = x^2 + \frac{2}{x}$; $x = -8$

$y = (-8)^2 + \frac{2}{-8}$

$= 64 - \frac{1}{4}$

$= 63\frac{3}{4}$

i. $y = x^2 + \frac{2}{x}$; $x = -2$

$y = (-2)^2 + \frac{2}{-2}$

$= 4 - 1$

$= 3$

125 More algebraic equations *continued*

Topic: Algebraic equations **Content links:** 78-79, 123-124
Grade 8 links: R8, 29-44 **Grade 9 links:** 37-38, 72-74, 81-85

135

Problem solving

a. What is the difference between the value of y in $y = x^2 + 2$, if you first replace y with 3 and then with –3?
b. y is equal to x squared plus four divided by x. If x is equal to eight. Substitute and calculate.
c. y is equal to p squared plus two divided by p. If p is equal to four. Substitute and calculate.
d. y is equal to b squared plus five divided by b. If b is equal to 10. Substitute and calculate.
e. y is equal to m squared plus three divided by m. If m is equal to four. Substitute and calculate.
f. y is equal to n squared plus nine divided by n. If n is equal to three. Substitute and calculate.

Answers:

a. $y = x^2 + 2 = 3^2 + 2 = 9 + 2 = 11$
$y = x^2 + 2 = (-3)^2 + 2 = 9 + 2 = 11$
The difference $= 0$

b. $y = x^2 + \frac{4}{x}; x = 8$
$y = 8^2 + \frac{4}{8} = 64 + \frac{1}{2} = 64\frac{1}{2}$

c. $y = p^2 + \frac{2}{p}; p = 4$
$y = 4^2 + \frac{2}{4} = 16 + \frac{1}{2} = 16\frac{1}{2}$

d. $y = b^2 + \frac{5}{b}; b = 10$
$y = 10^2 + \frac{5}{10} = 100 + \frac{1}{2} = 100\frac{1}{2}$

e. $y = m^2 + \frac{3}{m}; m = 4$
$y = 4^2 + \frac{3}{4} = 16 + \frac{3}{4} = 16\frac{3}{4}$

f. $y = m^2 + \frac{9}{n}\ ; n = 3$
$y = 3^2 + \frac{9}{3} = 9 + 3 = 12$

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

126 Data collection

Topic: Collect, organize and summarise data **Content links:** R16, 127-128
Grade 8 links: R16, 92 **Grade 9 links:** R16, 123

Objectives

- Design and use simple questionnaires to answer questions with:
  – yes/no type responses
  – multiple choice responses
- Pose questions relating to social, economic, and environmental issues in own environment
- Select appropriate sources for the collection of data

Dictionary

Research: any gathering of data, information, and facts for the advancement of knowledge
Data: factual information (as measurement or statistics) used as a basis of reasoning, discussion, or calculation
Questionnaire: a research instrument consisting of questions and other prompts for the purpose of gathering information from respondents
Hypothesis: This is an assumption or statement that might be true, which can then be tested. Example: the hypothesis that on average the boys in the class are taller than the girls can be tested.

136

Introduction
Explain that the first stages of the data cycle are to develop the questions and decide how you are going to collect the data which answers these questions.

Example:
Before collecting any research data you need to know what question or questions you are asking.

A good way of starting is to come up with a hypothesis. A hypothesis is a specific statement or prediction. The research will determine whether it is true or false.

Here are some examples of a hypothesis:

- Everybody in Grade 7 owns a cell phone.
- All Grade 7s understand square roots.
- All Grade 7s like junk food.

Q1 Where would you look to find data to give you answers to questions a. to j. at the top of page 137?
Answer: Learner's own answers

Q2 Is it always possible to collect data directly from the source?
Answer: No, it depends on the source you need to collect from.

Q3 In order to collect the data for Question 1, would you do primary or secondary research or both?
Answer: You would use both methods for the research, for example, primary research for question b. (such as a survey in the class), secondary research for question a. (using the internet or an encyclopedia)

126 Data collection *continued*

Topic: Collect, organize and summarise data **Content links:** R16, 127-128
Grade 8 links: R16, 92 **Grade 9 links:** R16, 123

Q4 Let's say you want to know the favourite colours of people at your school, but don't have the time to ask everyone, how will you go about finding the information?
Answer: You give out a questionnaire related to the information. You may also use a representative sample to reduce the number of people asked for information.

Q5 So how can we make sure that the result is not biased?
Answer: You need to ask different (groups of) people, not related to each other.

Q6 How would you design a questionnaire?
Answer: learner's own answer

139

Problem solving

How much water do learners in the school drink?

a. Write a hypothesis.
b. How will you find the data to prove or disprove the hypothesis? Will this be primary or secondary data?
c. Find any secondary research data on this topic.
d. Who should you ask?
e. What will the data tell you? (What questions will you ask about the data?)
f. Do you think the data can help you to answer the research question?
g. Think of some appropriate questions. Write them down.
h. Design a simple questionnaire that allows for both Yes/No type responses and multiple–choice responses.

Learners' own answers. Some possible answers are:

a. Learners in the school drink about a litre a day.
b. Using a simple questionnaire to gather primary data.
c. Check if there is any research on this on the internet. Ask at the tuck shop about the number of bottles of water sold a day.
e. How much water is drunk by the learners.
f. Yes.
g. Name? Grade? How much water do you estimate you drink at a time? How often do you drink water?
h. E.g. Name:
   Grade:
   How much water do you drink at time? Less than 250 ml/ Between 250 ml and 500 ml/ More than 500 ml
   How often do you drink water? Less than 3 times a day/ Between 4 and 6 times a day/ More than 6 times a day
   Do you drink other drinks in addition to water? Yes/No

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

127 Organise data

Topic: Collect, organize and summarise data **Content links:** R16, 126, 128
Grade 8 links: R16, 93 **Grade 9 links:** R16, 124

Objectives

- Organize (including grouping where appropriate) and record using:
  - – Stem and leaf displays
  - – Tallies
  - – Tables
- Group data into intervals

Dictionary

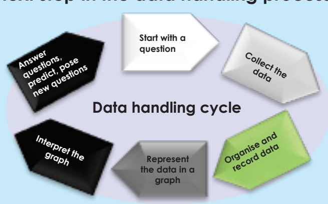
Tallies: marks used in recording a number of acts or objects, most often in series of five, consisting of four vertical lines cancelled diagonally or horizontally by a fifth line

Stem and leaf display: a plot where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits). For example "32" would be split into "3" (stem) and "2" (leaf). The "stem" values are listed down, and the "leaf" values are listed next to them.This way the "stem" groups the scores and each "leaf" indicates a score within that group.

140

Introduction

In the previous worksheet we looked at asking a question and collecting data. The next step in the data handling process is to organise the collected data.



We can organise the data using .

Tallies 𝍸 𝍷𝍷𝍷 = 8

Tallying is a way of counting data to make it easy to display in a table. A tally mark is used to keep track of counting.

Frequency tables

A frequency table has rows and columns. When the set of data values is spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we group the data into class intervals (or groups) to help us organise, analyse and interpret the data.

Stem–and–leaf tables

Stem–and–leaf tables (plots) are special tables where each data value is split into "leaf" (usually the last digit) and a "stem" (the other digits). The "stem" values are listed down, and the "leaf" values go right (or left) from the stem values. The "stem" is used to group the scores and each "leaf" indicates the individual scores within each group.

Q1 Learners present given test marks data in a frequency table.
Answers:

Score	Tally	Frequency
1 – 2		0
3 – 4	𝍷𝍷	2
5 – 6	𝍸 𝍷	6
7 – 8	𝍸 𝍷𝍷𝍷𝍷	9
9 – 10	𝍷𝍷𝍷	3

Q2 Learners present data in a frequency table.
Answer:

Score	Tally	Frequency
1 – 40	𝍷𝍷	2
41 – 80	𝍸	5
81 – 120	𝍸 𝍸 𝍷𝍷𝍷𝍷	14
121 – 160	𝍸	5
161 – 200	𝍷𝍷𝍷𝍷	4
201 –240	𝍷	1

127 Organise data *continued*

Topic: Collect, organize and summarise data **Content links:** R16, 126, 128
Grade 8 links: R16, 93 **Grade 9 links:** R16, 124

Q3

Compile a stem–and–leaf table of the examination data from the example on the previous page (page 141).
Answer:

Stem	Leaf
2	5
3	0
4	5679
5	1359
6	00345
7	379
8	5
9	0

Do at home

1. Compile a table showing tally and frequency.
Answer:

Favourite sport	Tally	Frequency
Soccer	𝍸	5
Rugby	𝍸	5
Basketball	|||	3
Netball	|||	3
Cricket	||	2
Tennis	||	2

143

Do at home

2. a. Set up a frequency table for this set of data values, using grouped data, grouped in six groups with intervals of two. Answer:

Temperature range	Tally	Frequency
16 – 18		0
19 – 21	||	2
22 – 24	𝍸 |	6
25 – 27	𝍸 𝍸	10
28 – 30	𝍸 𝍸 |	11
31 – 33	||	2

b. Compile a stem–and–leaf table of the recorded data.
Answer:

Stem	Leaf
1	9 9
2	2 2 3 4 4 4 6 6 6 7 7 7 7 7 7 7 8 8 8 8 9 9 9
3	0 0 0 0 1 1

128 Summarise data

Topic: Collect, organize and summarise data **Content links:** R16, 127-128
Grade 8 links: R16, 93 **Grade 9 links:** R16, 124

Objectives

- Summarize and distinguishing between ungrouped numerical data by determining the mode, mean, and median and identify the largest and smallest score in order to determine the spread of the data (range)

Dictionary

Mode: the value that appears the most.
Mean: the total of the numbers divided by how many numbers are.
Median: the middle value.
Range: the difference between the biggest and the smallest value.

Introduction

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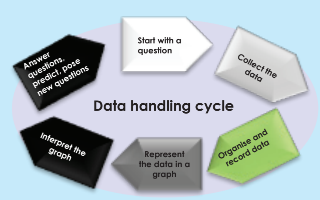

There are three different types of average that we generally use to understand data:

The **mean** is the total of the numbers divided by how many numbers there are.

The **median** is the middle value.

The **mode** is the value that appears the most often.

We also use the **range** of a set of numbers to see what the difference is between the biggest and the smallest numbers.

Q1 Use data set below to calculate the range, mean, median and mode: 3, 13, 7, 5, 21, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29
Answers:
a. The range $56 - 3 = 53$
b. The mean $331 \div 15 = 22{,}1$
c. The median is 23
d. The mode is 23

Q2 Sipho wrote seven maths tests and got scores of 68, 71, 71, 84, 53, 62 and 67. What were the median and mode of his scores?
Answer: Mode= 68 Median= 71

Q3 What is the mean of these numbers: 18, 12, 10, 10, 25?
Answer: Mean $= 75 \div 5 = 15$

Q4 The mean of three numbers is 8. Two of the numbers are 11 and 7. What is the third number?
Answer:
$8 \times 3 = 24$
$24 - 11 = 13$
$13 - 7 = 6$
The third number is 6.

Q5 The temperature in degrees Celsius over four days in July was 21, 21, 19 and 19. What was the mean temperature?
Answer:
$21 \times 2 = 42$
$19 \times 2 = 38$
$42 + 38 = 80$
$80 \div 4 = 20\ ^0C$

Q6 What is the mode of these numbers: 75, 78, 75, 71, 78, 25, 75, 29? Answer: 75

128 Summarise data *continued*

Topic: Collect, organize and summarise data **Content links:** R16, 127-128
Grade 8 links: R16, 93 **Grade 9 links:** R16, 124

Q7 Five children have heights of 138 cm, 135 cm, 140 cm, 139 cm and 141 cm. What is the range of their heights?
Answer: $141 - 135 = 6$ cm

Q8 What is the median of these numbers: 2,4; 2,8; 2,3; 2,9; 2,9?
Answer: 2,8

Q9 The cost of five cakes is R28, R19, R45, R45, R15. What is the median cost?
Answer: R28

Q10 What is the range of this group of numbers: 75, 39, 75, 71, 79, 55, 75, 59?
Answer: $79 - 39 = 40$

Q11 What is the median of these numbers: 10, 3, 6, 10, 4, 8?
Answer:
3; 4; 6; 8; 10; 10
$8 + 6 = 14$
$14 \div 2 = 7$

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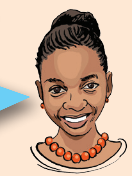
Do it on your own

These are the test results of 20 learners presented in a stem–and–leaf display.

Stem	Leaf
2	5
3	0
4	5 6 7 9
5	1 3 5 9
6	0 0 3 4 5
7	3 7 9
8	5
9	0

Note:
with an even amount of numbers the median will be the value that is halfway between the middle pair of numbers arranged from small to big.



1. Use this data to find the:
   a. Range Range $= 90 - 25 = 75$
   b. Mean $= 58,8$
   c. Median $= \dfrac{59 + 60}{2} = 59,5$
   d. Mode $= 60$

2. Draw a grouped frequency table showing a tally and frequency column.

Test mark	Tally	Frequency
20 - 39	II	2
40 - 59	~~IIII~~ III	8
60 - 79	~~IIII~~ III	8
80 - 99	II	2

129 Bar graphs

Topic: Represent data **Content links:** None
Grade 8 links: 96 **Grade 9 links:** 127

Objectives

- Critically read and interpret data represented in:
  – bar graphs
  – Words
- Draw a variety of graphs by hand/ technology to display and interpret grouped and ungrouped data on bar graphs

Dictionary

A bar graph/chart: a graphical display of data using bars of different heights that can be used to show the relative quantities or sizes of many things, such as what type of cars people have, how many customers a shop has on different days and so on

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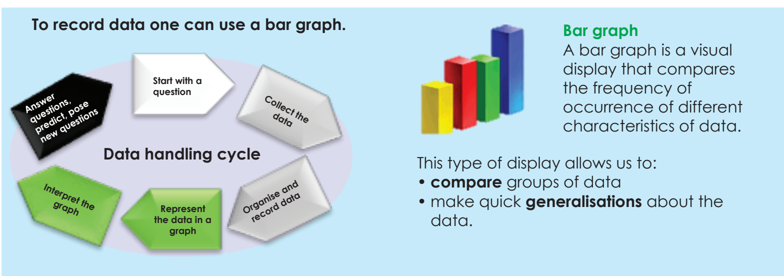
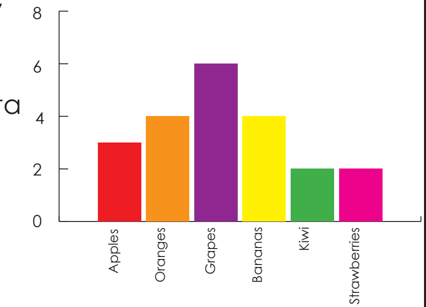
Introduction



Q1

Use the frequency table below to draw a bar graph. Use your bar graph and write three observations regarding the data represented in the graph.



Answer: Possible observations are:grapes are the most popular fruit; kiwis and strawberries are the least popular; several fruit are equally popular.

Q2

Critically read and interpret data represented in the bar graph.

a. How many learners are in the class?
   Answer: 22
b. Which method of transport is the most popular?
   Answer: car
c. Which method is the least popular?
   Answer: taxi
d. How many more learners use the bus than the taxi?
   Answer: $5 - 1 = 4$ learners
e. Why do you think more learners use the bus than the taxi?
   Answer: More space and it may be cheaper
f. Do you think most learners live far from or close to the school? Answer: learners own answer. Ask the learners to explain.
g. What percentage of the learners use public transport?
   Answer: $\frac{8}{22} = 0{,}36 \times 100 = 36\%$

129 Bar graphs *continued*

Topic: Represent data **Content links:** None
Grade 8 links: 96 **Grade 9 links:** 127

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Now try it by yourself

Use the data collected during a survey on learners' favourite subjects.

a. Compile a frequency table using tallies.

b. Draw a bar graph using your frequency table.

c. Interpret your graph and write at least five conclusions.

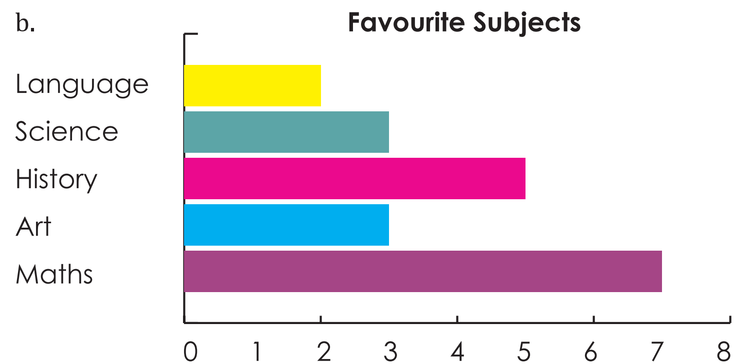
Name	Favourite subject
Peter	Maths
John	Arts
Mandla	History
Bongani	Sciences
Nandi	Sciences
David	Maths
Gugu	History
Susan	Arts
Sipho	Maths
Lebo	Maths
Ann	History
Ben	Maths
Zander	Sciences
Betty	History
Lauren	Arts
Alice	Maths
Veronica	Language
Jacob	Maths
Alicia	History
Thabo	Language

Remind the learners of the steps in drawing a bar graph (which are listed at the bottom of page 148 in the workbook).

a.

Favourite subject	Tally	Frequency
Maths	~~||||~~ ||	7
Art	|||	3
History	~~||||~~	5
Science	|||	3
Language	||	2

b.



c. Maths is the favourite subject.
Language is the least loved subject.
Equal number of learners love science and art.
10 learners favour Maths and Science.
10 learners favour Art, History and Language.

130 Double bar graphs

Topic: Represent data **Content links:** None
Grade 8 links: 96 **Grade 9 links:** 127

Objectives

- Critically read and interpret data represented in double bar graphs and draw a variety of graphs by hand/ technology to display and interpret data (grouped and ungrouped) in them

Dictionary

Double bar graph: A double bar graph shows two bars side by side that compare the quantities of two different but related things. Each bar on the graph represents a certain value, so you can easily see the difference between two related things .

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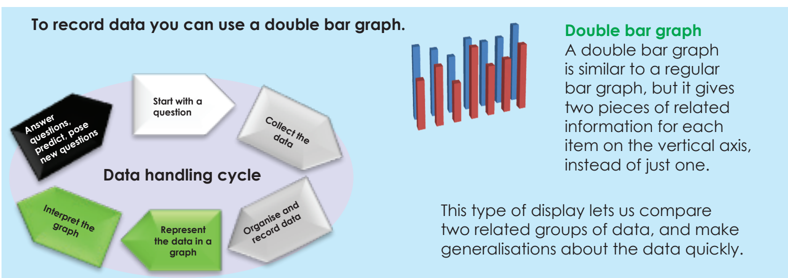
Introduction



Q1

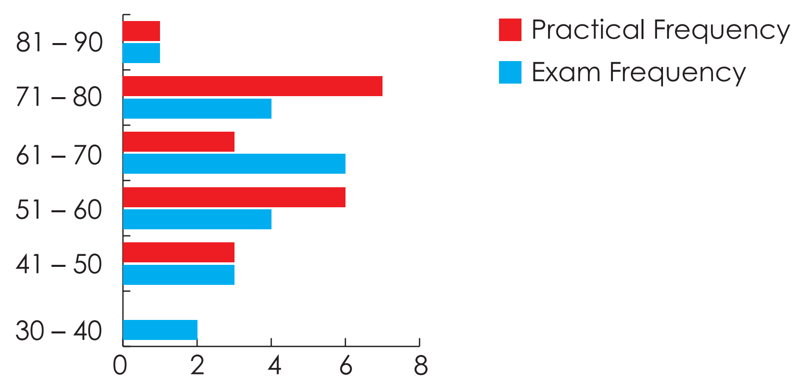
The results of your class exam and practical work is reflected in the table below.

a. Compile a frequency table using tallies.

Answer:

Range	Practical	Tally	Exam	Tally
30 – 40	2	||	0	
41 – 50	3	|||	3	|||
51 – 60	4	||||	6	~~||||~~ |
61 – 70	6	~~||||~~ |	3	|||
71 –80	4	||||	7	~~||||~~ ||
81 – 90	1	|	1	|

b. Draw a double bar graph comparing the learners' practical marks with their exam marks.



c. Interpret your graph and write down five conclusions.

- All learners got a mark in the practical work.
- No learner got less than 40% in the exam.
- 40% of learners got 70% and above in the exam
- 15% of learners failed the exam.
- Most learners did well in the class exam than in the practical.

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Do it by yourself

Use the data collected during the survey on learners' favourite subjects.

a. Compile a frequency table using tallies, splitting the different subjects between girls (green) and boys (blue).

b. Draw a double bar graph using your frequency table, comparing the preferences of boys with those of girls.

c. Interpret your graph and write down at least five conclusions.

d. How do your conclusions compare with the previous problem–solving activity where we used the same data?

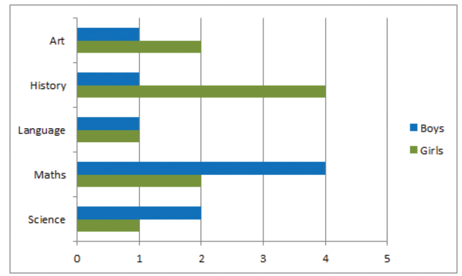
Name	Favourite subject	Name	Favourite subject
Peter	Maths	Ann	History
John	Arts	Ben	Maths
Mandla	History	Zander	Sciences
Bongani	Sciences	Betty	History
Nandi	Sciences	Lauren	Arts
David	Maths	Alice	Maths
Gugu	History	Veronica	Language
Susan	Arts	Jacob	Maths
Sipho	Maths	Alicia	History
Lebo	Maths	Thabo	Language

Answers:

a.

Subject	No. of boys	Tally	No. of girls	Tally
Maths	5	卌	2	ǀǀ
Art	1	ǀ	2	ǀǀ
History	1	ǀ	4	ǀǀǀǀ
Science	2	ǀǀ	1	ǀ
Language	1	ǀ	1	ǀ

b. **Boys' and girls' favourite subjects**



c. Learners's own interpretation and conclusions:
Here is an example answer. In this class:

- Boys and girls have different favourite subjects
- The most popular for boys was Maths
- The most popular for girls was History
- Overall Maths was the favourite subject of the class
- Language was the least favourite subject

131 Histograms

Topic: Represent data **Content links:** None
Grade 8 links: 97 **Grade 9 links:** 128

Objectives

- Critically read and interpret data represented in histograms and draw a histogram with given intervals y to display and interpret data (grouped and ungrouped)

Dictionary

Histogram: a graphical display of data using bars of different heights (or lengths) in different ranges

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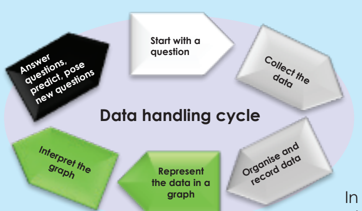
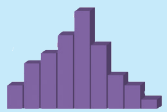
Introduction

To record data you can use a histogram.





Histogram
A **histogram** is a particular kind of **bar graph** that summarises data points falling in various ranges.

The main difference between a normal bar graph and a histogram is that a bar graph shows the frequency of each element in a set of data, while a histogram shows you the frequencies of a range of data.

In a histogram the bars must touch, because the data elements we are recording are **numbers** that are **grouped**, and form a **continuous range from left to right.**

Q1

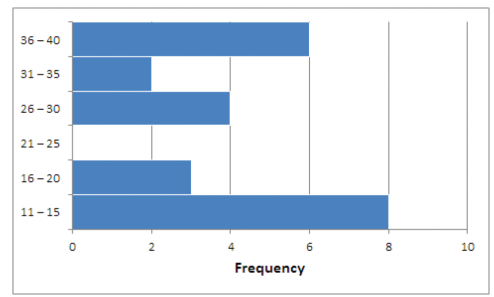
Use the following data to draw a histogram:
30, 32, 11, 14, 40, 37, 16, 26, 12, 33, 13, 19, 38, 12, 28, 15, 39, 11, 37, 17, 27, 14, 36

a. What is the mean, median, and the mode?
Answers:
Mean: $557 \div 23 = 24{,}5$ Median: 26 Mode: 37

b. Complete the frequency table. Make the bins 5 in size ranging from 11 to 40.

Range	Tally	Frequency
11 – 15	~~||||~~ |||	8
16 – 20	|||	3
21 – 25		0
26 – 30	||||	4
31 – 35	||	2
36 – 40	~~||||~~ |	6

c. Draw the histogram.



The histogram can also be drawn upright.

131 Histograms *continued...*

Topic: Represent data **Content links:** None
Grade 8 links: 97 **Grade 9 links:** 128

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Problem solving

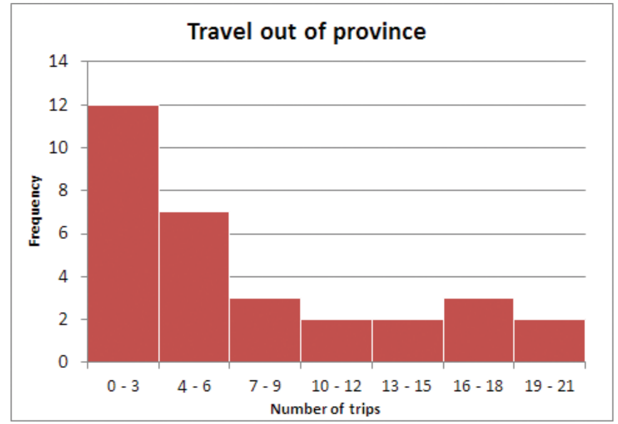
You surveyed the number of times your classmates have travelled to another province. The data you gathered is:

21, 0, 0, 7, 0, 1, 2, 12, 2, 3, 3, 4, 4, 6, 9,10, 25,18,11, 20, 3, 0, 0, 1, 5, 6, 7,15,18, 21, 25

Compile a frequency table and then draw a histogram using this data set. Make the bins 3 in size.

What can you tell us about the results of your survey by looking at the histogram?

Range	Tally	Frequency
0 – 3	~~IIII~~ ~~IIII~~ II	12
4 – 7	~~IIII~~ II	7
8 – 11	III	3
12 – 15	II	2
16 – 19	II	2
20 – 23	III	3
24 – 27	II	2



The histogram can also be drawn sideways.

The largest group (12) have never or seldom been out of the province. A majority (19) have left the province more than 4 times.

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

132 More about Histograms

Topic: Represent data **Content links:** None
Grade 8 links: 98 **Grade 9 links:** 129

Objectives

- Draw a histogram by hand/ technology to display and interpret data (grouped and ungrouped) with given intervals

Dictionary

Histogram: a graphical display of data using bars of different heights (or lengths) in different ranges

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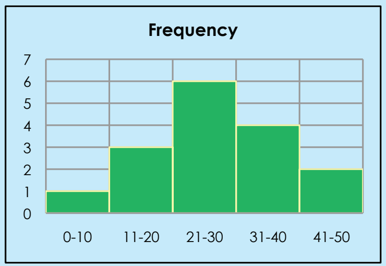
Introduction

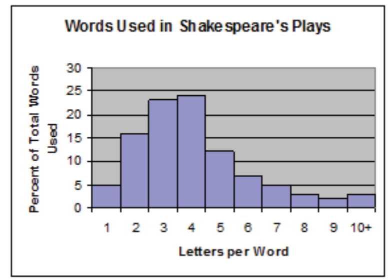
Part of the power of histograms is that they allow us to analyse extremely large sets of data by reducing them to a single graph that can show the main peaks in the data, as well as give a visual representation of the significance of the statistics represented by those peaks.

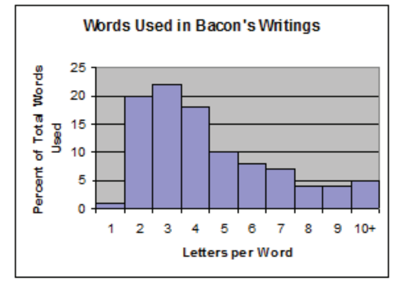


This graph represents data with a well–defined peak that is close to the median and the mean. While there are "outliers," they are of relatively low frequency. Thus it can be said that deviations from the mean in this data group are of low frequency.

Q1

These two histograms were made in an attempt to determine whether William Shakespeare's plays were actually written by Sir Francis Bacon. A researcher decided to count the lengths of the words in Shakespeare's and Bacon's writings. If the plays were written by Bacon the lengths of words used in these writings should be very similar.





a. What percentage of all Shakespeare's words are four letters long?
Answer: 24%

b. What percentage of all Bacon's words are four letters long?
Answer: 18%

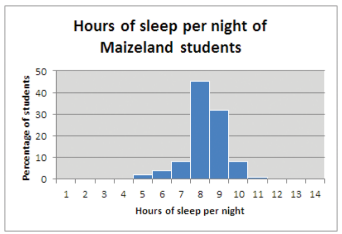
c. What percentage of all Shakespeare's words are more than five letters long?
Answer: 18%

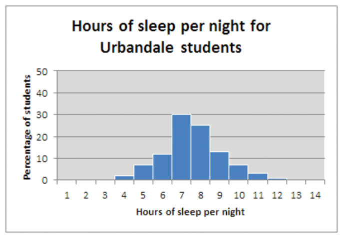
d. What percentage of all Bacon's words are more than five letters long? Answer: 28%

e. Based on these histograms, do you think that William Shakespeare was really just a pseudonym for Sir Francis Bacon? Explain. Answer: No. There is a different pattern, particularly in the number of one and two-letter words.

Q2

The two histograms show the sleeping habits of the teenagers at two different high schools. Maizeland High School is a small rural school with 100 learners and Urbandale High School is a large city school with 3 500 learners.





a. About what percentage of the students at Maizeland get at least eight hours of sleep per night? Answer: 86% [at least means 8 hours and over, so: 45% + 32% + 8% + 1% = 86%]

b. About what percentage of the students at Urbandale get at least eight hours of sleep per night? Answer: 49% [at least means 8 hours and over, so: 25% + 13% + 7% + 3% + 1% = 49%]

c. Which high school has more students who sleep between nine and ten hours per night? Answer: Maizeland [40% as against 20 %]

d. Which high school has a higher median sleep time? Answer: Maizeland [8 hrs as against 7 hrs]

e. Maizeland's percentage of students who sleep between eight and nine hours per night is 39% more than that of Urbandale. [Maizeland 77% and Urbandale 38%, so the difference is 39%. The Maizeland figure is 202,63% larger than the Urbandale one.]

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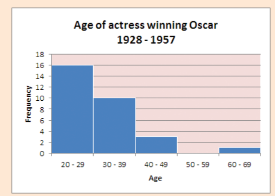
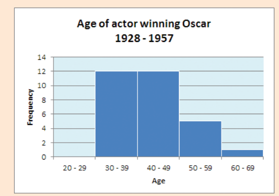
Problem solving

The table below shows the ages of the actresses and actors who won the Oscar for best actress or actor during the first 30 years of the Academy Awards. Use the data from the table to make two histograms (one for winning actresses' ages and one for winning actors' ages). Use bin widths of ten years (0–9; 10–19; 20–29 etc.)

Answers:





Write a short paragraph discussing what your two histograms reveal.

The age of winning actors is generally later than for actresses. Actresses tend to win in their 20s, actors in their 30s and 40s.

133 Pie charts

Topic: Represent data **Content links:** None
Grade 8 links: 99 **Grade 9 links:** 130

Objectives

- Critically read, analyse, and represent data on a pie chart and answer questions on them

Dictionary

Pie chart: A pie chart is a circular chart in which the circle is divided into sectors, the 'pie slices' that show the relative sizes of the data. They are useful to compare different parts of a whole amount.

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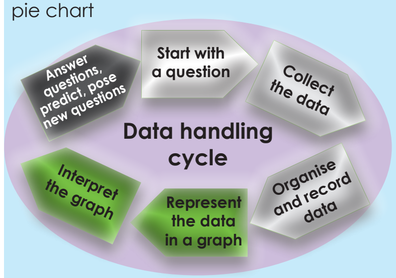
Introduction

To record data one can use a pie chart



Pie chart

A **pie chart** is a circular chart in which the circle is divided into sectors.

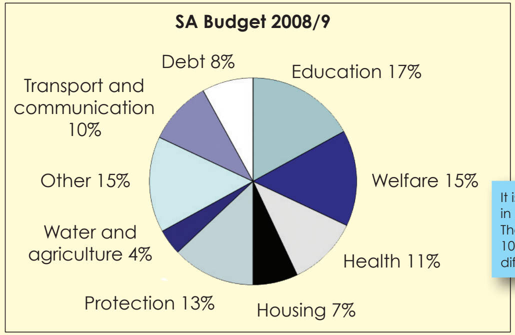
Each sector visually represents an item in a data set. The size of the sector is in proportion to the amount of the item as a percentage or fraction of the total data set.

Pie charts are useful to compare different parts of a whole amount. They are often used to present budgets and other financial information.

Q1

Answer the following questions.

Example: Look at this example of South Africa's National budget of 2008/9.

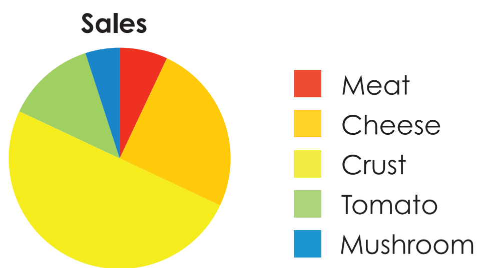


a. Will the sectors always be in percentage?
   Answer: No it can be in degrees.
b. Will it always add up to 100% ?
   Answer: Yes
c. What was the biggest expense in the South African budget?
   Answer: Education
d. What was the smallest expense in the South African budget?
   Answer: Water and agriculture.

133 Pie charts *continued*

Q2

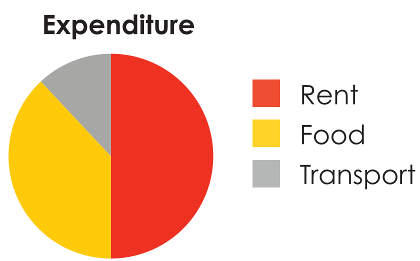
The ingredients of a mushroom pizza are the following:
Meat 75 g
Cheese 250 g
Crust 500 g
Tomato 125 g
Mushrooms 50 g
Draw a pie chart that shows the different ingredients.
Answers:



Q3

Represent Butho's expenditure on a pie chart.
Answers:

Expense	Value
Rent	300
Food	225
Transport	75



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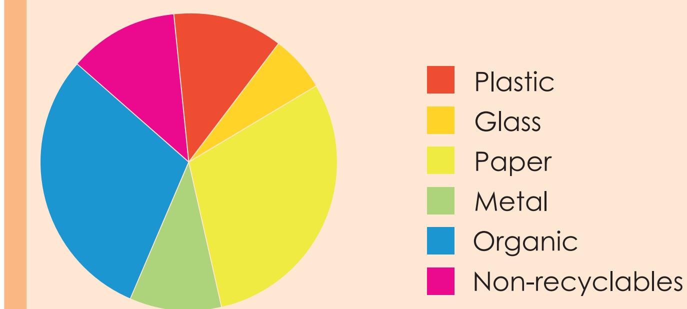
Waste!

Currently every person in South Africa generates about 2 kg of solid waste per day.

This table shows the different categories of solid waste and the amount in grams generated per day.

Draw a pie chart to display this information.

Waste category	Waste generated per person per day (grams)
Plastic	240
Glass	120
Paper	600
Metal	200
Organic	600
Non–recyclables	240



Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

134 Report data

Topic: Analyse, interpret and report data **Content links:** 134
Grade 8 links: 102 **Grade 9 links:** 134

Objectives

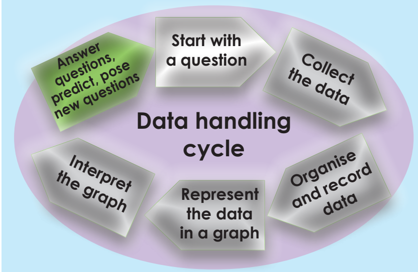
- Summarize data in short paragraphs that includes: aim, hypothesis, plan, analysis (including appropriate summary statistics for the data (mean, median, mode and range), interpretation and conclusions

Dictionary

Aim: the purpose or desired outcome that you hope to achieve by doing something
Hypothesis: This is a statement that might be true, which can then be tested. Example: the hypothesis that on average the boys in the class are taller than the girls can be tested.
Plan: a document, programme or diagram that shows how to proceed
Analysis: the careful examination, by looking at or breaking down the parts of something, to understand its structure or function
Interpretation: the process of making sense of numerical data that has been collected, analysed, and presented
Conclusion: a position or opinion or judgment reached after consideration
Appendices: Appendices are the sections at the end of a book that gives additional information on the topic explored in the contents of the text.
References: A reference is someone or something which is a source of information about a subject.

166

Introduction



To report on the data you have collected and analysed you need to remember the shape of a research report:

- Aim
- Hypothesis
- Plan
- Analysis of data
- Interpretation of data
- Conclusions
- Appendices
- References

Q1

Use the information from the favourite colour survey and write a report summarizing the data and draw conclusions.
Answers: These are possible answers.

a. Aim:

To determine the favourite colour among learners

This is the general aim of the project.

b. Hypothesis

Blue is the favourite colour among students?

A specific statement or prediction that you can show to be true or false.

134 Report data *continued*

Topic: Analyse, interpret and report data **Content links:** 134
Grade 8 links: 102 **Grade 9 links:** 134

c. Plan:

What data do you need? Answer: Number of learners favouring each colour.
Who will you get it from? Answer: From the students.
How will you collect it? Answer: Through a questionnaire
How will you record it? Answer: On a tally sheet with the data then transferred to a frequency table.
How will you make sure the data is reliable? Answer: Choose a random sample.
Why? Give reasons for the choices you made. Answer: Need to make sure the data is representative and not biased.

d. Analysis:

Number (and percentage) of students who like each colour listed is used to draw a bar graph and a pie graph. Determine the most popular and unpopular colours. Check if blue is indeed the most popular colour (as was stated in the hypothesis).

- This is where you do the calculations and draw charts.
- Graphs are good for representing data visually.
- Note mean and median (not appropriate in this study)
- Note the range as a measure of how spread out the group is (not appropriate in this study).

e. Conclusions:

Do your results agree with the hypothesis? Answer: Yes because most learners prefer blue as their favourite colour.
How confident are you? Answer: Confident because it was a random sample and the question asked was simple.
What went wrong? How did you deal with it?
Answer: Nothing went wrong.
What would you do differently if you did the research again?
Answer: I would increase the sample size.

f. Appendices

- Copy of questionnaire
- Instruction to data collector (including on how to get random sample)
- Tally sheet

It is good practice to include a copy of the questionnaire if there is one. The appendices may also include tables related to sample selection, instructions to interviewers, and so on.

g. References

No secondary data was used.

If you used any secondary data or research you must acknowledge your sources here.

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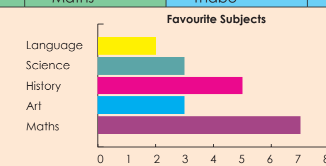
Now try this!

Use this favourite subject survey and write a report on the findings. Include a frequency table, graphs and conclusions.

Name	Favourite subject	Name	Favourite subject
Peter	Maths	Ann	History
John	Arts	Ben	Maths
Mandla	History	Zander	Sciences
Bongani	Sciences	Betty	History
Nandi	Sciences	Lauren	Arts
David	Maths	Alice	Maths
Gugu	History	Veronica	Language
Susan	Arts	Jacob	Maths
Sipho	Maths	Alicia	History
Lebo	Maths	Thabo	Language

Favourite subject	Tally	Frequency
Maths	卌 ||	7
Art	|||	3
History	卌	5
Science	|||	3
Language	||	2



Conclusion: Maths is the favourite subject among boys. History is the favourite subject among girls. Language is the list favoured subject.

135 Data handling cycle

Topic: Data handling **Content links:** R16
Grade 8 links: R16,103-104 **Grade 9 links:** R16, 135-137

Objectives

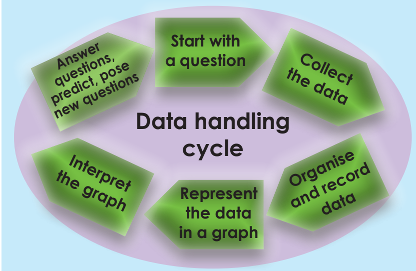
- Collect, organise, record, represent, interpret, and make conclusions on a set of data

Dictionary

Aim: the purpose or desired outcome that you hope to achieve by doing something
Hypothesis: This is a statement that might be true, which can then be tested. Example: the hypothesis that on average the boys in the class are taller than the girls can be tested.
Plan: a document, programme or diagram that shows how to proceed
Analysis: the careful examination, by looking at or breaking down the parts of something, to understand its structure or function
Interpretation: the process of making sense of numerical data that has been collected, analysed, and presented
Conclusion: a position or opinion or judgment reached after consideration
Appendices: Appendices are the sections at the end of a book that gives additional information on the topic explored in the contents of the text.
References: A reference is someone or something which is a source of information about a subject.

170

Introduction



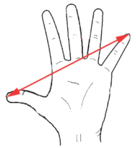
Data handling

Data handling is a process of collecting, organising, representing, analysing and interpreting data.

The visual representation of data is of major importance.

This assignment will go over two worksheets.

Is the hand span of Grade 7 girls smaller than that of boys in the same grade?



Q1 Choose your research team.
Answer: Names of the learner's research team

Q2 What is the aim of you research?
Answer: To compare hand spans between boys and girls. (Learners write what they hope to achieve by doing the research)

Q3 What is your hypothesis?
Answer: Boys have bigger hand spans than girls. (Learners state what results they anticipate from the research – an educated guess of research results.)

135 Data handling cycle *continued*

Topic: Data handling **Content links:** R16
Grade 8 links: R16,103-104 **Grade 9 links:** R16, 135-137

Q4

Questions that might help you to plan:

a. What data do you need?
   Answer: The length of the girls and boys' hand span.
b. Who will you get it from?
   Answer: From the grade seven boys and girls in my class.
c. How will you collect it?
   Answer: I will measure randomly girls and boys' hand span through measuring.
d. How will you record it? Answer: I will use a tally sheet, frequency table and double bar graph.
e. How will you make sure the data is reliable?
   Answer: By making a random selection of equal numbers of boys and girls and double checking the measurements on a percentage of these.
f. Why? Give reasons for the choices you made.
   Answer: To double check, and prove the authenticity of the results.

Your group will get an opportunity to present your aim, hypothesis and plan to the rest of the class.

Q5

Once all the research teams have presented their plans, you will get the opportunity to change your plans based on what they heard from the other teams.
Answer:
Our changes are:
Learners change their plans in consideration to the feedback they got from the other teams.

Q6

Your revised plan is:
Answer: Learners write down their revised plan for the team.

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Preparing

Now your plan is submitted you should start collecting and recording the data you need.

Answer: Learners measure, record and represent their data on tally sheet, frequency table and bar graph

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

Data handling cycle continued

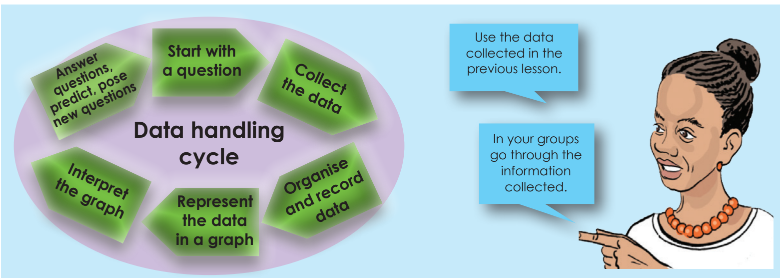
Topic: Data handling **Content links:** R16
Grade 8 links: R16,103-104 **Grade 9 links:** R16, 135-137

Objectives

- Collect, organise, record, represent, interpret, and make conclusions on a set of data

172

Introduction



Follow this sequence:

1. Learners collect data using questionnaires and other forms of data collection.
2. Now the learners organise their data accordingly.
3. Then they record and represent it in frequency tables and graphs.
4. Now they can analyse the data through the measures of central tendency(mean, median, mode, and range)
5. Lastly, they can plan and conclude on their findings.

Is the hand span of Grade 7 girls smaller than that of boys in the same grade?

Q1

Use the data you collected and recorded to:

a. Organise your data in a frequency table.
Answers: Learners use the tips given in the previous page. They measure hand spans of girls and boys and record the data on a tally sheet and then transfer this to a frequency table.

b. Calculate the mean, median and mode.
Mean: the sum of scores divided by the total number of terms
Median: the middle term after arranging the scores in an ascending order
Mode: the term that appears the most often in the set of numbers

c. Calculate the data range.
Answer: Data range = the biggest score – the smallest score

d. Draw a stem-and-leaf display. Answer: Stem and Leaf Display

For example, in a Stem and Leaf Display for the numbers 12 14 14 20 12 18 13 17 32 45 47 19 22, the leaf contains the last digit and the stem the remaining numbers (in this case all tens)

Stem	Leaf
1	2 4 4 2 8 7 9
2	0 2
3	2
4	5 7

136 Data handling cycle continued *cont...*

Topic: Data handling **Content links:** R16
Grade 8 links: R16,103-104 **Grade 9 links:** R16, 135-137

e. Represent your data in a graph. You may use more than one type of graph.

Answers:

Learner can use:

- Bar graphs
- Histograms
- Pie charts

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Interpreting your graphs and writing a report

Interpret you graphs and tables and write a report under the following headings:

1. Aim
2. Hypothesis
3. Plan
4. Analysis
5. Interpretation
6. Conclusions
7. Appendices
8. References

Answer: Learner's own answer based on the data the group collected.

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

137 Possible outcomes

Topic: Probability **Content links:** R15, 138-140
Grade 8 links: 135 **Grade 9 links:** 138-143

Objectives

- Perform simple experiments where the possible outcomes are equally likely and list the possible outcomes based on the conditions of the activity

Dictionary

Probability: Probability is the chance that something will happen - how likely it is that some event will happen.

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Introduction

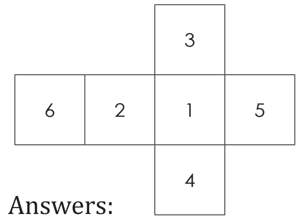
What are the possible outcomes when you throw this dice. What are the possible numbers the die can land on?

The possible outcomes are:
1, 2, 3, 4, 5 and 6.

a. What is your chance to land on ______? Write it as a fraction.

Answer: the probability on landing on any number on the dice is always $\frac{1}{6}$

Q1



Answers:

a.

i. 2 —— $\frac{1}{6}$

ii. 5 —— $\frac{1}{6}$

iii. 3 —— $\frac{1}{6}$

iv. 6 —— $\frac{1}{6}$

v. 4 —— $\frac{1}{6}$

vi. 1 —— $\frac{1}{6}$

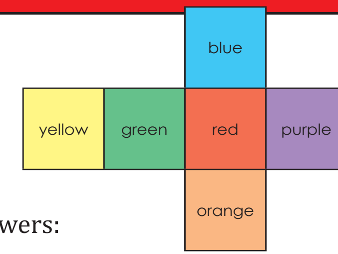
vii. 2 and 3 —— $\frac{2}{6}$ $\frac{1}{3}$

viii. 1,3 and 4 —— $\frac{3}{6}$ $\frac{1}{2}$

ix. 2, 4 and 7 —— $\frac{3}{6}$ $\frac{1}{2}$

x. 1, 2, 3, 4 and 5 —— $\frac{5}{6}$

query



Answers:

a.

i. Blue —— $\frac{1}{6}$

ii. Red —— $\frac{1}{6}$

iii. Purple —— $\frac{1}{6}$

iv. Orange —— $\frac{1}{6}$

v. Yellow —— $\frac{1}{6}$

vi. Green —— $\frac{1}{6}$

vii. Blue & Yellow —— $\frac{2}{6}$ $\frac{1}{3}$

viii. Green & Red —— $\frac{3}{6}$ $\frac{1}{2}$

ix. Purple, Red &Blue —— $\frac{3}{6}$ $\frac{1}{2}$

x. Orange, red, blue, green and yellow —— $\frac{5}{6}$

137 Possible outcomes *continued*

Topic: Probability **Content links:** R15, 138-140
Grade 8 links: 135 **Grade 9 links:** 138-143

Q2

If the possible outcomes are the following, how many faces will your dice have?

a. 1, 2, 3, 4, 5, 6, 7, 8
Answer: 8

b. Green, blue, yellow and red
Answer: 4

c. The probability is $\frac{1}{6}$ to land on 3.
Answer: 4

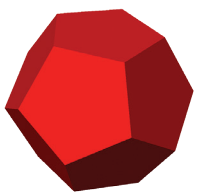
d. The probability is $\frac{1}{12}$ to land on 6.
Answer: 12

Q3

Make your own dice that will have _____ possible outcomes.
Answers:

a. 4

b. 12



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Problem solving

I have a circle that is divided into a number of sectors. Each sector has a number. What could the possible outcomes be for the following:

- circle divided into six equal parts Answer: 6
- circle divided into eight equal parts Answer: 8
- circle divided into two equal parts Answer: 2

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

138 Definition of probability

Topic: Probability **Content links:** R15, 137, 139-140
Grade 8 links: 136 **Grade 9 links:** 138-143

Objectives

- Perform simple experiments where the possible outcomes are equally likely and list the possible outcomes based on the conditions of the activity with the understanding of probability concept

Dictionary

Probability: Probability is the chance that something will happen - how likely it is that some event will happen.

Introduction

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This is a probability scale:

unlikely likely

impossible even chance certain

Read the following statements. Where would you place them on the probability scale?

a. The sun will rise tomorrow.
b. I don't have to study much for maths.
c. When I flip a coin it will land on tails.

When I flip a coin the probability is $\frac{1}{2}$, 0,5 or 50% to land on heads or tails. What does this mean?

We can use words, fractions and/or decimals to show the probability of something happening.

A fraction probability line is shown like this.

$\frac{1}{2}$

0 $\frac{1}{4}$ $\frac{3}{4}$ 1

Q1 Put these words in the correct place on top of the probability line: certain, impossible, likely, unlikely, even chance.

Answers:

impossible unlikely even chance likely certain

Q2 Put these numbers in the correct place on the probability line: 50%, 75%, 25%, 100% and 0%

0% 25% 50% 75% 100%

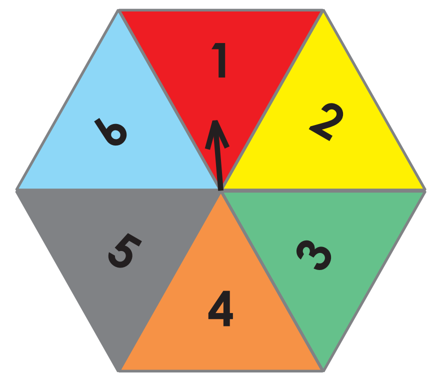
Q3 What is the probability of landing on each number on the spinner?

Answers:

$1 = \frac{1}{6}$ $4 = \frac{1}{6}$

$2 = \frac{1}{6}$ $5 = \frac{1}{6}$

$3 = \frac{1}{6}$ $6 = \frac{1}{6}$



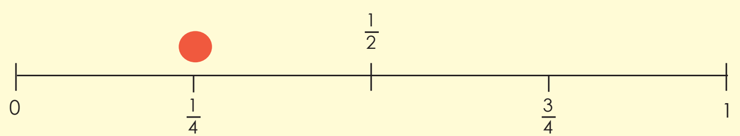
a. What number are you most likely to land on?
   Answer: All

b. What are the chances of landing on an even number?
   Answer: Half $\frac{1}{2}$

Q4

Show the following on the probability scale.

Example: The probability to land on 4 on a spinner with four equal sections



a. The probability of landing on heads when tossing a coin.
   Answer: $\frac{1}{2}$

b. The probability of a single ball randomly chosen from a bucket of four balls.
   Answer: $\frac{1}{4}$

c. The probability of three sweets chosen from a packet with four sweets.
   Answer: $\frac{3}{4}$

Write the above as decimals and then percentage.

Answers:

$$\frac{1}{2} = 0{,}5 = 50\%$$

$$\frac{1}{4} = 0{,}25 = 25\%$$

$$\frac{3}{4} = 0{,}75 = 75\%$$

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Problem solving

What is the probability of a person drawing one sweet from a packet of four sweets? Write it in words, fractions, decimals and percentages.

Answer:

$$\frac{1}{4} = 0{,}25 = 25\% = \text{a one in four chance}$$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

139 Relative frequency

Topic: Probability **Content links:** R15, 137-138, 140
Grade 8 links: 137 **Grade 9 links:** 138-143

Objectives

- Determine the relative frequency of actual outcomes for a series of trials

Dictionary

Relative frequency: This is how often something happens divided by all the outcomes. For example: if your team has won 9 games from a total of 12 games played: the frequency of winning is 9 and the number of possible outcomes is 12.
The Relative Frequency of winning is $\frac{9}{12} = 75\%$

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Introduction

Sometimes we cannot tell who will win a game, but we can look at previous results to estimate the probability.

Let us look at this example: the blue and red teams have played 50 matches.
The red team won 30 of the 50 matches.
The blue team won 10 of the 50 matches.
The two teams drew 10 matches.

- What is the probability of the red team winning the next match?
  The chance probability is $\frac{30}{50} = \frac{3}{5}$ or 60%
- What is the probability of the blue team winning the next match?
  The chance probability is $\frac{10}{50} = \frac{1}{5}$ or 20%

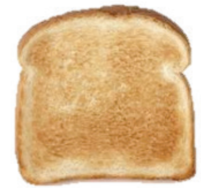
This is the formula for relative frequency.

$$\text{Relative frequency} = \frac{\textit{number of successful trials}}{\textit{total number of trials}}$$

Q1

Calculate the relative frequency.

a.

Dropped a piece of buttered toast 20 times	Landed 16 times with buttered side down.	Landed four times with buttered side up.
	$\frac{16}{20} = \frac{80}{100}$ or 80%	$\frac{4}{20} = \frac{20}{100}$ or 20%

Answers:

i. What is the relative frequency for the bread to land with its buttered side down?
   80% Likely

ii. What is the relative frequency for the bread to land with its buttered side up?
   20% Unlikely

Relative frequency *continued*

Topic: Probability **Content links:** R15, 137-138, 140
Grade 8 links: 137 **Grade 9 links:** 138-143

b.

Coin tossed 100 times	Landed 60 times on heads	Landed 40 times on tails
	Relative frequency $\frac{60}{100} = 60\%$	Relative frequency $\frac{40}{100} = 40\%$

c.

A six–sided dice was rolled 100 times.	The 1 occurred 21 times.	The 2 occurred 18 times.	The 3 occurred 17 times.	The 4 occurred 25 times.	The 5 occurred 10 times.	The 6 occurred 9 times.
	Relative frequency $\frac{21}{100}$ $= 21\%$	Relative frequency $\frac{18}{100}$ $= 18\%$	Relative frequency $\frac{17}{100}$ $= 17\%$	Relative frequency $\frac{25}{100}$ $= 25\%$	Relative frequency $\frac{10}{100}$ $= 10\%$	Relative frequency $\frac{9}{100}$ $= 9\%$

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Problem solving

What is the relative frequency when a drawing pin lands point up 23 times out of 100?

Answer:
Relative frequency $= \frac{23}{100} = 23\%$

Reflection questions

Did learners meet the objectives?

Common errors

Make notes of common errors made by the learners.

140 Probability and relative frequency

Topic: Probability **Content links:** R15, 137-139
Grade 8 links: 138 **Grade 9 links:** 138-143

Objectives

- Perform simple experiments where the possible outcomes are equally likely and determine the relative frequency of actual outcomes for a series of trails and calculate the difference between the relative frequency and the actual probability

Dictionary

Related frequency: How often something happens divided by all outcomes.

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Introduction

Let us look at the examples and compare.

Probability

A probability scale can have percentage intervals

What is the probability of a coin landing on heads?

$\frac{1}{2}$ or 50%

Relative frequency

You and your friend tossed a coin 100 times. It landed 58 times on heads and 42 times on tails. What is the relative frequency for each?

- Heads: $\frac{58}{100} = 58\%$
- Tails: $\frac{42}{100} = 42\%$

The difference between the probability and the relative frequency is 58% – 50% = 8%

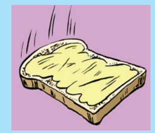
Will this always be the case? Answer: no

Q1

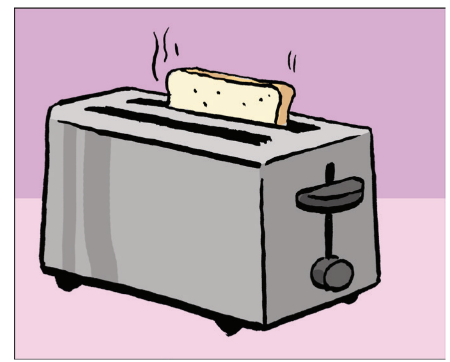
What is the difference between the probability and relative frequency? Give your answer in percentages.

Answers:

a.

Dropped a piece of buttered toast 50 times	Landed with buttered side down 29 times.	Relative frequency: $\frac{29}{50} = 58\%$
		Probability: $\frac{25}{50} = 50\%$

Difference: 8%



140 Probability and relative frequency *cont...*

Topic: Probability **Content links:** R15, 137-139
Grade 8 links: 138 **Grade 9 links:** 138-143

b.

Tossed a coin 100 times 	Landed tails up 52 times. 	Relative frequency: $\frac{52}{100} = 58\%$
		Probability: $\frac{50}{100} = 50\%$

Difference: 2%

c.

Rolled a 10–sided dice 100 times. 	Landed 12 times on 5.	Relative frequency: $\frac{12}{100} = 12\%$
		Probability: $\frac{10}{100} = 10\%$

Difference: 2%

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Problem solving

Give five everyday life examples of probability.

Answer:
1) predicting weather
2) predicting a lotto win
3) predicting the goals at a soccer match
4) predicting an election
5) predicting a census

Reflection questions
Did learners meet the objectives?

Common errors
Make notes of common errors made by the learners.

Topic: Revision **Content links:** 142-144
Grade 8 links: 140-141 **Grade 9 links:** 144

141 Revision: number, operations and relationships

In this worksheet we are going to revise number, operations and relationships.

This table will give you information on where to go and revise your work.

Tick yes or no.

Number operations and relationship concepts	Worksheet numbers	Do you need support?	
		Yes	No
Whole numbers	R1, R2, R3, R4, R5, 8		
Exponents	14, 15, 16, 17, 18, 19		
Integers	105, 106, 107, 108, 109, 110, 111, 112, 113		
Fractions	**Common fractions:** R7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43 **Decimal fractions:** R8, 40, 41, 42, 43, 44, 45, 46, 47		
Multiples and factors	R6, 5, 6		
Properties of numbers	R9, 1, 2, 3, 4		
Financial mathematics	9, 10, 11, 12, 13		
Ratio and rate	7, 8		

Q1

Learners to go through all the worksheets per topic above and make their own notes and summary.

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Whole numbers	Exponents
Multiples and factors	Properties of number
Financial mathematics	Ratio and rate

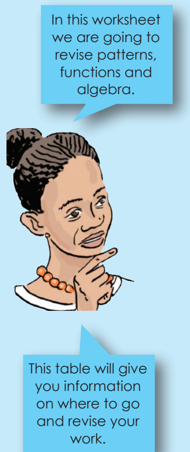
Go through the learner's notes and make a list of the topics they are struggling with. Revise these topics with them.

Provide the learners with extra paper if they need it.

What do you understand now?

After doing this worksheet, share with your teacher and/or friends what you understand now that you didn't understand before.

141 Revision: number, operations and relationships *continued*



Tick yes or no.

Patterns, functions and algebra	Worksheet numbers	Do you need support?	
		Yes	No
Functions and relationships	48, 49, 50, 51, 72, 73, 118, 119		
Numeric and geometric patterns	65, 66, 67, 68, 69, 70, 71, 114, 115, 116, 117		
Algebraic expressions	74, 75, 76, 120, 122		
Algebraic equations	74, 77, 78, 79, 123, 124, 125		
Graphs	80, 81, 82, 83, 84, 85		

Q1

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Learners to go through all the worksheets per topic above and make their own notes and summary.

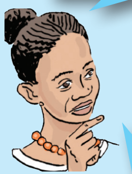
Functions and relationships	**Numeric and geometric patterns**

Algebraic expressions and equations	**Graphs**

What do you understand now?

After doing this worksheet, share with your teacher and/or friends what you understand now that you didn't understand before.

142 Revision: shape and space (geometry)



Tick yes or no.

Shape and space (geometry)	Worksheet numbers	Do you need support?	
		Yes	**No**
Construction of geometric figures	R10, 20, 23, 24, 25, 97, 119, 120		
Geometry of 2–D shapes	R10, 20, 21, 22, 24, 26, 27, 28, 29		
Transformation geometry	R11, 86, 87, 88, 89, 90, 91, 92, 93, 94		
Geometry of 3–D objects	R10, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104		

Q1

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Learners to go through all the worksheets per topic above and make their own notes and summary.

Functions and relationships	Numeric and geometric patterns

Q1

Algebraic expressions and equations	Graphs

What do you understand now?

After revising this lesson, share with your teacher and/or friends what you understand now that you didn't understand before.

143 Revision: measurement

Topic: Revision **Content links:** 141-142, 144
Grade 8 links: 143 **Grade 9 links:** 144

In this worksheet we are going to revise measurement

This table will give you information on where to go and revise your work.

Tick yes or no.

Measurement	Worksheet numbers	Do you need support?	
		Yes	No
Area and perimeter of 2–D shapes	R12, 52, 53, 54, 55		
Surface area and volume of 3–D objects	R14, 56, 57, 58, 59, 60, 61, 62, 63, 64		

Q1 Learners to go through all the worksheets per topic above and make their own notes and summary.

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Area and perimeter of 2–D shapes

Space to make some drawings.

Area and perimeter of 2–D shapes

Space to make some drawings.

What do you understand now?

After finishing this worksheet, share with your teacher and/or friends what you understand now that you didn't understand before.

Reflection questions

Did learners meet the objectives?

144 Revision: data handling

Topic: Revision **Content links:** 141-143
Grade 8 links: 144 **Grade 9 links:** 144

In this worksheet we are going to revise data handling.

This table will give you information on where to go and revise your work.

Tick yes or no.

Data handling	Worksheet numbers	Do you need support?	
		Yes	No
Collect, organize and summarise data	R16, 126, 127, 128		
Represent data	129, 130, 131, 132, 133		
Analyse, interpret and report data	129, 130, 131, 132, 133, 134, 135, 136		
Probability	R15, 137, 138, 139, 140		

Q1

Learners to go through all the worksheets per topic above and make their own notes and summary.

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Collect, organize and summarise data	Geometry of 3–D objects

Space to make some drawings or more notes.

Analyse, interpret and report data	Probability

Space to make some drawings or more notes.

2. Add some everyday life examples of data handling.

What do you understand now?

After revising this lesson, share with your teacher and/or friends what you understand now that you didn't understand before.

Teacher's notes

Teacher's notes

Teacher's notes

Teacher's notes

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